DEPARTMENT OF THE ARMY TECHNICAL MANUAL TM11-661

DEPARTMENT OF THE AIR FORCE TECHNICAL ORDER TO 16-1-218

ELECTRICAL FUNDAMENTALS

(DIRECT CURRENT)

DEPARTMENTS OF THE ARMY AND THE AIR FORCE

JUNE 1951

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This manual supersedes so much of TM 11-490, 10 August 1944, as pertains to direct-current electricity

ELECTRICAL FUNDAMENTALS (DIRECT CURRENT)

For sale by the Superintendent of Documents, U. S. Government Printing Office
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Price $2.00

United States Government Printing Office
Washington : 1951

DEPARTMENTS OF THE ARMY AND THE AIR FORCE
WASHINGTON 25, D. C., *22 June 1951*

TM 11–661 is published for the information and guidance of all concerned.
[AG 300.7 (27 Feb 51)]

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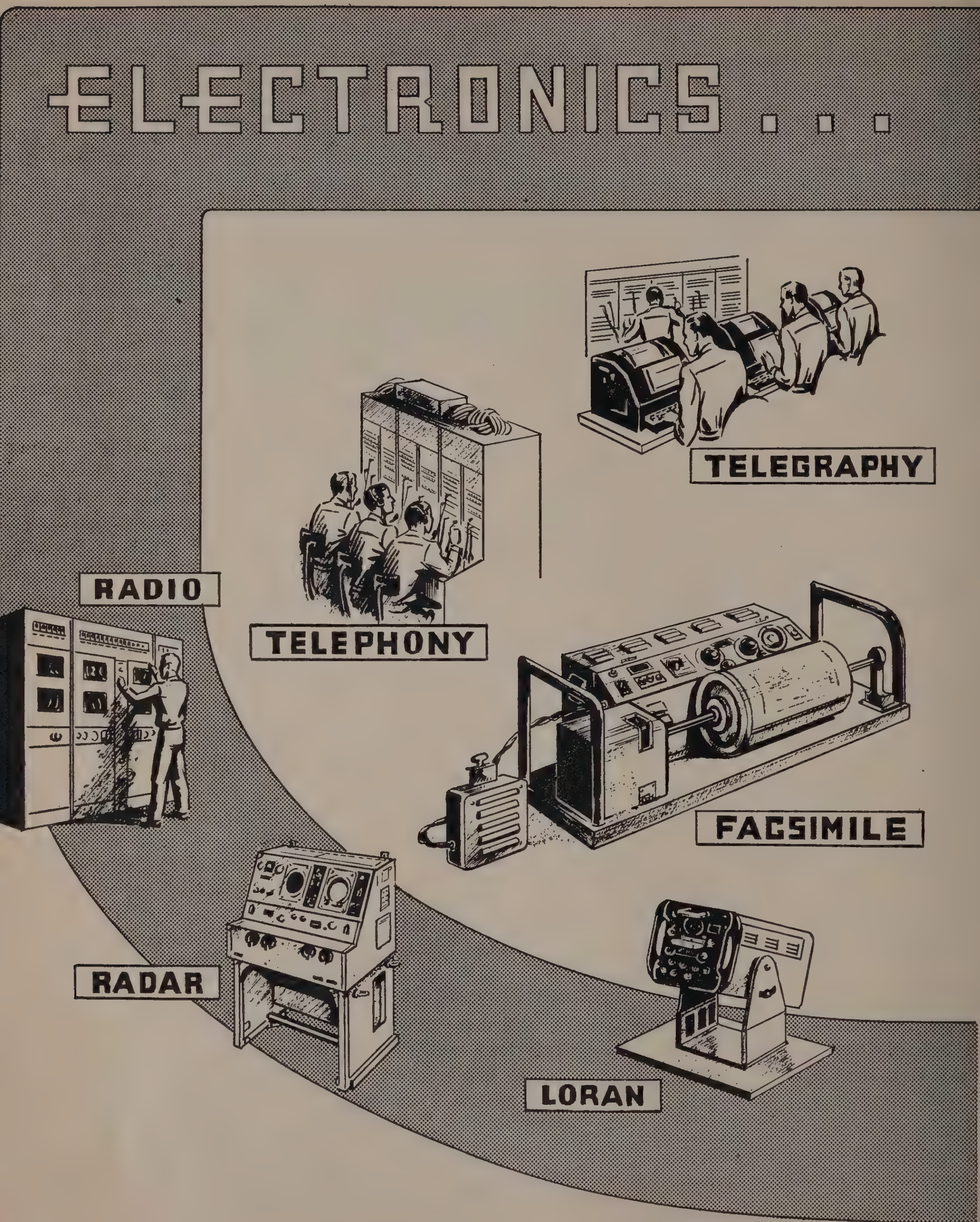
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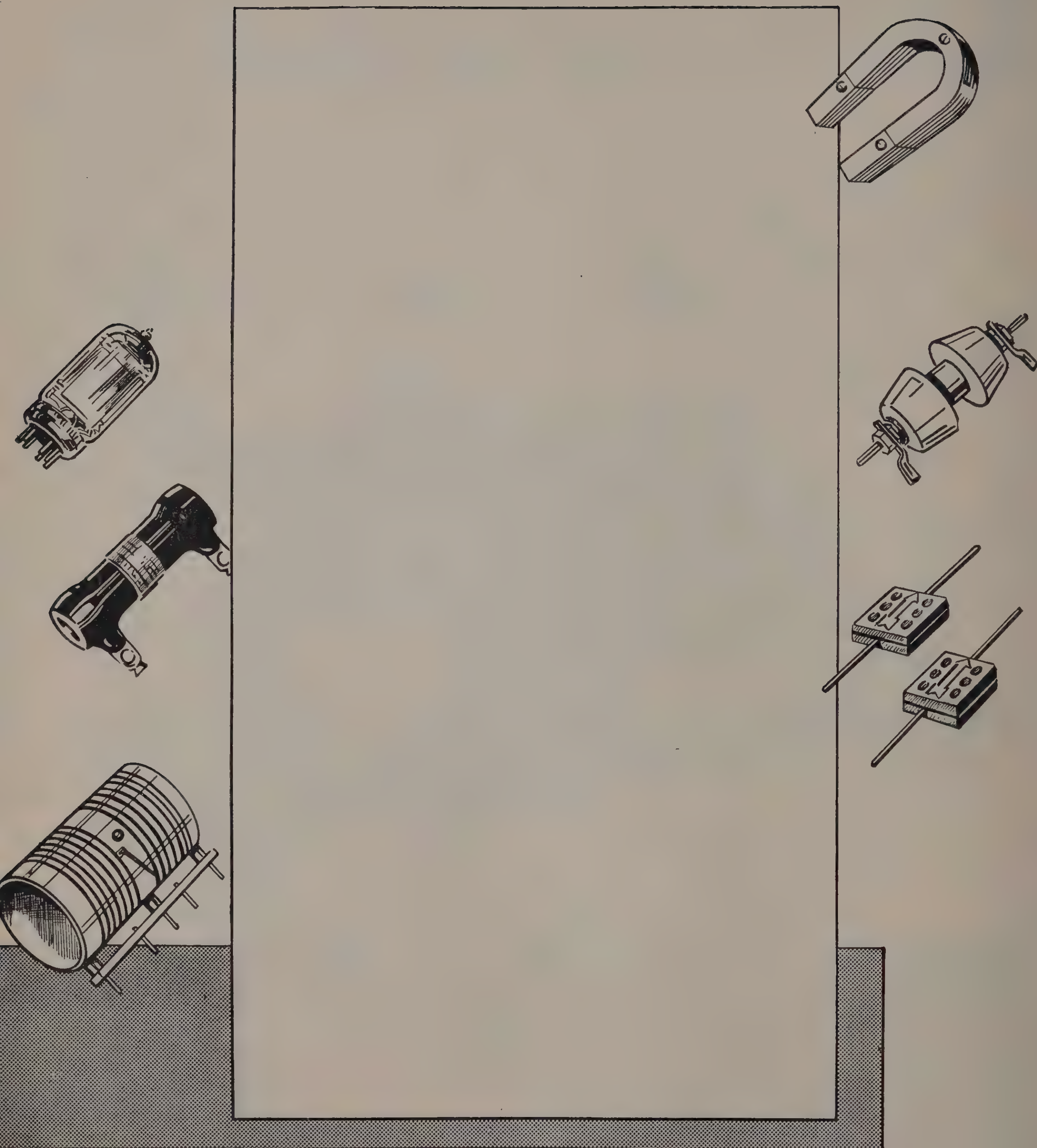

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CHAPTER 1

MAGNETISM

1. Invisible Forces

The two fundamental—and invisible—forces which are responsible for the wonders of electronics are *electric force* and *magnetic force*. These are the forces which make possible the operation of electric motors, generators, lights, doorbells, measuring instruments, and other electrical apparatus; they are the forces which comprise those invisible electromagnetic waves which travel through space at the speed of light to give us radio, television, radar, and the other electronic communication systems. This section of the manual presents many of the effects associated with *magnetic force*. It is magnetic force which attracts small bits of iron and steel to the end of the ordinary horseshoe magnet. It is magnetic force which swings a compass needle toward the North. Although most of the devices which utilize magnetic force can be classified as "modern," the more fundamental aspects of magnetic force—those dealing with magnetism—are as ancient as history itself.

2. Early History of Magnetism

The ancient Greeks knew that certain stones, found in the town of Magnesia in Asia Minor, had the property of attracting bits of iron. Quite appropriately, they called these stones *magnetite*. Legend tells of a shepherd, who thrust his iron staff into a hole containing magnetite and found to his dismay that he was unable to remove it. Another story dating back 2,300 years relates how Ptolemy Philadelphos had the entire dome of a temple at Alexandria made of magnetite in order that he might suspend a statue in mid-air. The experiment was a failure. Today, it is known that magnetite is an iron ore possessing magnetic qualities. In other words, magnetite is an unrefined product of nature, and is a *natural magnet*.

3. Magnetic Compass

The orientals learned that if a piece of magnetite were mounted or suspended in a horizontal plane and allowed to rotate, it would turn so that one particular end always pointed toward the North. The Europeans later learned of this discovery and used it as a magnetic compass to aid in navigation. Because of this property, the piece of magnetite came to be known as a *leading stone* or *lodestone*.

4. Magnetism and Electricity

Early students of science believed that magnetism was an independent phase of science, similar to electricity but quite a different thing. Today, it is generally recognized that the study of magnetism is a branch of the study of electricity.

5. Experiment: Attraction of Iron Filings to a Bar Magnet

For this experiment, iron filings are poured into a small pile on a table top (A of fig. 1). These filings are a powdered form of iron similar in appearance to black pepper.

a. A of Figure One. One end of a bar magnet (either end may be used) is brought close to the pile of filings. In this position, no filings can be observed on the magnet.

b. B of Figure One. Although the end of the magnet has not reached the pile, the filings closest to the magnet are pulled upward to the magnet. *An invisible force (a magnetic force) is at work.*

c. C of Figure One. The end of the magnet is thrust into the pile.

d. D of Figure One. When the magnet is lifted out of the pile, magnetic force causes the filings which were near the magnet to leave the pile and bunch in the space surrounding the end of the magnet.

e. Conclusion. From this experiment, it can be concluded that an invisible magnetic force exists in the space surrounding a magnet, a force capable of attracting iron. Actually, this magnetic force extends far out into space, but its strength decreases greatly with its distance from the magnet. For example, in this experiment the magnetic force is strong enough to attract those

Figure 1. Experiment showing attraction of iron filings to a bar magnet.

filings which are close to the magnet but is not strong enough to attract other filings a little farther away.

6. Artificial Magnets

Although pieces of magnetite are natural magnets when taken from the earth, they now have only historic value. Better magnets can be made from pieces of iron or steel by artificial means. Artificial magnets are made in a wide variety of sizes and shapes and are used extensively in electrical apparatus. The bar magnet (fig. 1), horseshoe magnet (fig. 18), and compass needle (fig. 12) are common types of artificial magnets.

a. Temporary Magnets. It has been found that pieces of iron and steel become magnetized when they are brought in contact with, or close to a strong magnet. This process is called inducing magnetism. For example, in D of figure 1, the iron filings which are shown bunched in the space surrounding the end of the bar magnet are magnetized, each of the filings becoming a tiny magnet by induction. However, these filings, which are made from *soft* iron, retain their magnetism only as long as they remain in the space about the bar magnet. When the filings and bar magnet are separated, the filings lose their magnetism very rapidly. For this reason, magnets (either small or large) made from *soft* iron are called temporary magnets. In fact, *any magnet which loses its magnetism rapidly is called a temporary magnet.*

b. Permanent Magnets. If the filings used in this experiment had been made from steel instead of soft iron, the same results would have been obtained, except that the steel filings would have retained their magnetism for a longer period of time after having been separated from the bar magnet. *Any magnet which retains its magnetism over a long period of time is called a permanent magnet.* Thus, in the case of the steel filings, hundreds of tiny permanent magnets would have been made. The amount of magnetism retained by a material after the magnetizing force has been removed is called *residual magnetism.*

c. Inducing Magnetism by Stroking. Another method of making an artificial magnet by induction is by stroking an iron or steel bar with a strong magnet (fig. 2). When this method is used, it is important that all strokes be made in the same direction.

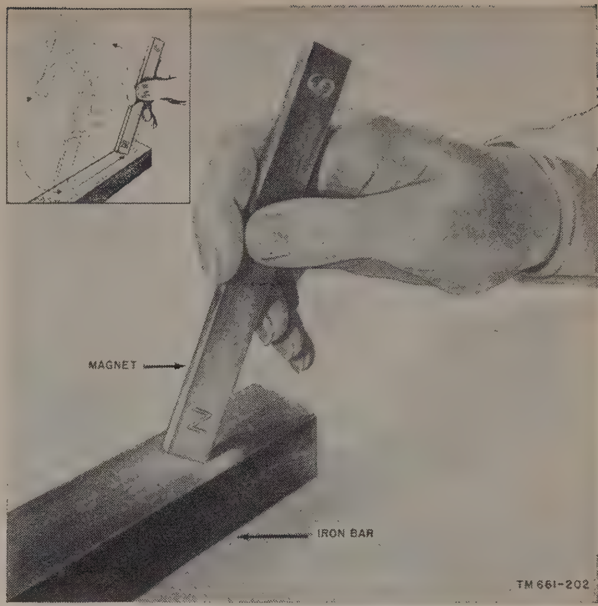


Figure 2. Stroking to make a magnet.

d. Electrical Method. The best method of making artificial magnets is by electrical means and is explained later in this manual.

7. Magnetic and Nonmagnetic Substances

a. It has been shown that the invisible magnetic force which exists in the space surrounding a magnet is capable of attracting pieces of iron and steel. For this reason, iron and steel are called *magnetic substances.*

b. There are other magnetic substances which are attracted by a magnet but not so strongly as iron and steel. The more common of these substances are cobalt, nickel, and manganese. However, it is interesting to note that some of the *best* permanent magnets are made from alloys of these substances. The ability of a material to retain its magnetism is called *retentivity.* Since steel retains its magnetism longer than soft iron, steel has greater retentivity than soft iron. It follows that a material with good retentivity will have a large amount of residual magnetism, and will make a good permanent magnet.

c. Most other substances are not attracted by a magnet and are said to be nonmagnetic. Examples of these *nonmagnetic substances* are air, wood, paper, glass, copper, aluminum, lead, tin, and silver.

d. Magnetic force *acts through any nonmagnetic substance,* as can be demonstrated by moving a

permanent magnet beneath a piece of paper or glass on which is sprinkled iron filings. As the magnet moves, movement of the filings can be observed.

e. Magnetic force is mutual to both a magnet and a magnetic substance. For example, a magnet can be attracted to a firmly held piece of iron or steel just as strongly as the iron or steel is attracted to the magnet.

8. Magnetic Poles

a. When iron filings are sprinkled over the entire area of a magnet, it can be noticed that those filings which fall near the ends of the magnet will be attracted to form bunches or tufts; scarcely any filings which fall near the center of the magnet will be so attracted. Thus, it can be seen that the bar magnet has two distinct regions, or *poles,* each pole indicating the area or region where the magnetic force is greatest. The same is true of a horseshoe magnet which is, in effect, a bar magnet bent so that the poles are closer together.

b. When a bar magnet is suspended so that it is free to swing in a horizontal plane, as shown in figure 3, it is found that the magnet will swing around and then come to rest with one end of the magnet pointing nearly due north. Also, it can be found that regardless of the number of times this experiment is repeated, the same end of the magnet always comes to rest pointing toward north. Thus, it can be seen that there is a difference in the direction of the magnetic forces which act at the two poles of the magnet. Long ago, when this fact was first established, it was decided arbitrarily to call the North-seeking pole of the magnet the north pole. Likewise, the South-seeking pole of the magnet was called the south pole. These designations for the poles of a magnet are still used. In fact, permanent magnets frequently are marked "N" at the north pole and "S" at the south pole.

c. When a bar magnet (A of fig. 4) is broken into two parts, and the pieces brought into contact with iron filings, it will be found that the filings

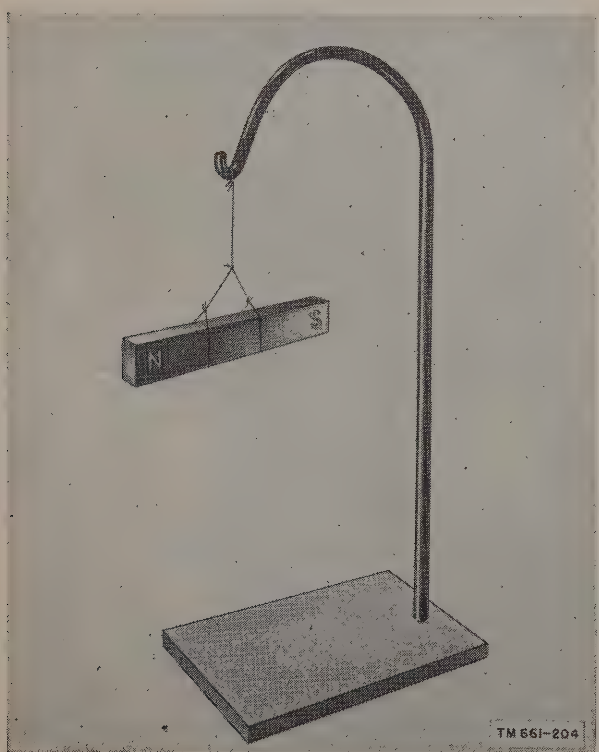


Figure 3. Suspended bar magnet.

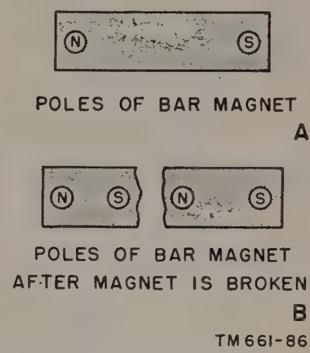


Figure 4. Bar magnet broken into two pieces.

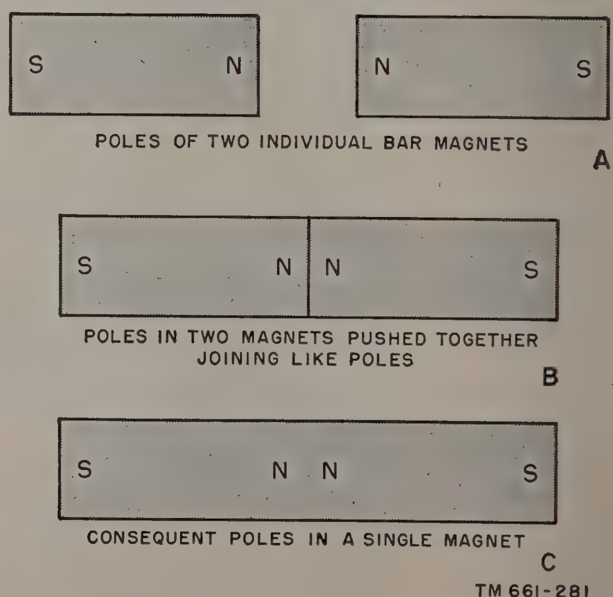


Figure 5. Bar magnet with two north and two south poles.

will bunch at both ends on each of the pieces, thus proving that each piece has two poles (B of fig. 4). It can also be found that one of the poles is a north pole and the other a south pole. The piece which contains the north end of the original magnet will have a south pole at the break; and the piece containing the south pole of the original magnet will have a north pole at the break. Regardless of the number of times a magnet is broken, each piece will have its own north and south pole.

d. A magnet may have more than one north and more than one south pole, but it must have at least one of each. A of figure 5 shows two bar magnets placed with their north poles facing each other. If these magnets are joined (B of fig. 5), they will form a magnet with four poles, two south poles at the ends, and two north poles in the middle. If a single bar of steel (C of fig. 5) can be magnetized (usually by electricity) to have poles in this position, they are called consequent poles.

9. Earth as a Magnet

The fact that a suspended magnet, anywhere upon the surface of the earth, always points toward the North indicates that the earth itself is a huge natural magnet. This is illustrated in figure 6. Note that the poles of the magnet are far below the surface of the earth, and that the magnet is somewhat inclined from the north and south geographic poles. Since the pole of any compass points to the North, that magnetic pole of the earth which is near the north geographic pole must be of south magnetic polarity, and the magnetic pole near the south geographic pole must have north magnetic polarity, as indicated in figure 6. The north geographic pole is referred to as *true North*, and the south geographic pole as *true South*. The deviation from the true North, which at some places is very large, is called the *magnetic declination* of the station. Remember that *magnetic North* is merely the direction in which the north end of a compass needle points.

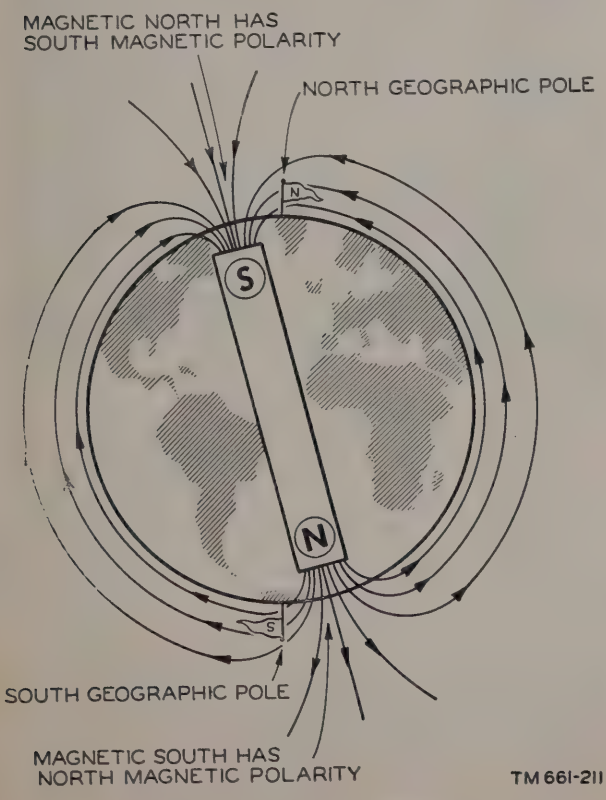


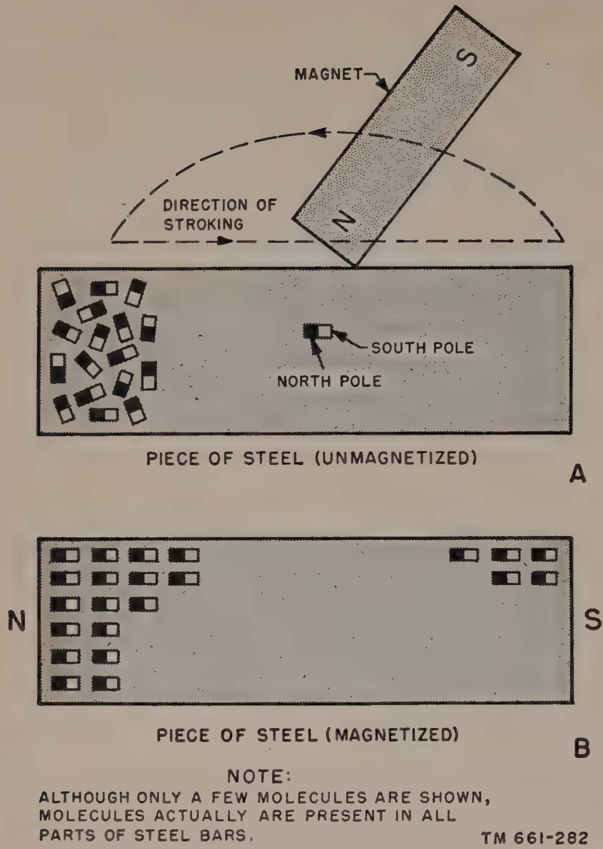
Figure 6. Earth as a magnet.

10. Molecular Theory of Magnetism

A common and one of the simple theories of magnetism is that a piece of iron or steel consists of millions of tiny elementary magnets. These tiny magnets, which are so small that they cannot be seen with a microscope, may consist of atoms or molecules so alined as to form iron or steel crystals. Before a piece of iron or steel has been magnetized, these tiny magnets may be thought of as being jumbled at random with no definite order (A of fig. 7). If the north pole of an inducing magnet is drawn over the bar, it attracts the south poles of the tiny magnets and turns them so that they will aline themselves in a given direction (B of fig. 7). This definite alinement of molecular magnets will give the bar a definite north pole at one end and a south pole at the other end.

11. Attraction and Repulsion of Magnetic Poles

a. If a bar magnet is suspended so that it is free to swing about in a horizontal plane, the magnet normally comes to rest with its north pole pointing toward North (par. 8*b*). However, if the north pole of a second magnet is brought towards the north pole of the suspended magnet (fig. 8), the latter magnet will be pushed away. The same results will be obtained if the south pole of the second magnet is brought towards the south pole of the suspended magnet. Thus, it can be seen that the magnetic forces existing in the



A. Bar showing jumbled condition of tiny magnets before magnetization.
B. Bar showing orderly alinement after magnetization.

Figure 7.

Figure 8. Repulsion of like magnetic poles.

Figure 9. Experiment showing the repulsion of like magnetic poles.

space surrounding *like* magnetic poles cause mutual *repulsion* of the poles.

b. Another demonstration of the repulsion of like magnetic poles is shown by the following experiment: Two strong permanent magnets are placed on a table top in a parallel position (fig. 9), so that the north pole of one magnet is directly opposite the north pole of the other magnet. The south poles of the magnets are also opposite each other. As one bar magnet is pushed steadily toward the second magnet, invisible magnetic

force pushes, or repels, the second magnet. Thus, both magnets move across the table top, as shown in four steps in A, B, C, and D of figure 9, one being pushed manually and the other by magnetic forces.

c. If a bar magnet is again suspended so that it is free to swing about in a horizontal plane, it will be observed that when the north pole of a second magnet is brought towards the south pole of the suspended magnet, the two poles will be pulled together (fig. 10) and will cling to each other until they are separated by manual force. If the south pole of the second magnet is brought close to the north pole of the suspended magnet, it will be observed that these two poles will also be pulled together, and manual force is required to separate the poles. Thus, it can be seen that the magnetic forces about the unlike poles of two magnets will cause the two magnets to be attracted.

d. The above facts are used as a basis for the fundamental law of magnetic forces which is: LIKE MAGNETIC POLES REPEL EACH OTHER; UNLIKE MAGNETIC POLES ATTRACT EACH OTHER. Or, THE FORCE BETWEEN TWO LIKE POLES IS ONE OF MUTUAL REPULSION; THE FORCE BETWEEN TWO UNLIKE POLES IS ONE OF MUTUAL ATTRACTION.

Figure 10. Attraction of unlike poles.

12. Factors Affecting Magnetic Force

a. Strength of Magnets. The amount of magnetic force existing in the space surrounding a magnet can be estimated roughly by measuring its lifting power. However, there are various conditions, other than the magnet itself, which will cause the lifting power to vary. Some of these conditions are: the kind of magnetic material to be lifted, the shape of the material to be lifted, the manner in which the material is applied to the magnet, and the shape of the magnet. A more accurate method for measuring the strength of a magnet, and the method most commonly used, is by measuring the force of attraction or repulsion that the magnet has on another magnet of known strength. That is, *the effects that magnets have upon one another is a measure of their strength.*

b. Distance Factor. The force of attraction or repulsion between two magnetic poles varies with the distance between them. This fact is brought out in the experiment explained in paragraph 11. When the poles are separated by a considerable distance, no visible effects are apparent. It is only after the like poles are brought close to each other that the suspended pole is repelled. Likewise, it is only after the two unlike poles are brought close together that the suspended pole is attracted. Thus, it can be seen that the magnetic force of attraction or repulsion between two magnetic poles increases very rapidly as the distance between the two poles is decreased. With suitable measuring equipment, it can be shown that this force varies *inversely* with the distance squared. This expression simply means that if the distance between the two poles is halved, the force becomes four times as great; if the distance is reduced to one-third, the force becomes nine times as great; if the distance is reduced to one-fourth, the force becomes sixteen times as great; if the distance is reduced to one-fifth, the force becomes twenty-five times as great, and so on. This expression holds true whenever the two magnetic poles are separated in a vacuum.

c. Strength Factor. The force of attraction and repulsion between two magnets also varies with the amount of force that the individual poles of the magnets are capable of exerting. In other words, the force of attraction and repulsion varies with the strength of the poles. The strength of a pole, in turn, varies with its size, the

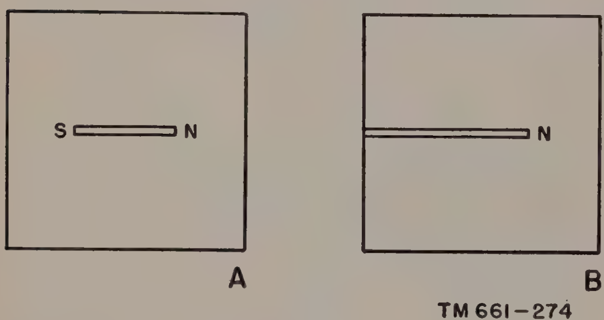
material from which it is made, and its degree of magnetization.

d. Point Poles. Before giving a mathematical equation expressing the force between two point magnetic poles of charges (strength) m_1 and m_2, it is necessary to explain what is meant by a *point pole.* A point pole is any amount of magnetic charge concentrated at a point. This is a purely imaginary concept and is never realized in nature for two reasons:

(1) Poles always appear in pairs. This means that for any north pole there must be a mate south pole, as has been previously explained.

(2) Magnetic charges are always found distributed over finite areas—not concentrated at points.

Note. The idea of a point pole can be gotten in the following way. Imagine a bar magnet being placed in a certain position (A of fig. 11). Now suppose it is stretched to the left indefinitely while holding the north pole fixed (B of fig. 11). When the bar has been sufficiently stretched, the south pole is too far away to exert any effect on magnetic substances in the vicinity of the north pole. Consequently, the magnetic force in the region near the north pole is, for all practical purposes, due to the north pole alone. Such a north pole is often referred to as an isolated north pole.



A. Bar magnet.
B. North pole of elongated bar magnet.

Figure 11.

e. Mathematical Formula. By experiment, scientists have discovered that the mutual force between two point poles of magnitude m_1 and m_2 can be expressed by a mathematical formula in the following way:

$$F = \frac{m_1 m_2}{\mu_a d^2}$$

F is the force between the two poles, m_1 and m_2 are the magnitudes of the poles, d is the distance between the poles, and μ_a is a constant (called *permeability*) that depends on the medium in which the poles are located; μ_a is equal to 1 in a vacuum and approximately 1 in air (par. 16). The force can be one of attraction or repulsion, depending on whether m_1 and m_2 have opposite signs or the same signs. This equation is used to define what is meant by a *unit* magnetic pole. In the cgs (centimeter-gram-second) system, the constant μ_a is arbitrarily taken to be *one* in a vacuum. A *unit* magnetic pole in this system is now defined as one which when separated by a distance of one centimeter in vacuum from another unit magnetic pole experiences a force of one dyne. (A dyne is a unit of force which acting on a mass of 1 gram (.035 ounce) for one second will increase its velocity by one centimeter per second.) From the formula it is seen that when m_1, m_2, μ_a, and d are each equal to 1, then F is 1. This is the reason for using a vacuum for the definition. In any medium other than a vacuum, the permeability, μ_a, has a value greater than unity or 1. In air, $\mu_a = 1.000004$ which, for practical purposes, can be considered as unity. In water, μ_a is equal to 81. Thus, in water, the forces on the charges are 81 times smaller than the forces on the charges in air or vacuum.

f. Summary. The formula states that the force of attraction or repulsion between two point magnetic poles varies *directly* with the product of the strengths of the poles and *inversely* with the square of the distance between them.

13. Magnetic Field

It has been shown previously that an invisible magnetic force exists in the space surrounding a magnet and that this force is capable of acting on other magnets or magnetic substances. The space which surrounds a magnet is called the *external magnetic field.* (The complete field consists of the external field plus the field through the substance of the magnet.) Thus, a *magnetic field may be defined as a region wherein magnetic forces act.*

a. Exploring Magnetic Field with Compass. Certain facts concerning the nature of the magnetic field about a magnet can be obtained by exploring such a field with an ordinary compass. (The needle of any compass is a small permanent magnet.)

(1) When a compass is placed near the south pole of a bar magnet, the compass needle will swing about and come to rest with the *north* pole of the needle as close as

possible to the south pole of the bar magnet. In this position, the south pole of the compass needle is as far away as possible from the south pole of the bar magnet (A of fig. 12). This action is easily explained by the first law of magnetic forces which states that like magnetic poles repel each other and unlike magnetic poles attract each other.

(2) When the compass is placed at the other, or north pole, end of the bar magnet, the needle swings and then comes to rest with the south pole of the needle as close as possible to the north pole of the bar magnet (B of fig. 12).

(3) When the compass is placed near the center of the bar magnet, the force existing in the magnetic field causes the needle to come to rest in the position shown in C of figure 12,

(4) In D of figure 12, a number of compasses have been placed at various positions in

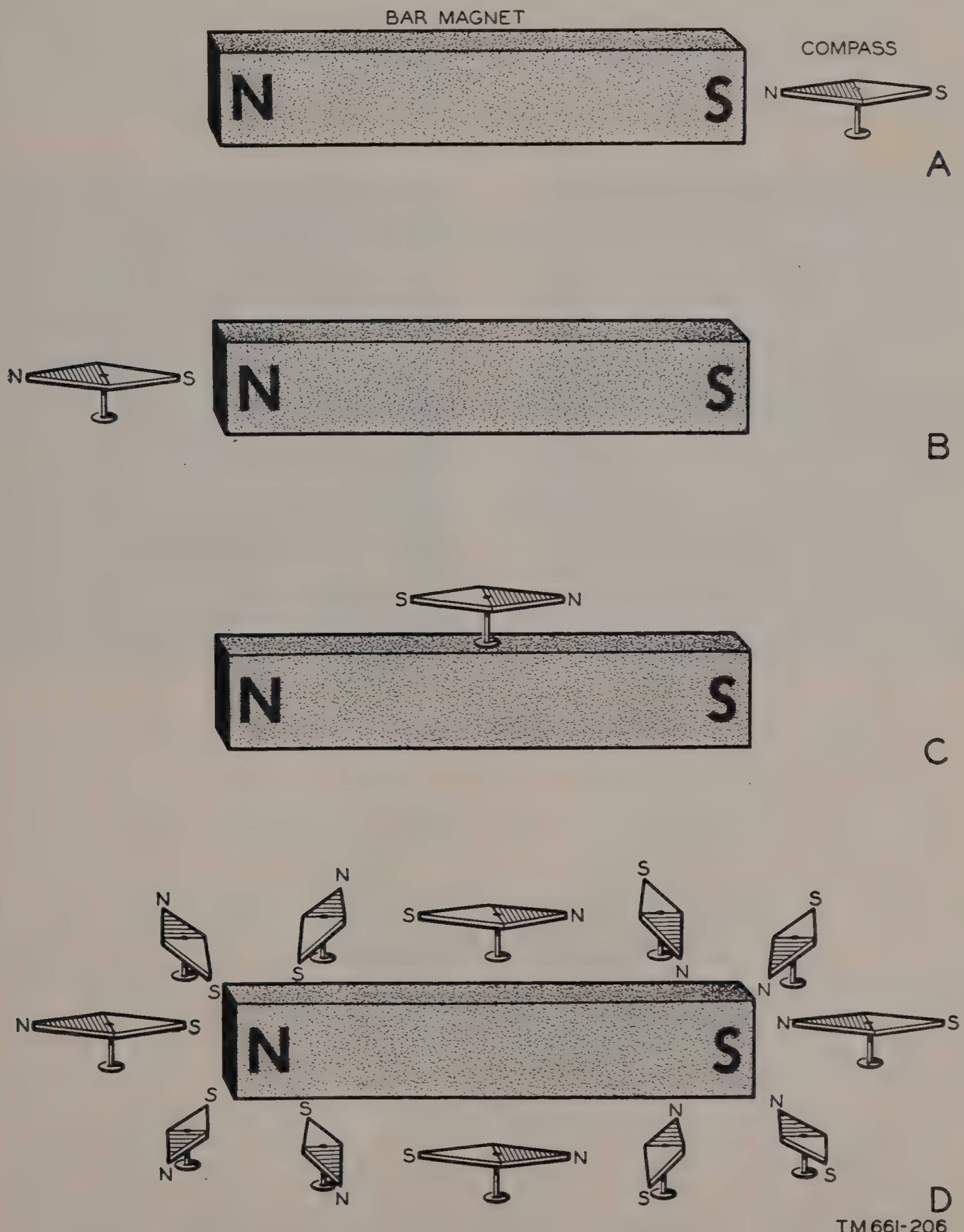


Figure 12. Effects of magnetic field on compass needle.

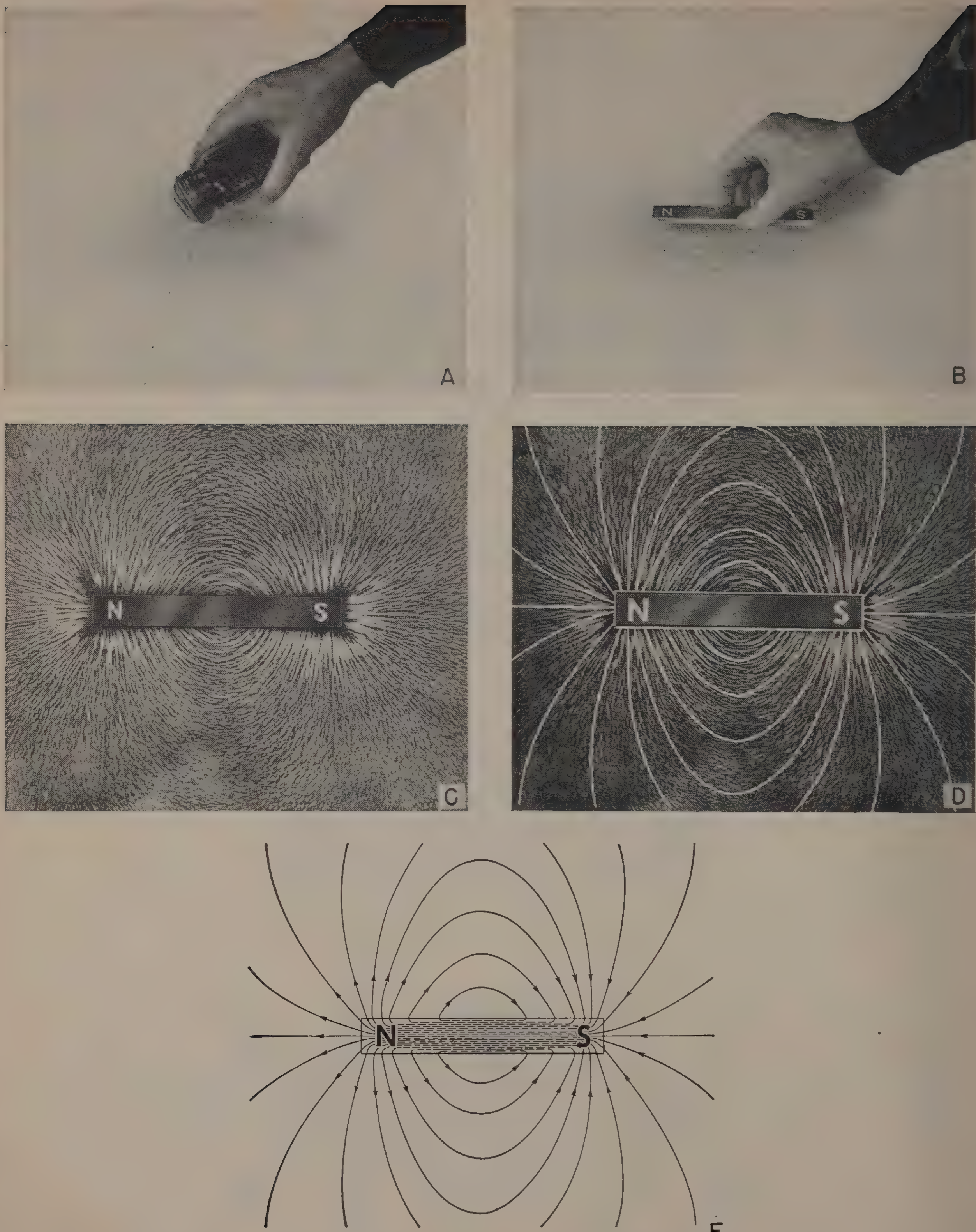


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Figure 13. Pattern formed by iron filings about a bar magnet represents the magnetic field about this magnet.

the magnetic field of a bar magnet. Note that the compass will point in different directions as its position in the magnetic field is changed.

(5) Thus, by exploring the magnetic field about a bar magnet, it is found that one important characteristic of a magnetic field is that it has *direction* which varies from point to point.

(6) Another important characteristic of the magnetic field about a bar magnet is that the *intensity* of the field decreases very rapidly with distance from the poles. For example, when the exploring compass is placed a few feet away from the bar magnet, the compass needle is not visibly affected by the presence of the magnetic field. Only when the compass is brought to within a certain distance of the bar magnet will the magnetic forces in the field be sufficiently strong to cause the compass needle to swing.

(7) From the above experiments with an exploring compass, it is found that the magnetic field about a bar magnet is characterized by a force which varies in *direction and intensity* from point to point in the field.

b. Iron Filings Experiment. A representation of the magnetic field about a bar magnet can be obtained by means of an experiment using iron filings.

(1) This experiment can be performed by placing a piece of paper over a table top and sprinkling some iron filings over a large area of the paper (A of fig. 13).

(2) A bar magnet is then dropped into the center of the area containing the filings (B of fig. 13). As the magnet reaches the table, it will be noted that many of the filings near the poles of the magnet are attracted to the magnet, and slight movement of most of the other filings takes place. In short, the filings rearrange themselves because of the magnetic field about the magnet.

(3) If the table is gently tapped, the filings completely rearrange themselves and the pattern about the magnet then appears similar to that shown in C of figure 13. Regardless of the number of times that this experiment is repeated, the filings will always arrange themselves in approximately the same pattern as that shown in C of figure 13.

(4) The explanation of why this pattern always forms is quite simple. In paragraph 6 it is explained that pieces of iron or steel become magnetized by induction when they are brought in contact with, or close to, a magnet. Thus, each of the tiny iron filings about the bar magnet becomes a small magnet with a north and south pole of its own. Each of these filings (or magnets) is affected by the direction and intensity of the magnetic field about the bar magnet, and alines itself in the same manner as a compass needle, in response to the magnetic field.

(5) Observe that the force of the magnetic field is greatest near the poles; many of the filings are attracted, and adhere to the magnet showing distinct alinement. As the distance from the poles increases, the field becomes less intense, and the action of the force on the iron filings is less apparent. At the edge of the pattern, as shown in C of figure 13, the effect on the filings is hardly discernible.

(6) This iron filings experiment has long been used to represent the nature of the magnetic field about a magnet and it gives a conception of the direction and intensity of the magnetic forces which act in the field. However, it should be remembered that the pattern formed in this experiment *is not the magnetic field itself* (the magnetic field is invisible) but only a representation of the manner in which the magnetic forces in the field act on magnetic substances within the field.

14. Lines of Force

Frequently, it is desired to represent, by a drawing the direction and intensity of the magnetic field about a magnet. This can be done by drawing a picture of the pattern obtained when iron filings are placed about the magnet and, in the case of a bar magnet, such a drawing would be similar to that shown in C of figure 13. However, this method is difficult and time consuming. A simpler and more commonly used method is that of arbitrarily representing the forces in a magnetic field by drawing a few lines called *lines of force*.

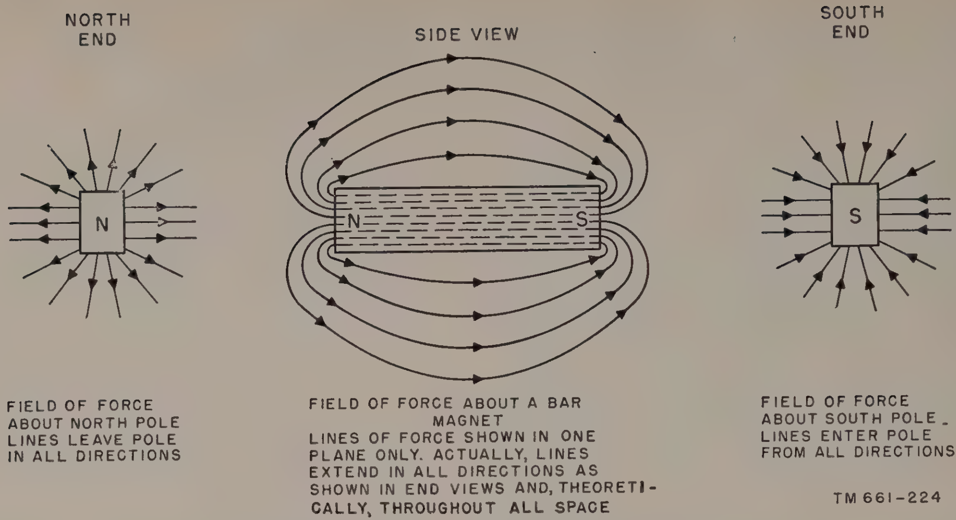


Figure 14. Field of force about a bar magnet.

a. E of figure 13 shows lines of force as they are usually drawn to represent the magnetic field about a bar magnet. By studying these lines of force, it can be seen that their over-all pattern is similar to the over-all pattern formed by the iron filings about the magnet.

b. Note that arrowheads have been placed on each of the lines of force shown in E of figure 13. Also note that the direction of each arrowhead is away from the north pole and toward the south pole of the magnet. In other words, the arrowheads indicate that lines of force leave the magnet at the north pole and enter the magnet at the south pole. Within the magnet, the direction of the force is assumed to be from the south pole to the north pole, so that a continuous loop is formed by each line of force. The direction of these lines was defined arbitrarily long ago as the direction in which the north pole of a compass needle will point if placed at any point along a line of force.

c. Actually, the magnetic field completely fills the space about a magnet and does not exist only in a single plane, such as along a table top. This field can also be shown to extend out to great

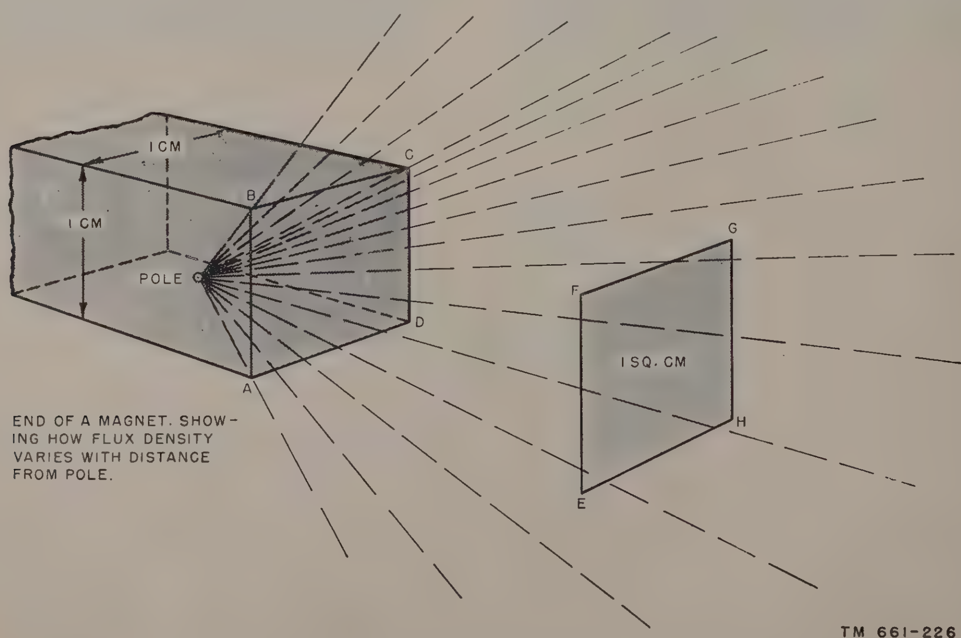


Figure 15. End of a magnet showing how field intensity varies with distance from pole.

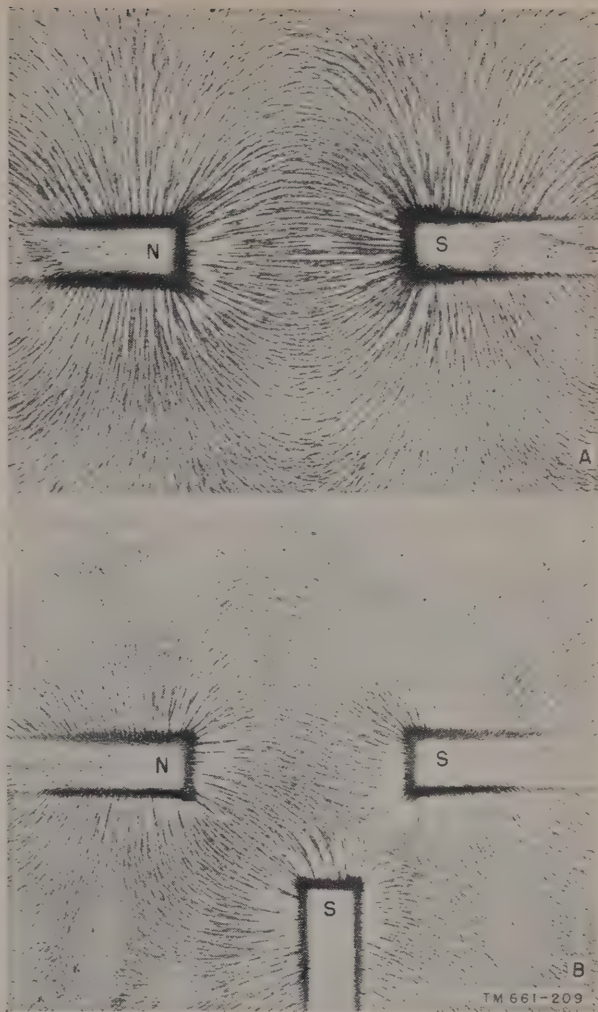
distances and, theoretically at least, throughout all space, with the intensity of the field decreasing very rapidly as the distance is increased. Figure 14 shows how the field of force extends in all directions about a bar magnet. The end views show the line of force leaving the north pole and entering the south pole.

d. Lines of force are also used as an indication of the strength or intensity of the magnetic field. Paragraph 12*e* gives the formula which shows that magnetic force (field strength) varies inversely with the square of the distance. This law is not strictly true in practice since it is based on the assumption that the lines of force from a pole face emanate from a point somewhere within the magnet. Actually, the lines of force do not converge to a single point within the magnet. Figure 15 shows a magnet face of 1 square centimeter area. It is obvious that there are more lines of force passing through the square centimeter area *ABCD* than pass through the same size area *EFGH* which is located the same distance from the pole face. This means that the field intensity is greater in area *ABCD* than in *EFGH* as indicated by the greater number of lines passing through *ABCD*. The farther away from the pole one goes, the less the field intensity will be. When the lines emanate from a point, the number of lines through a given area will vary inversely with the square of the distance that the area is from the point—not from the pole face. With a long magnet the field intensity will be nearly as shown in figure 15, and the law, the inverse as the square of the distance, will be almost true. But if the magnet is short and thick, the field will be more uniform for quite a distance from the pole and the distance-square law will be less accurate.

15. Characteristics of Magnetic Fields

Thus far, only the pattern of the magnetic field associated with a single bar magnet has been considered. Actually, this pattern of the magnetic field is only one of an unlimited number of possible patterns which can be produced by using one or more magnets.

a. If the area surrounding the north pole of one bar magnet and the south pole of a second bar magnet is sprinkled with iron filings, the pattern shown in A of figure 16 will be obtained. Note the similarity between this pattern and the pattern of filings about a single bar magnet. B of figure 16 shows the distortion of this magnetic field which occurs when a third magnetic pole is brought into the region. Note that the intensity of the magnetic field is greatest near the poles of the magnets. In B of figure 16, note the small area midway between the two like poles where the filings appear to lie as if unmagnetized. This is actually the case, because midway between the two like poles the magnetic force produced by one pole is equal to, and in opposition with, the magnetic force produced by the other pole.



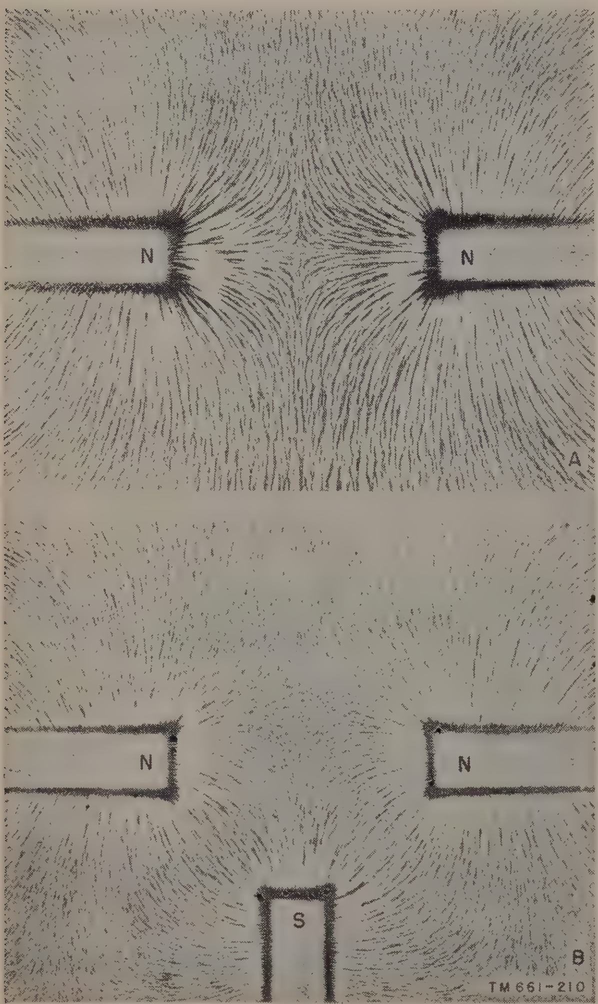
A. Iron filings pattern about unlike magnetic poles.
B. Distortion caused by insertion of a third magnetic pole

Figure 16.

b. A of figure 17 further illustrates the manner in which magnetic forces act to produce a magnetic field. In this case, the magnetic field contains forces of repulsion. Again note that the magnetic force is strongest near the poles and decreases very rapidly as the distance from the poles is increased, while at a point midway between these like poles there is a magnetic field of zero intensity.

c. B of figure 17 shows the distortion of the field which occurs when a third pole is brought into the region. Again, it can be observed how the magnetic field is more intense between unlike poles and less intense between like poles. Close scrutiny of B of figure 17 will reveal another small area where the magnetic forces act in opposition to produce a resultant force of low or zero intensity.

d. Figure 18 shows the lines of force associated with a horseshoe magnet. The pattern formed by these lines of force is different from the pattern for the single bar magnet, because of the physical position of the poles with respect to the magnet and to each other. Although these lines of force have a different pattern, or configuration, the characteristics of the magnetic field remain the same. That is, the magnetic field varies in intensity and direction from point to point in the field.



A. Iron filings pattern about like magnetic poles.
B. Distortion caused by insertion of a third magnetic pole.

Figure 17.

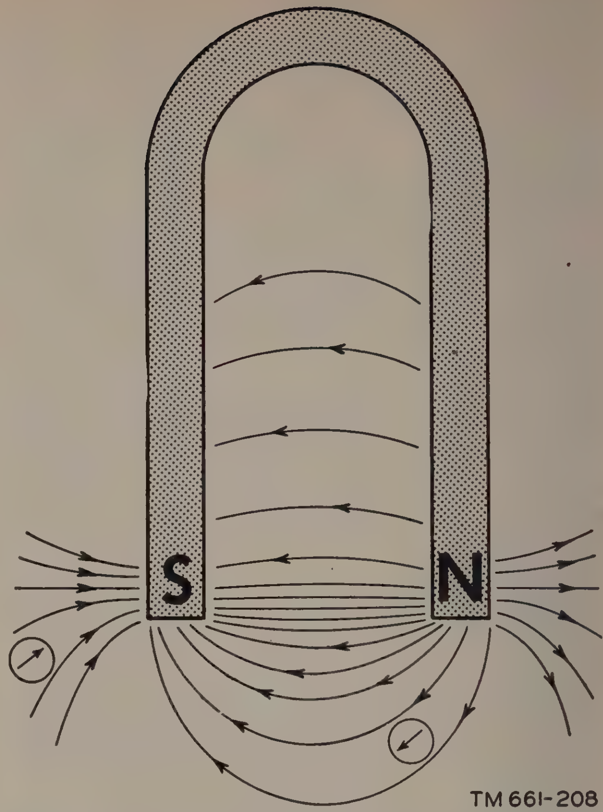


Figure 18. Lines of force associated with a horseshoe magnet.

16. Permeability

a. Field Strength. The field strength at a point is defined as *the force exerted on a unit north pole at that point.* The letter H is used as a symbol for field strength.

b. Permeability. As given in paragraph 12*e*, the force F between two magnetic poles is expressed by the equation

$$F=\frac{m_1 m_2}{\mu_a d^2}$$

The quantity μ_a is a constant involving the nature of the medium in which the two poles m_1 and m_2 are situated. The constant μ_a is called the permeability and it is assumed to have the value of unity in empty space; for air it is nearly 1 (1.000004); for iron it is very high (about 10,000). Figure 19 shows the distortion of the magnetic field produced by placing a soft iron bar in the field.

c. FLUX DENSITY. As explained in paragraph 14, lines of force are used to represent a field of force. In a magnetic field, the field strength or intensity H is defined as the force (in dynes) acting on a unit positive (north) pole. Thus, H indicates both magnitude and direction. The *flux density* in free space is equal *numerically* to the intensity; that is, in free space where the intensity is H, there are H lines of flux. All materials influence the force between magnetic poles; therefore, the flux density in materials is not equal to H. Using the letter B to represent the flux density in any material, we get—

$$B=\mu_a H$$

Thus, the permeability of a material tells us what the flux density will be relative to the flux density in empty space. For example, assume that the field intensity H at a point in free space is represented by one line of force. When an iron bar with a permeability of 2,000 is placed in this field, it becomes magnetized by induction; that is, it becomes a magnet. The resultant flux density is—

$$B=\mu_a H$$
$$B=2{,}000 \times 1=2{,}000 \text{ lines of flux.}$$

d. APPLICATION. The property of permeability is utilized commercially in protecting sensitive instruments such as meters, watches, and compasses from external magnetic forces. Figure 20 shows how the magnetic field between a north and a south magnetic pole is distorted by providing a more permeable path through the soft iron ring. A compass placed inside the ring would not be affected by the magnetic field. This method of protecting instruments is known as *shielding.*

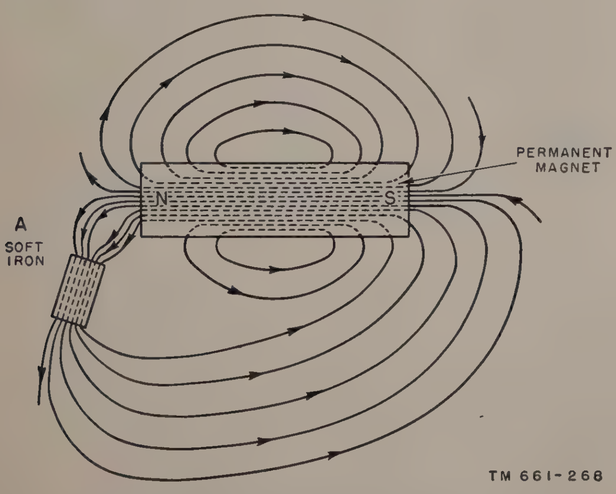


Figure 19. Distortion of the magnetic field about a permanent magnet produced by insertion of soft iron bar.

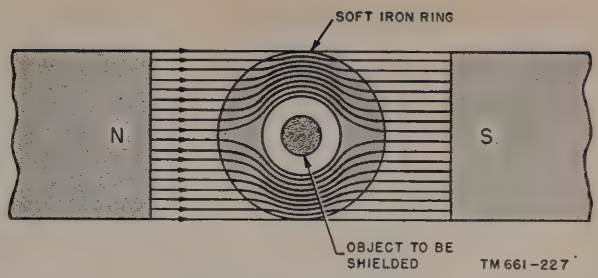


Figure 20. An object shielded from magnetic field by presence of soft iron ring.

17. Handling and Care of Magnets

a. A piece of steel that has been magnetized may be demagnetized by any external force that will disturb the symmetrical position of the molecules (par. 10). Striking a magnet with a hammer jars the molecules, causing them to move and return to their original jumbled position. Heating a magnet expands the metal and will also allow the molecules to move. Therefore, care must be used in handling equipment containing permanent magnets to prevent them from losing their magnetism. They should never be dropped or placed in high temperatures.

b. The proper method of storing permanent magnets is very important, if they are to retain their magnetism. Horseshoe magnets should always have a soft iron bar, called a keeper, placed across the poles to reduce leakage. See A of figure 21 and notice how the flux can pass from the north pole to the south pole through a very permeable path, thus reducing the leakage to a minimum. Bar magnets should always be stored in pairs, putting a north pole and a south pole together. See B of figure 21 and notice how the magnets make a complete circuit path for the flux.

18. Theory of Magnetization

The material presented in this section of the manual has told much about magnets and magnetic force in terms of the visible effects magnetic force has on substances such as iron and steel. However, the reason still remains why magnetic force is produced by a magnet—why an unmagnetized piece of iron or steel is not accom-

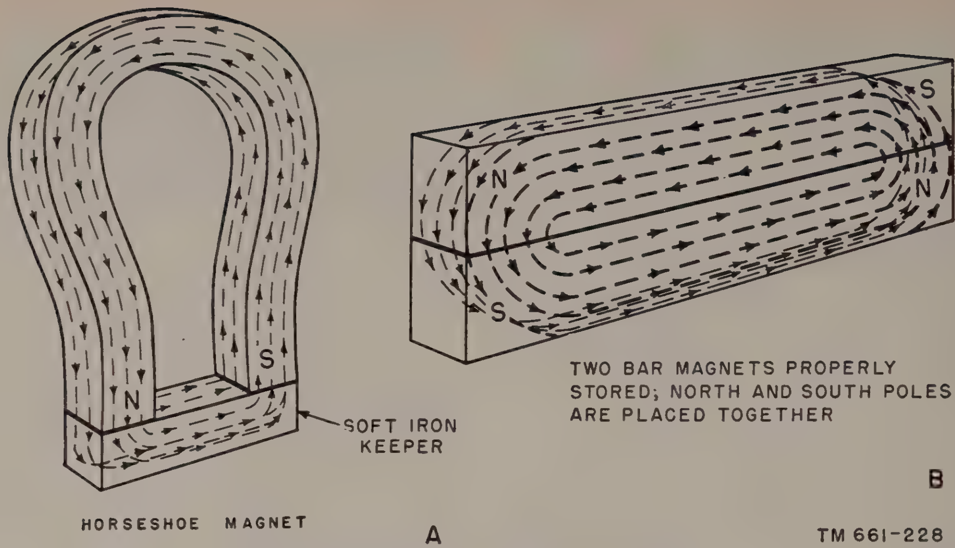


Figure 21. Storage of horseshoe and bar magnets.

panied by magnetic forces and a magnetized piece of iron or steel is accompanied by magnetic forces. Most scientists now agree that the theory of magnetization is best explained by means of the electron theory. Chapter 2 of this manual presents the electron theory.

19. Summary

a. An invisible magnetic force exists in the space surrounding a magnet. This force is capable of attracting pieces of iron and steel.

b. Although pieces of magnetite possess magnetic properties when taken from the earth, better magnets can be made by artificial means.

c. Pieces of iron or steel become magnetized by induction when they are brought close to, or in contact with, a magnet.

d. Any magnet which loses its magnetism rapidly is called a temporary magnet.

e. Any magnet which retains its magnetism over a long period of time is called a permanent magnet.

f. Artificial magnets can be made by stroking an unmagnetized piece of iron or steel with a magnet. The best method of making a magnet is by electrical means.

g. Any substance which is attracted to a magnet is known as a magnetic substance. Most substances are nonmagnetic.

h. Magnetic force is mutual to both a magnet and to magnetic substances.

i. Every magnet has at least one north pole and one south pole.

j. Like magnetic poles repel each other; unlike magnetic poles attract each other.

k. The force of attraction or repulsion between two magnetic poles varies directly with the product of the strengths of the poles and inversely with the square of the distance between them.

l. The space which surrounds a magnet is called a magnetic field. A magnetic field is defined as a region wherein magnetic forces act.

m. A magnetic field is characterized by forces which vary in intensity and direction from point to point in the field.

n. Graphically, magnetic fields are commonly represented by lines of force.

o. The fact that a suspended magnet always points towards North indicates that the earth itself is a huge magnet.

p. The amount of magnetism retained in a material after the magnetizing force has been removed is known as *residual magnetism*.

q. The ability of a material to retain magnetism is called *retentivity*.

r. The best permanent magnets are made from hard materials, such as steel and its alloys.

s. The ratio of the number of lines of force which pass through a given area when it is occupied by a substance, to the number of lines of force passing through that area when it is occupied by air, is called the *permeability* of the substance.

t. Magnets may lose their magnetism by jarring or heating.

20. Review Questions

a. What is a natural magnet?

b. What is a magnetic substance?

c. Distinguish between a magnetic substance and a magnet.

d. What are two methods of making an artificial magnet?

e. What is the difference between a temporary and a permanent magnet?

f. What is a magnetic pole?

g. How many kinds of magnetic poles are there and what are they called?

h. What is residual magnetism? What is retentivity?

i. What are the fundamental laws of magnetic forces?

j. What two factors govern the force of attraction or repulsion between two magnetic poles?

k. What is permeability? What is reluctance?

l. What is the mathematical equation for the force of attraction and repulsion between two magnetic poles?

m. What is a magnetic field?

n. What are two characteristics of a magnetic field?

o. What are lines of force? What is flux?

p. How is the intensity of a magnetic field represented by lines of force? What is the unit of flux density?

q. Do lines of force cross each other?

r. Why is it believed that the earth is a huge magnet?

s. What is the angle of declination?

t. Iron filings become magnetized when they are placed in a magnetic field. Explain.

CHAPTER 2

ELECTRIFICATION

21. Early History of Electrification

The ancient Greeks learned that a magnetized piece of iron is not the only substance about which exists an invisible force capable of attracting objects. They found that a yellowish resin called amber, if rubbed, will attract small bits of wood shavings. They also learned that the invisible force about a piece of amber so rubbed is *different* from the force about a magnetized piece of iron; that is, the force about the amber, a nonmagnetic substance, does not attract magnetic substances; while a magnetic force does not attract a nonmagnetic substance, such as wood shavings. Later, it was discovered that many other substances such as glass and rubber, after being rubbed with a piece of fur, wool, silk, etc., exhibit the same characteristics as amber; that is, they attract bits of paper, wood, and certain other light objects (fig. 22).

a. In an attempt to explain the nature of this force, William Gilbert (1600) divided all substances into two classes: electrics and nonelectrics. He classified an electric as being any substance possessing the property of amber, and a nonelectric as being any substance *not* possessing the property of amber. Although such a classification is not acceptable today, the word *electric* from the Greek *elektron* has been handed down and is still commonly used. If a piece of amber is rubbed, it is said to be *electrified*, or *charged* with electricity, and the invisible force about an electrified piece of amber is called an *electric force*.

Figure 22. Attraction of paper to electrically charged glass rod.

b. In 1733, a Frenchman named DuFay observed that when a piece of glass is rubbed with cat's fur, the glass and the cat's fur both become electrified, but that the glass will attract some charged objects that are repelled by the cat's fur and vice versa. From this observation, he concluded correctly, that *there are two exactly opposite kinds of electricity.*

c. Benjamin Franklin introduced the terms *positive* (+) and *negative* (−) into the science in order to distinguish between the two different kinds of electricity. Franklin defined a positively charged body as one which exhibits the same kind of charge as that associated with a piece of glass after it is rubbed with silk (fig. 23). He defined a negatively charged body as one which exhibits the same kind of charge as that associated with a rubber rod after it is rubbed with cat's fur. He defined as electrically *neutral* all bodies which exhibit no charge.

d. Further study and experimentation since Franklin's time have added much information regarding the characteristics of electric charges and electric forces.

22. Attraction and Repulsion

a. The forces of attraction and repulsion between electrically charged bodies may be demonstrated as follows: One end of a rubber rod is rubbed with fur and then suspended by a piece of string (A of fig. 24). When a second rubber rod has been electrified the same way and its charged end

brought near the charged end of the suspended rod, the latter turns away as in A of figure 24, showing repulsion. If, instead of the second rod, the fur is brought near the charged end of the suspended rod, the latter turns toward the fur, as in B of figure 24. If a glass rod, rubbed with silk, is brought near the charged end of the suspended rod, there is attraction, as in C of figure 24, but when the silk is used, there is repulsion, as in D of figure 24. Since the fur and the glass both attract, these have the same kind of electricity and are said to be positively charged. The rubber and silk are said to be negatively charged. This experiment shows that two kinds of charges or electricity exist. It also demonstrates a rule concerning the action of one kind of charge on another.

b. A of figure 24 shows that two negative charges repel each other. C of figure 24 shows that positive and negative charges attract each other, and E of figure 24 shows that two positive charges repel each other. This attraction or repulsion is mutual and is expressed in the following fundamental laws: LIKE CHARGES REPEL EACH OTHER; UNLIKE CHARGES ATTRACT EACH OTHER, or, THE FORCE BETWEEN TWO LIKE CHARGES IS ONE OF MUTUAL REPULSION; THE FORCE BETWEEN TWO UNLIKE CHARGES IS ONE OF MUTUAL ATTRACTION. Note the similarity between the laws of electricity and the laws of magnetism as stated in paragraph 11*d*.

c. Also, it has been found that the force of attraction or repulsion between two electrical charges varies *directly* with the product of the quantities of the charges and *inversely* with the square of the distance between them. This may

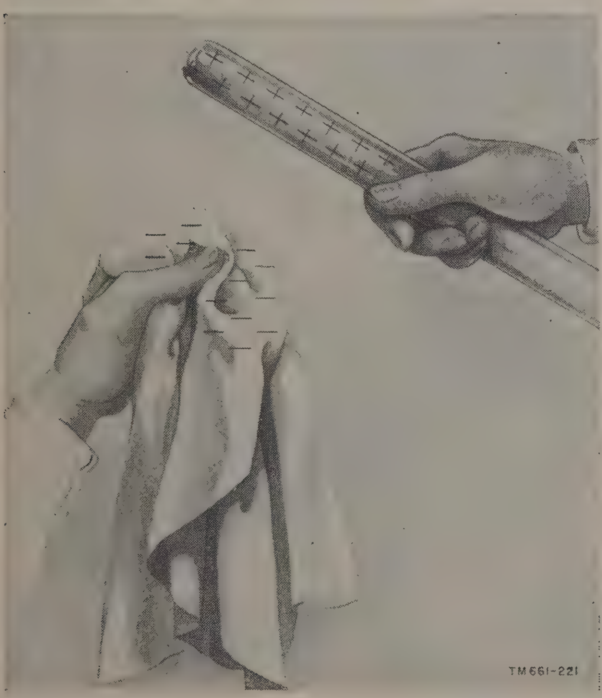


Figure 23. After glass and silk are rubbed together, they become charged with electricity.

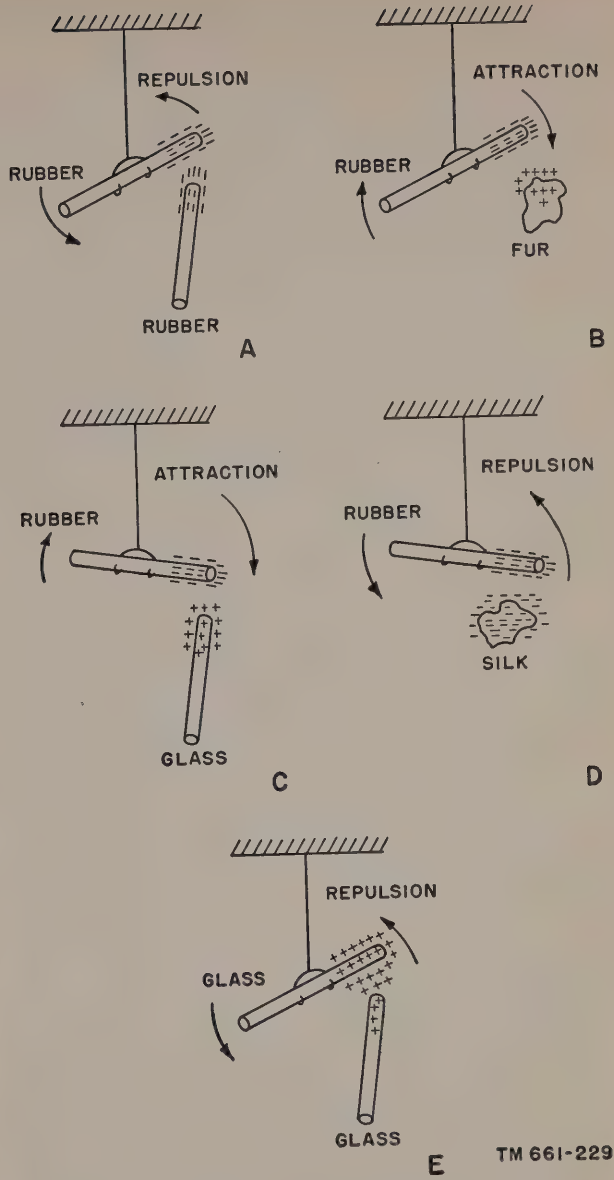


Figure 24. Attraction and repulsion of charged bodies.

be expressed by the following mathematical equation:

$$F=\frac{q_1 q_2}{\epsilon d^2}$$

where F=force,

d=distance between charges,

q_1=quantity of one charge,

q_2=quantity of second charge, and

ϵ=dielectric permittivity of the medium in which the charges are located. In vacuum, $\epsilon=\epsilon_0=1$.

The force is one of attraction if the charges are unlike and one of repulsion if the charges are alike. Again note the similarity between this equation and the equation for the magnetic force of attraction or repulsion as stated in paragraph 12*e*.

23. Theory of Electrification

For a long time it was believed that only substances such as amber, glass, wax, silk, and cat's fur were capable of being electrified. It is now known that under the proper conditions *any* substance can be charged electrically. That is, all substances possess the property of electrification. The question then arises: if all substances are capable of being electrified, how is electrification related to matter? The answer to this question is to be found in the study of the structure of matter.

24. Structure of Matter

Matter may be defined as any substance that has weight (mass) and occupies space. Examples of matter are the air we breathe, water, cars, the clothing we wear, and our own bodies. From these examples, we can conclude that matter may be found in any one of three states; namely, solid, liquid, or gaseous.

a. Elements. All matter consists of one or more basic materials which we call *elements.* Scientists have definite proof that 97 elements exist and believe that there are several other elements. In chemistry, an element is defined as a substance that can be neither decomposed (broken up into a number of substances) by ordinary chemical changes nor made by chemical union of a number of substances. Copper, iron, aluminum, and gold are examples of metallic elements; oxygen, hydrogen, and sulphur are nonmetallic elements.

b. Compound. A substance containing more than one constituent element and having properties different from those of the elemental constituents is called a *compound.* For example, water is made up of two parts hydrogen and one part oxygen. Therefore, water is a compound.

c. Molecules. A *molecule* is defined as the smallest particle of matter which can exist by itself and still retain all the properties of the original substance. If we take a drop of water, a compound, and divide it until we have the smallest particle possible and still have water, that particle is known as a molecule. An idea of the size of molecules may be obtained by imagining that a stone is first broken into two pieces, that the two pieces are then broken into four pieces, and that this process is carried on indefinitely. The smallest particle of stone which could be obtained by this process would be a molecule. Actually, it is impossible to crush a stone into its molecules; we can only crush it into dust. One small particle of dust is composed of thousands of molecules.

d. Atoms. An atom is defined as the smallest part of an element that can take part in ordinary chemical changes. The atoms of a particular element are of the same average mass, but their average mass differs from that of the atoms of all other elements. For simplicity, the atom may be considered to be the smallest particle that retains its identity as part of the element from which it is divided. Figure 25 shows that the molecule of water is made up of two atoms of hydrogen and one of oxygen. Since there are 97 known elements, there must be 97 different atoms or a *different* atom for each element. All substances are made of one or more of these atoms. Just as thousands of words can be made by combining the proper letters of the alphabet, so thousands of different materials can be made by chemically combining the proper atoms.

e. Subatomic Particles. Although it was formerly believed that the atom was the smallest particle of matter, it is known now that the atom itself can be subdivided into still smaller, or subatomic particles. Paragraph 25 explains the nature of these subatomic particles.

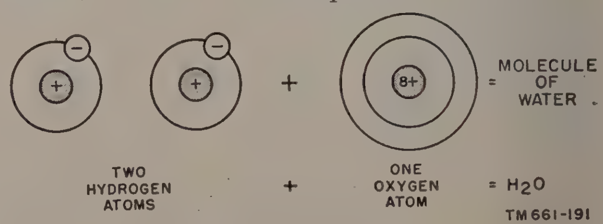


Figure 25. Water molecule.

25. Structure of the Atom

In order to become acquainted with the relationship of the atom to matter, and the atom to its subatomic particles, consider the diagrams of figure 26. All of these diagrams are imaginary, except A and B. The reason that they are imaginary is that the portions of matter which they represent are so small that they are invisible to the eye, even with the aid of the most powerful microscope. However, the fact that they are invisible does not prevent us from knowing the characteristics of such tiny bits of matter. In fact, many things with which we are quite familiar are also invisible. For example, no one has ever seen the wind, yet we know of its existence because of the visible effects associated with it—a flag unfurled, the movement of leaves, and the presence of whitecaps on the sea. So it is with the atom and with the subatomic particles of which it is made. Although no one has ever seen them, by experiment and study scientists have been able to learn facts concerning subatomic particles and, from these facts, have been able to give us an understandable explanation of the make-up of the atom. Thus, diagrams D, E, and F of figure 26 can be called representations of what scientists believe one of the elements, aluminum, might look like if tiny bits of aluminum could actually be seen. To pursue this line of approach still further, let us picture that an *imaginary microscope* is available which will allow us to examine tiny particles of matter. Furthermore, let us imagine that this microscope has a control knob which can be rotated in order to allow any magnifying power desired. With such a device, we could *see* into the make-up of matter and into the structure of the atom. In all likelihood, observations made through this microscope would indicate that the representations of figure 26 are correct if the matter under scrutiny is a chunk of aluminum, which we know to be a medium-hard, silvery-white metal.

a. In A of figure 26, the magnification is unity, that is, we are simply looking at a piece of aluminum.

b. In B of figure 26, the magnification has been increased to 100 diameters. It is now apparent that the aluminum is not a perfectly smooth surface but rather a crystalline substance, with the size of the crystals being dependent on the previous heat and mechanical treatment of the metal. The average size of one of these crystals might be 1/1,000 of an inch in diameter.

c. When the control knob of the imaginary microscope is adjusted to a magnification of 100,000 diameters, the aluminum appears to have a regular structure (C of fig. 26), but no evidence of the presence of individual atoms or subatomic particles can be detected. At this magnification, the size of the aluminum under observation is smaller than the wavelength of light and cannot be seen in any light microscope. However, some evidence of a structure such as that shown in C of figure 26 has been obtained with a device using X-rays instead of light.

d. In D of figure 26, the diagram is completely imaginary. The magnification is 10 million diameters, or far beyond the capabilities of any known microscopic apparatus. At this magnification, individual dots or little bodies of approximately spherical shape and a rather fuzzy outline would be observed. These dots are the atoms of aluminum and, no matter how hard we try, we can detect no difference between any of them. We could, if we wished, measure the diameter of one of the aluminum atoms but we would have trouble because the edges of the atoms are fuzzy and it is difficult to say just where one atom ends and the next atom begins.

e. In an attempt to eliminate this fuzziness, we again adjust the microscope until only one of the aluminum atoms fills the viewing area, or to a magnification of 100 million diameters, an unbelievably small area of view. We can then see that a single atom of aluminum resembles our mental picture of the solar system, for there is a central body, called a *nucleus*, about which a number of smaller particles move in approximately elliptical orbits (E of fig. 26). Each of the moving particles, called *electrons*, is so tiny and moves so fast, seeming to wave as it passes by, that we cannot see clearly where it is at any given instant, but we guess that it is also spherical in shape. We try to count the number of electrons in the atom but find that this task is exceedingly difficult because of the way the electrons race around their elliptical orbits. However, after a great deal of effort, we determine that there are 13 electrons in the aluminum atom. By carefully examining the atom, we find that each of the electrons has a charge of electricity that is exactly the same as the charge on any other electron. We also note that the charge associated with an electron is the smallest electrical charge which has ever been discovered, and so we call it the *elemental charge* for electricity. The charge on an electron is the

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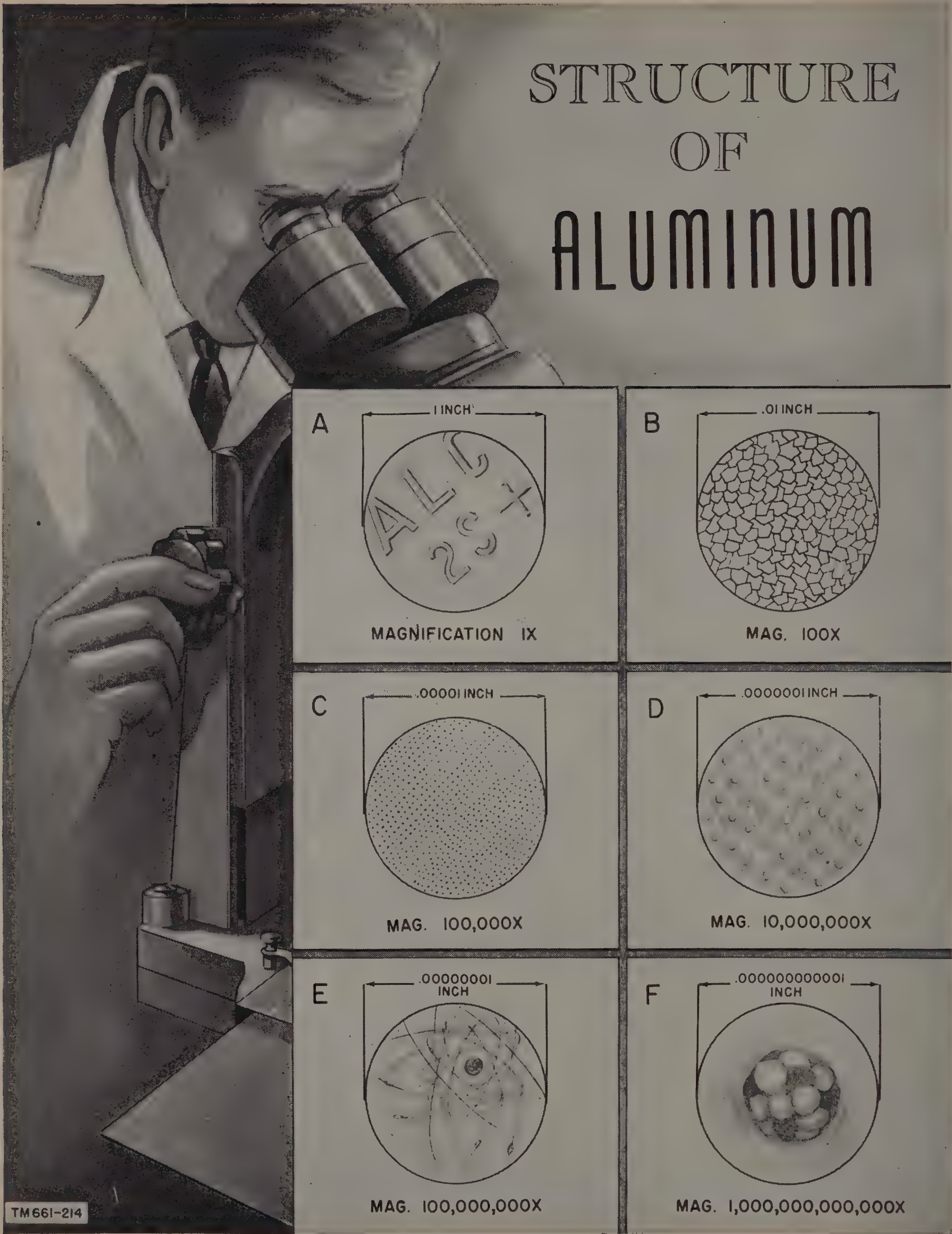


Figure 26. Structure of aluminum.

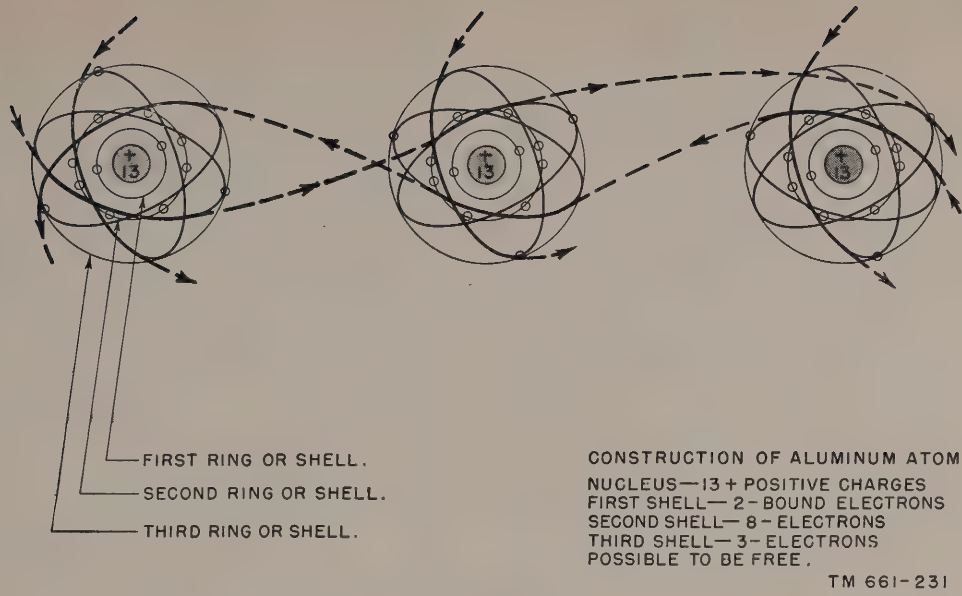


Figure 27. Three atoms of aluminum.

same kind of charge that appears in sealing wax after it is rubbed with fur and is arbitrarily called negative. (The charge on the electron was first measured by R. A. Millikan, an American physicist.) Thus, we say that the aluminum atom contains 13 elemental charges of negative electricity. Further examination of the atom reveals that the nucleus has a positive electrical charge, and that the quantity of this positive charge is 13 times as great as the quantity of the negative charge on one electron. Therefore, we say that there are 13 elemental positive charges on the nucleus, thus making the entire atom electrically neutral. That is, *the aluminum atom contains an equal number of positive and negative charges—an equal amount of opposite kinds of electricity*.

f. By twisting the dial of the imaginary microscope still further, the magnification is increased until only the nucleus fills the viewing area. It is then observed that the nucleus looks like a bunch of grapes (F of fig. 26) consisting of 27 particles, 13 of which carry an elemental positive charge of electricity; they are called *protons*. The other 14 particles in the nucleus are uncharged and are called *neutrons*. At first, we wonder why the nucleus does not explode as a result of the repulsion of the positive charges (like charges repel each other), but then we notice a sort of gelatinous material which exerts a force (nuclear force) capable of binding the particles of the nucleus together. This binding force is so strong that tremendous energy, such as that developed by a giant cyclotron, is required to dislodge the particles in the nucleus.

g. After this observation of the nucleus, we reduce the magnifying power of the microscope until a few atoms of aluminum are observed in the viewing area (fig. 27). It can then be noticed that the electrons which revolve in the outer orbits (near the edges of the atoms) do not always remain in the same atom. Instead, some of these electrons move haphazardly from atom to atom. Therefore, we conclude correctly that the electrons in the outer orbit of the aluminum atom are not tightly bound to the atom. We shall come to know these outer-orbit electrons as *free electrons*.

h. The branch of science which deals with the nucleus of atoms is called nuclear physics, and the scientists who study this subject are called nuclear physicists. It is nuclear physics which has given us atomic power and the atomic bomb. The study of electronics, on the other hand, is related to the behavior of those easier-to-control particles of the atom, namely electrons.

26. Electrons, Protons, and Neutrons

In paragraph 25*e* the aluminum atom is shown to be comprised of a positively charged nucleus and negatively charged electrons that revolve at very great speed around the nucleus. The electron theory explains that the atoms of *all* elements (copper, gold, oxygen, etc.) are similarly constructed of a central nucleus and revolving electrons.

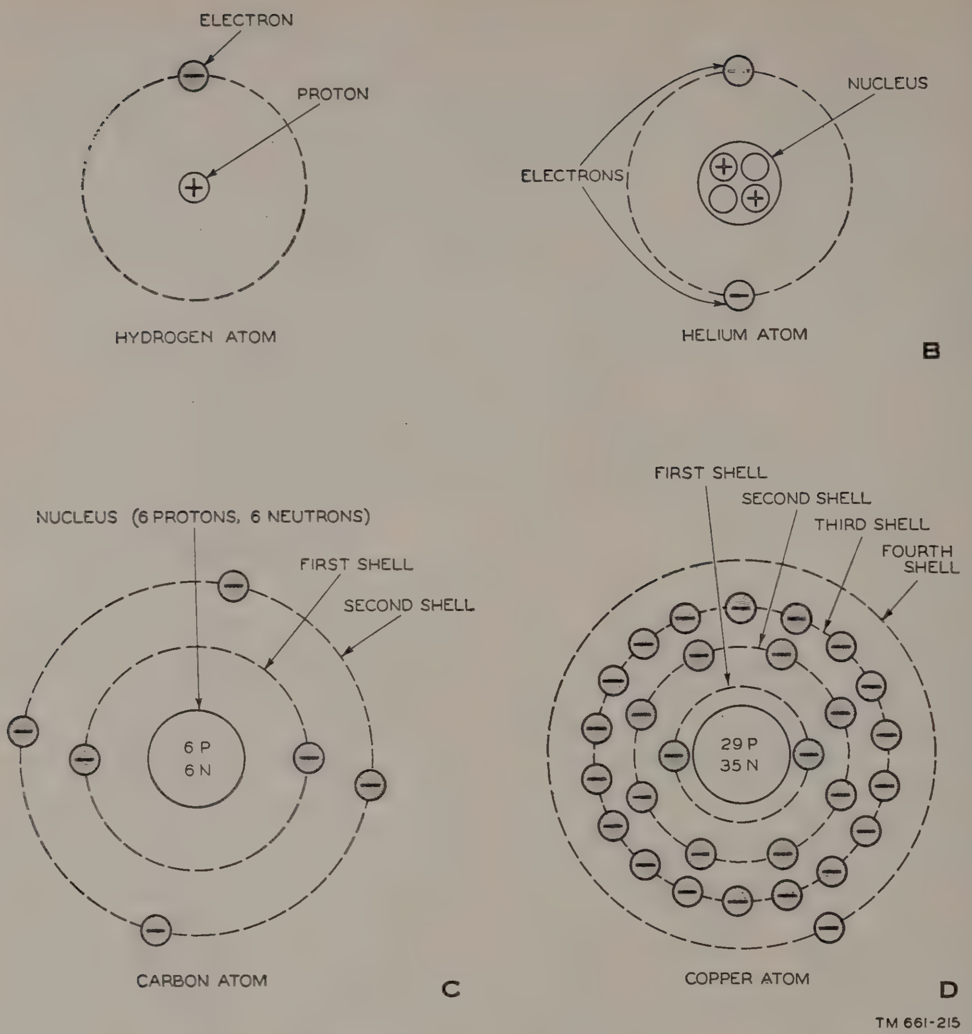


Figure 28. Structure of atoms.

a. Examples of Atomic Structure.

(1) A of figure 28 represents the atomic structure of the simplest of all atoms, the hydrogen atom. It contains one electron revolving around one proton which acts as a nucleus. Because the negative charge on the electron is exactly equal to the positive charge on the proton, the atom is electrically balanced or neutral.

(2) B of figure 26 represents the helium atom. The nucleus of the helium atom contains two neutrons and two protons. The positive charges of the two protons are just balanced by the negative charges of the two revolving electrons and the electrical charge on the entire atom is again neutral.

(3) Atoms of other elements are more complex than the hydrogen and helium atoms. For example, C of figure 26 represents the structure of the carbon atom. Note that the six orbital electrons revolve in two separate rings or *shells*.

(4) In D of figure 26 an even more complex atom is shown, namely, the copper atom. The nucleus is composed of 29 protons and 35 neutrons. The 29 orbital electrons revolve in four separate shells; only one electron revolves in the outer shell.

b. The Building Blocks. The electron theory shows that the only difference among the various elements is in the *number* and *arrangement* of the electrons, protons, and neutrons of which each atom is composed. There is no difference between

the electron in an atom of copper and the electron in an atom of aluminum, or any other element. There is no difference between a proton in one atom and a proton in another atom of a different element. Likewise, the neutrons in the atoms of various elements are thought to be identical. Since all matter is composed of atoms and all atoms are composed of positively charged particles called protons, negatively charged particles called electrons, and uncharged particles called neutrons, it follows that the proton, electron, and neutron are the fundamental building blocks of the universe.

c. CHARACTERISTICS OF SUBATOMIC PARTICLES.

(1) *Electrical.* The electrical charge of the proton is exactly equal and opposite to that of the electron; that is, the proton and the electron are exactly equal amounts of opposite kinds of electricity. Because it is believed that no smaller amount of electricity exists, the charge on the electron or proton is the elemental unit of electrical charge. However, the elemental unit is too small a quantity of electricity for practical purposes and a larger unit of charge called the coulomb is commonly used. One coulomb of electricity contains over 6 million, million, million (6.28×10^{18}) electrons. Neutrons are uncharged particles.

(2) *Physical.* Electrons and protons are approximately spherical particles of matter. The diameter of an electron, approximately 0.00000000000022 inch, is about three times the diameter of a proton. Despite its smaller diameter, a proton has a mass 1,850 times greater than the mass of the electron; that is, a proton is 1,850 times heavier than an electron. The diameter and mass of a proton and a neutron are approximately the same. Relatively speaking, there are great distances between the electrons and the protons of an atom even in solid matter. It has been estimated that if a copper one-cent piece could be enlarged to the size of the earth's path around the sun (approximately 186,000,000 x 3.14 miles), the electrons would be the size of baseballs and would be about 3 miles apart. What then keeps the electrons in their orbits? In order to better understand this phenomenon, we must continue our study of energy as evidenced through the forces in nature.

27. Forces Associated With Matter in the Universe

a. MAGNETIC FORCE. Magnetic force was discussed in detail in chapter 1 of this text.

b. ELECTRIC FORCE. In many ways, electric force is similar to magnetic force. It will be discussed more thoroughly throughout this text.

c. GRAVITATIONAL FORCE. Many everyday phenomena are subject to the laws of gravitational force. It will be discussed more fully in *e* below.

d. NUCLEAR FORCE. Nuclear force is the force that is latently present in the nucleus of the atom and is the subject of much present-day research.

e. LAW OF GRAVITATION. Sir Isaac Newton, an English physicist, experimented with gravity and developed the law which stated that every object attracts every other object with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This is called Newton's Law of Gravitation. Notice the similarity between this law and the law of attraction of charged bodies (par. 22*b*). It is gravitation that holds the universe together. With no gravitational field, the planets, including the earth, would not revolve around the sun in their orbits, but would fly off at a tangent. The moon would cease to revolve around the earth and, because of the rotation of the earth, objects on the surface of the earth would be thrown into space like mud from a bicycle wheel.

f. FORCES WITHIN THE ATOM. Within the atom, the force of gravity is believed to be small compared to the electric, magnetic, and nuclear forces, but it acts on the particles composing the atom in the same way that it acts on the planets in the solar system, helping the other forces keep the electrons, which revolve at tremendous speeds around the nucleus, in their orbits.

g. ELECTRICAL BALANCE. Because the protons and electrons of the atom carry positive and negative charges of electricity, respectively, they are particles of energy. These charges form the electric field of force within the atom, and since the number of positive charges (protons) is always equal to the number of negative charges (electrons), the atoms are electrically balanced.

For example, every atom of aluminum has 13 positive and 13 negative charges and every atom of hydrogen has 1 positive and 1 negative charge. Since all matter in the universe is composed of atoms, it follows that all matter is normally electrically neutral.

h. Electrical Unbalance. As shown in paragraph 22*a* and figure 27, it is possible to transfer some of the *free electrons* from one substance to another. When this happens, the normally equal distribution of positive and negative charges in the two substances no longer exists, and because they contain more of one kind of electricity than of the other they are said to be electrically charged. For example, when a glass rod (fig. 23) is rubbed with silk, some of the electrons which are loosely held to the atoms (free electrons) in the glass are transferred to the silk. Then when the glass and silk are separated, the glass rod will have more positive charges (protons) than negative charges (electrons) and will be *positively* charged. The silk, on the other hand, will have more electron than protons and will be *negatively* charged. When a hard rubber rod is rubbed with cat's fur, the cat's fur loses electrons to the rubber rod. In this case, the cat's fur becomes positively charged and the rubber rod becomes negatively charged.

i. Summary. To summarize, positive and negative electricity exist together in normal matter in equal amounts. Because normal matter contains equal amounts of opposite kinds of electricity, there is no electrical effect; that is, normal matter does not attract or repel an electrically charged body. It is possible, however, to *separate* the two kinds of electricity in normal matter by means of *work*. Whenever positive electricity is thus separated, an equal amount of negative electricity is of necessity separated. In terms of electric *charge*, the production of one kind of charge necessitates the production of an equal and opposite charge.

28. Charging

a. Charging by Contact. A of figure 29 shows a neutral body with equal numbers of electrons and protons. If, as shown in B of figure 29, a negatively charged body is placed in contact with the neutral body, electrons will pass from the charged body to the neutral body. If the negatively charged body is then removed, the body that was originally neutral will possess an excess number of electrons and will, therefore, be negatively charged (C of fig. 29). If, during the above experiment, a positively charged rod had been used instead of the negatively charged rod, the neutral body would have lost some of its electrons to the positively charged body and the neutral body then would have acquired a positive charge. In either case, the neutral body is *charged by contact* and becomes charged with the same polarity or kind of charge as the charging body.

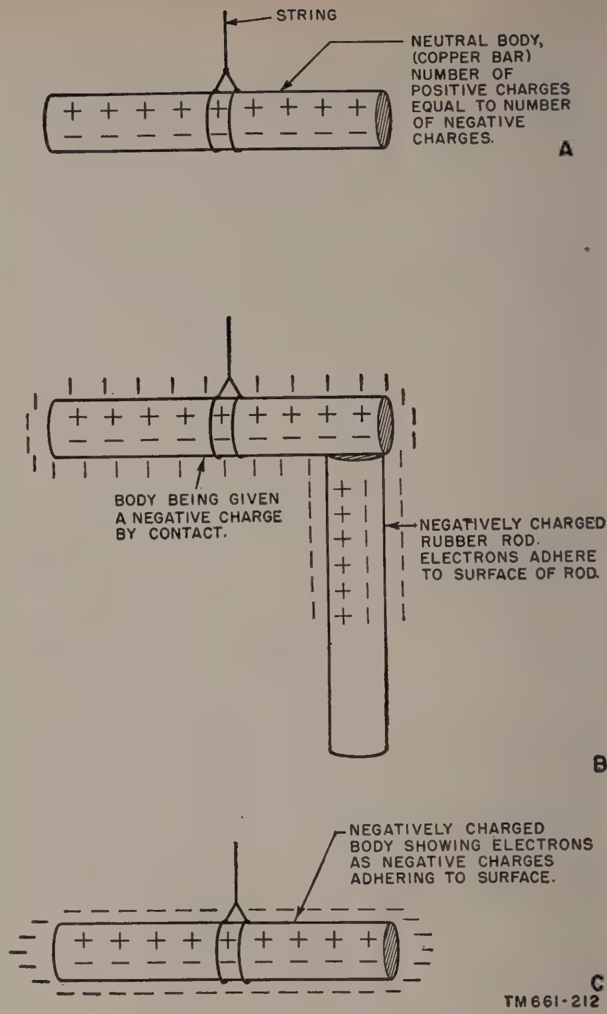


Figure 29. Charging by contact.

b. Charging by Induction. A second method of charging a neutral body is by *induction*.

(1) Suppose that a neutral body is again suspended by a piece of string (A of fig. 30) and suppose that one end of this neutral body is connected to another large neutral body, ground or earth, by means of a switch.

(2) If, as shown in B of figure 30, a negatively charged rod is brought near to, but not in actual contact with, the neutral body, the negatively charged body will repel electrons on the neutral body and will cause some of these electrons to flow into the ground.

(3) If the switch is opened before the rod is removed, the suspended body will have more protons than electrons and, therefore, will be positively charged (C of fig. 30).

(4) If a positively charged rod had been used instead of the negatively charged rod in the above experiment, electrons would have gone from the ground into the suspended body; consequently, the neutral body would have acquired a negative charge. In each instance there is no actual contact between the body to be charged and the charging body; no electrons pass from one to the other. This method of charging is called *charging by induction.*

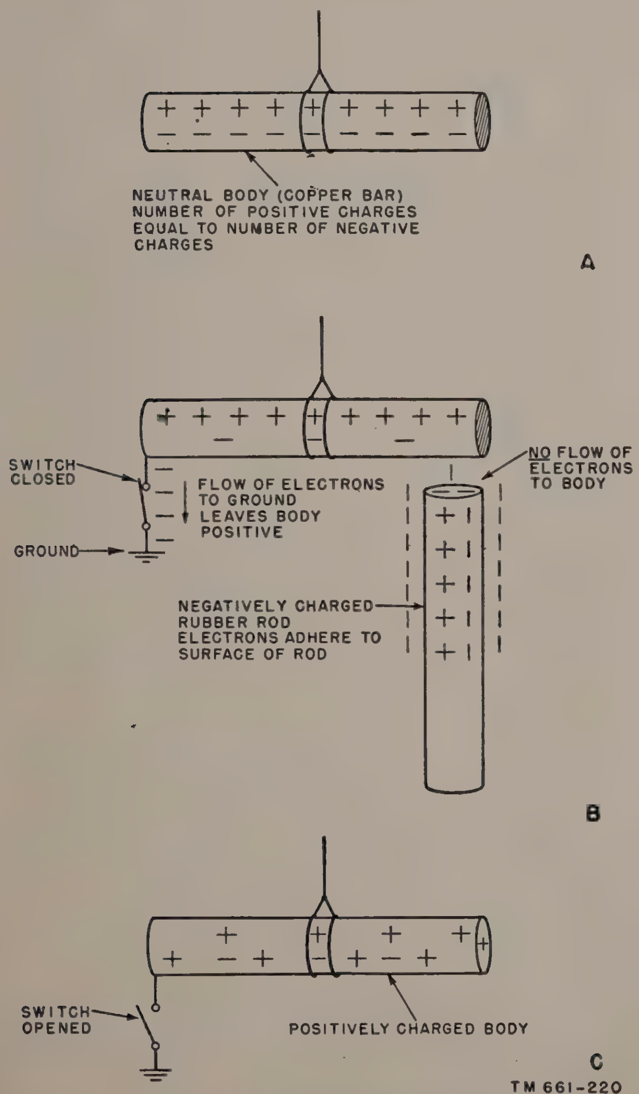


Figure 30. Charging by induction.

29. Summary

a. An invisible electric force exists in the space surrounding a charged body.

b. There are two exactly opposite kinds of electricity, called positive and negative.

c. Two positive charges repel each other; two negative charges repel each other; a positive and a negative charge attract each other. In other words, like charges repel each other and unlike charges attract each other.

d. The force of attraction or repulsion between two electrical charges varies directly with the quantities of the charges and inversely with the square of the distance between them.

e. All matter is composed of one or more of 97 elements.

f. The smallest part of an element which can take part in ordinary chemical changes is called an atom.

g. Atoms are comprised of positively charged particles called protons, negatively charged particles called electrons, and uncharged particles called neutrons.

h. Positive and negative electricity exist together in normal matter in equal amounts. Thus, normal matter exhibits no external electrical effect.

i. Matter or substances with an excess of electrons is said to be negatively charged. Matter, or substances, with a deficiency of electrons is said to be positively charged.

j. A neutral body can be charged by contact or by induction.

30. Review Questions

a. What are the two kinds of electricity called?

b. What are the fundamental laws of attraction and repulsion between two electrical charges?

c. What is the mathematical equation for the mutual force of repulsion and attraction between two electrical charges?

d. How many elements are there?

e. What is an atom?

f. What are the subatomic particles?

g. What is meant by the elemental charge of electricity?

h. Does all matter contain electricity?

i. How does normal matter differ from electrically charged matter?

j. What are free electrons?

k. What is substance?

l. What is a molecule?

m. Define a neutron.

n. What is the quantity and kind of charge on a proton? On an electron?

o. What is the difference between charging by contact and charging by induction?

CHAPTER 3

ELECTROSTATICS

31. Electric Field and Lines of Force

Just as lines of force are used to represent the direction of the magnetic force associated with one or more permanent magnets, they can also be used to represent the direction of the electric force about one or more charged bodies. For example, suppose that a charged body is placed under a piece of glass and some short brush bristles are then sprinkled over the top of the glass. When this is done, the bristles which fall close to the charged body and other bristles in the vicinity of the charged body move somewhat before coming to rest. It can be concluded, therefore, that the charged body is exerting a force on the bristles. Since there is no physical contact between the charge and the bristles, we say that the charge produces *action at a distance*. It is often said that the charge creates a field of force in space. Also, examination of the pattern formed by the bristles on the glass reveals that the bristles aline themselves in definite directions (a fact which leads us to attribute a *direction* to the forces in the electric field). No matter how many times the above experiment is repeated, it will be found that the bristles always arrange themselves into a radial pattern (provided that no other strong electric charges are in the vicinity to distort the electric field). This radial pattern is represented by radial lines of force in A or B of figure 31.

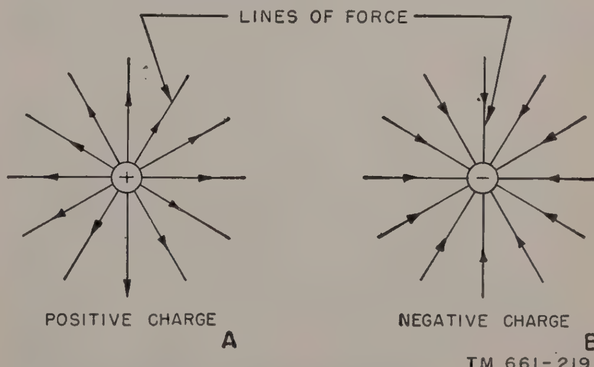
32. Exploring an Electrical Field

How a magnetic field about a magnet can be explored by the use of another small magnet or magnetic compass is shown in chapter 1. In a similar manner, an electric field about a charged body is explored by the use of another charged body known as a *test* charge. *By agreement, a unit positive charge is always used as a test charge.* A method of exploring an electric field with a test charge is as follows: suppose a test charge is carried from *point to point* in the vicinity of some charged body. Then, we find that at every point the test charge is acted on by a force having both *magnitude* and *direction*. That is, as it is carried from point to point, the test charge experiences a force which varies in strength and direction. Thus, the electric field intensity at any point in the field is defined as the force which a unit positive charge would experience if placed at this point. For example, if the unit positive test charge were carried in the vicinity of a negatively charged body, the force on the test charge would be one of attraction (mutual attraction of unlike charges). However, if the test charge were carried in the vicinity of another positive charge, the force acting on the test charge would be one of repulsion (mutual repulsion of like charges).

Note. The choice of a unit positive test charge is purely conventional. Actually, a unit negative charge would serve just as well. In fact, it would be a more logical choice because the electron, which is negative, is the basic unit of charge; positive charge, as previously indicated, is a lack of electrons.

33. Electric Lines of Force

An electric line of force is, by definition, a line which at every one of its points gives the direction of the *resultant* electric force acting on a unit positive charge if placed at this point. In other words, the tangent to the line at any point is the direction of the electric field intensity at that point. A and B of figure 31 are examples of lines of force representing electric fields.



A. Lines of force about a positive point charge.
B. Lines of force about a negative point charge.

Figure 31.

a. Coulomb discovered experimentally that the force of attraction or repulsion between two *point* charges of magnitude or strength q_1 and q_2 separated by a distance d is given by the formula—

$$F = \pm \frac{q_1 q_2}{\epsilon d^2}$$

where ϵ=a constant characterizing the medium in which the charges are located. The plus sign is used if q_1 and q_2 have like charges and the minus sign is used if q_1 and q_2 have unlike charges. By a *point* charge is meant a charge which can be considered to be concentrated at a point. ~~A way~~ trated at a point **which has no dimensions and occupies no space.** However, it must * * * dis- ~~charge would occupy negligible volume and might be treated as a point charge~~. However, it must be realized that a point charge is an idealization which cannot possibly be attained, since the smallest unit of charge is that of the electron which occupies a definite amount of space. Therefore, in view of what has been said, Coulomb's law can be interpreted as giving the force between two charges the physical dimensions of which are small compared with the distance between them.

b. A of figure 31 shows the lines of force which represent the electric field produced by a positive point charge. The field is radial, since the force on a unit positive test charge if placed in the field, would be along the line connecting it to the field charge. Since both charges are positive, the force is one of repulsion, which explains the direction of the arrows on the electric field lines. Since, by definition, the field intensity at any point is the *force* on a unit positive test charge placed at this point, we can find the magnitude of this force by using Coulomb's law and making q_2, the test charge, equal to 1. Thus, the electric field intensity at any point distant d from the point charge q_1 is—

$$F = \pm \frac{q_1 \times 1}{\epsilon d^2}.$$

This equation shows that *the field intensity varies inversely as the square of the distance from charge* q. The letter F is used to denote electric field intensity. It is customary, in mapping the field, to draw more lines in regions of greater intensity; with this convention, regions far away from charges would have a smaller number of lines drawn per unit area than regions closer to the charges. Notice that the field diagram for a negative point charge is essentially that of a positive point charge, except that the arrows are reversed. This follows since the force on the test charge would then be one of attraction.

34. Lines of Force Associated With Two Charged Bodies

In order to picture the field when more than one charge is present, use the principle of superposition.

a. This means that to find the force on a unit positive charge when two charges are present, the following steps are taken:

(1) Find the force caused by charge No. 1 acting alone.

(2) Find the force caused by charge No. 2 acting alone.

(3) Find the resultant by adding these two forces, taking into account the directions of these forces.

b. To grasp this more fully, consider the following illustration: Suppose it were possible to have two point charges of equal magnitude and opposite sign located at the same point (fig. 32). Then at any point in space a unit positive charge would experience two forces, one of repulsion caused by the positive charge and one of attraction caused by the negative charge. Since the superimposed charges have equal magnitudes, the seforces would be equal and thus would have a resultant force equal to zero. This condition is shown in figure 32.

c. Now, let us find by the same method the field due to two charges which are separated. It must be understood that, despite the fact that the test charge will be subjected to two forces, it can only move in response to the *resultant force.* (In the case of the two equal and opposite charges located at the same point (*b* above), the resultant force on the test charge at any point in the field was found to be zero. This means that the test charge will not tend to move but will remain at rest.)

(1) Figure 33 shows the test charge being acted upon by two forces: the force caused by the positive charge and the force caused by the negative charge. Since the test charge can only respond to the resultant force, and since it can only move in one direction, it actually experiences a displacement in the direction of the resultant force. Thus the direction of the electric field at the point

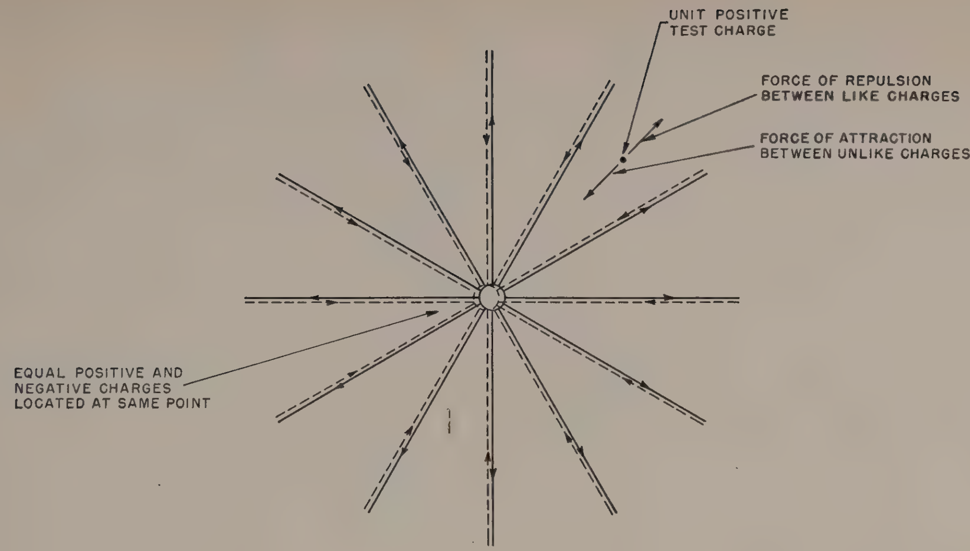


Figure 32. Superposition of unlike charges of equal magnitude.

of the test charge is shown by the direction of the arrow on the resultant force in figure 33.

(2) By using this method, it is possible to determine the direction of the electric force at any point on the field produced by two equal and unlike point charges or two equal and like point charges (A and B of fig. 34); the lines of force represent the direction of the *resultant force* at every point in an electric field. Note that in B of figure 34, the point midway between the charges experiences zero resultant force; a test charge placed at this point would remain at rest. In this respect, B of figure 34 is similar to A of figure 17.

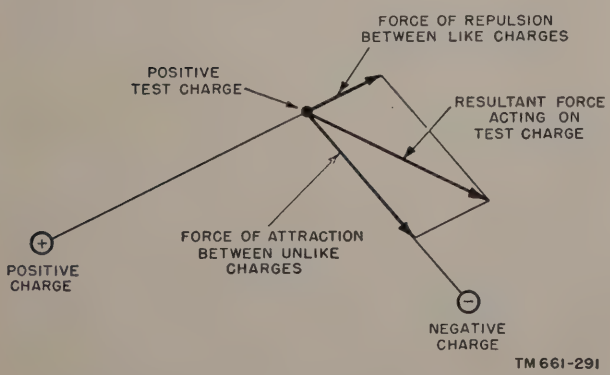
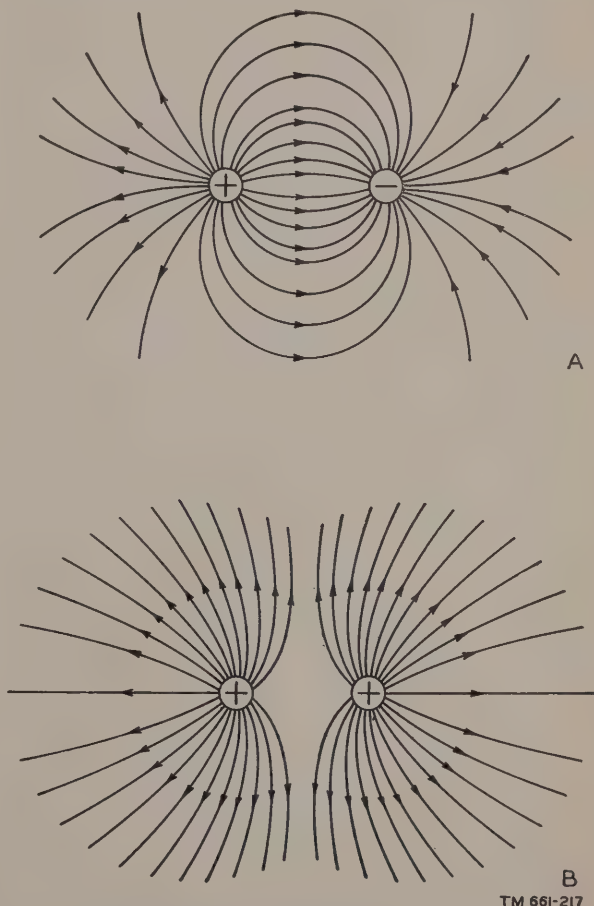


Figure 33. Resultant of forces exerted by unlike charges.



A. Lines of force associated with unlike charges.
B. Lines of force associated with like charges.

Figure 34.

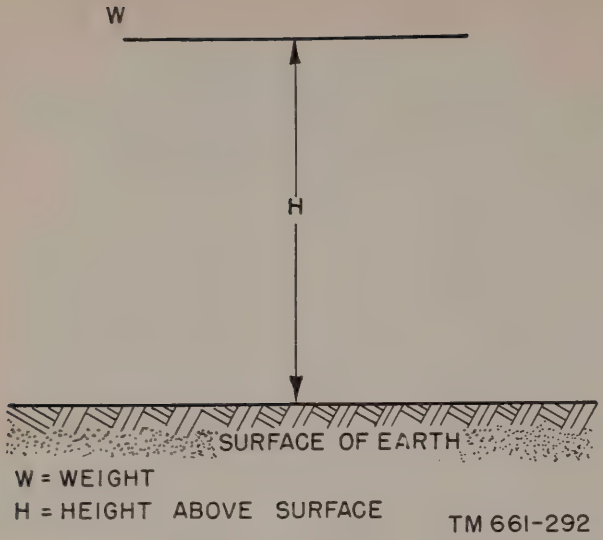


Figure 35. An object raised above the surface of the earth possesses potential energy.

35. Potential Energy in Gravitational Field

Before discussing the concept of potential and potential energy in an electrical field, let us look at conditions in the gravitational field produced by the attraction of the earth (fig. 35).

a. From everyday experience, we know that in order to raise a weight above the surface of the earth, a force must be exerted. That is, the earth produces a force on every piece of matter in the universe, tending to pull objects toward its center. This force is called gravity. Thus, to raise an object against the force of gravity, work must be performed. (For an exact definition of work, see appendix III.)

b. Although work must be done in raising a weight, it must not be thought that this work is lost. On the contrary, if the weight were allowed to drop, it could turn a wheel or a generator or even drive a nail into a piece of wood. All of this represents work performed. Thus, the work done in raising the weight is all returned in allowing it to drop. We shall learn that the electric field, from this point of view, is like the gravitational field.

c. The higher a body is raised above the earth's surface, the more work is required and, therefore, the greater will be the energy returned in allowing the body to drop. We say that objects at greater heights have greater *potential energy*, the word potential being used to signify the possibility of getting work done. Therefore, points at different heights have different potentials. It is said that a *difference of potential* exists between points which are in different energy levels.

36. Potential in an Electric Field

a. Suppose that an electric field is being created by a point-positive charge of strength or magnitude q (fig. 36). Then, as we have seen, a unit positive test charge will experience a force of repulsion no matter where it is placed in the field. Let us assume that our test charge is initially at point A in the field and that it is desired to move it to point B (fig. 36). Since the repelling force on the test charge is opposite in direction to the path of desired travel AB, it is obvious that a force equal and opposite to the electric field intensity must be exerted in order to move the unit positive charge from A to B. In other words, the electric field intensity tends to move the test charge in a direction opposite to the direction AB. Thus, in order to move the unit positive charge from A to B, *work must be done* by the agency moving the test charge to overcome the force of repulsion produced by the field of the charge q. Of course, it would not be necessary to do any work in moving the test charge from A to D, since the work required to move the charge toward D would be provided by the electric field.

b. From these considerations, we are led to the following definition: *The potential at any selected point in an electric field is the work done in moving a unit-positive point charge from any fixed point, which can be called point* Q, *to the selected point.* Once the fixed point Q has been chosen, it must remain the same for the evaluation of the potential at every selected point. Q, our fixed reference point, can be assigned any arbitrary value of potential. The definition given above is the one used in finding the potential of a point in *any* electric field regardless of how the field is created. Thus, the field may be that created by a point charge, by a number of like or unlike charges, by a system of charged conductors, etc. The reader may now raise the valid objection that since the location of point Q is arbitrary, different locations of Q will lead to different values of potential at the same *point* in the field, since the work done in taking our test charge from point Q to the field point in question will depend on the location of point Q. For example, if in figure 36 our test charge moved from A' to B, instead of from A to B, less work would be done. The explanation for this will be given in the next subparagraph in which it will be shown that

one is never interested in the *absolute potential* of a point as given by the definition, but merely in the *difference of potential* between any two points.

c. The dimension of potential is $\frac{\text{work}}{\text{unit charge}}$. The definition of dimension is given in appendix I. Thus, if the potential at a selected point in our field is expressed as V, the work done in taking a point charge of magnitude q from our reference point to this selected point is

$$V \times q \left(\frac{\text{work}}{\text{unit charge}} \times \text{charge} \right).$$

This represents the potential energy of the charge q *at the selected point.* If the charge q is negative, the product $V \times q$ will be negative. This means that the field has acutally done work in moving q from the reference point to the selected point. It has been stated that the definition given for potential is dependent on the location of the reference point. Suppose, however, that two different points of reference, X and Y, are chosen (instead of a single point Q), and the potential at a point B in the field is calculated first using X and then Y as the reference (fig. 37). The potential at B using Y as a reference is the work done on a test charge in moving it from Y to B. The potential at B using X as a reference is the work done in moving the test charge from X to B. However, we can get from X to B by first going from X to Y and then to B. Thus, the work done on a unit positive charge in going from X to B is the work done in going from Y to B plus the work done in going from X to Y. Therefore, the potential at B using X as a reference is equal to the potential at B using Y as a reference plus the work done in going from X to Y. From this we see that if the potentials are calculated at a point B using two different reference points, the results will differ by a constant amount. As stated before, what is important is the *difference of potential* between two field points.

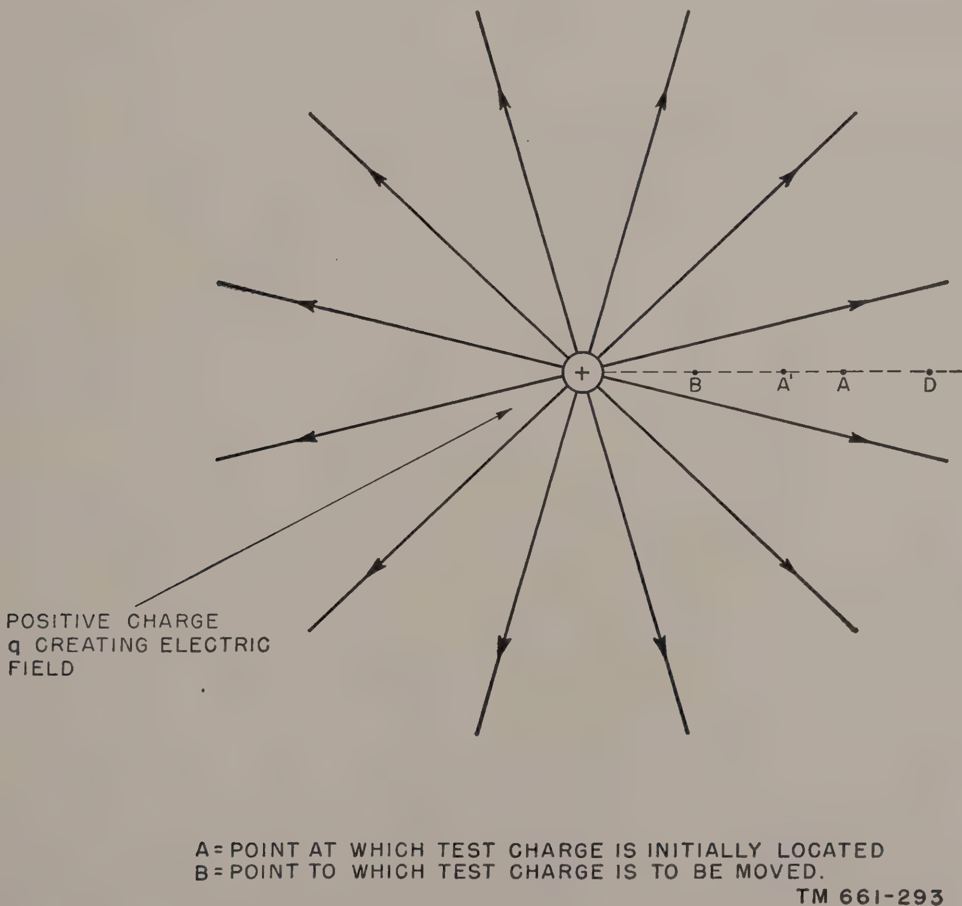


Figure 36. Potentials at points in an electric field.

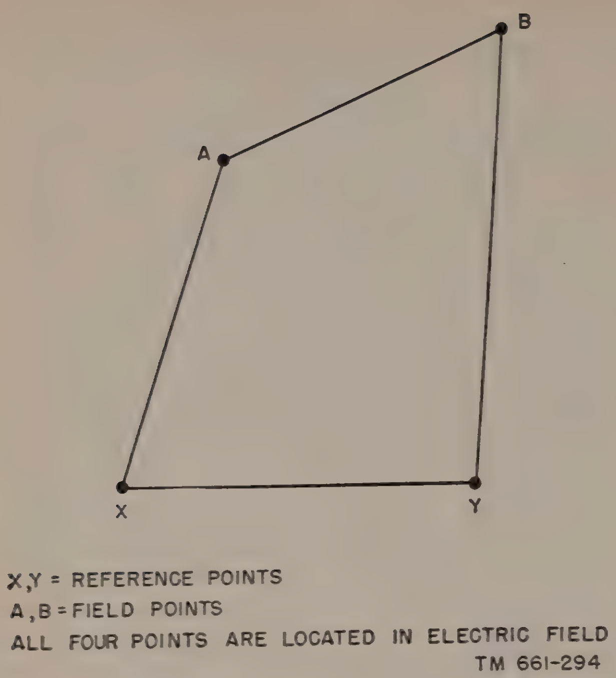


Figure 37. The work done in moving a test charge from point A to B equals the potential at B less the potential at A, regardless of whether point X or point Y is used as a reference point.

d. The rise in potential in going from point A to point B in the field is the work done in taking a unit positive point charge from point A to point B. Thus, the potential at B less the potential at A is the work done in taking the test charge from A to B. This is illustrated in figure 37. Let us first use X as our reference point; by definition V_{BX}=potential at B=work done in taking a test charge from X to B. However, the work done in going from X to B is the same as the work done in going from X to A plus the work done in going from A to B. But the work done in taking our test charge from X to A, is by definition, the potential at A using X as reference. Thus, it can be stated that the work done in taking a unit positive charge from A to B is the potential at B less the potential at A or the potential rise in going from A to B. Of course, both are calculated using the same reference point X. The question now arises, would the work done in going from A to B with our test charge be the same if all potentials were calculated using Y as a reference? As we have seen before, the potential at B using Y as reference is the potential at B using X as reference plus a constant. The same holds for the potential at A using Y as reference. Therefore, when the potential at A is subtracted from the potential at B, the consant term drops out leaving us with the same result as before. Symbolically, this can be written as—

$$V_{BY}=V_{BX}+\text{Constant}$$
$$V_{AY}=V_{AX}+\text{Constant}$$
$$V_{BY}-V_{AY}=-V_{BX}-V_{AX}+\text{Constant}-\text{Constant}=V_{BX}-V_{AX}$$

where V_{BY}=potential at B using Y as a reference,
V_{BX}=potential at B using X as a reference,
V_{AX}=potential at A using X as a reference,
and
V_{AY}=potential at A using Y as a reference.

This important result can be stated as: *the work done in moving a test charge from point* A *to point* B *in a field is the potential of* B *less the potential at* A *regardless of what reference point is used to calculate these potentials.* That the result should be the same is physically clear, since the work done in moving a unit point charge from one point to another in the field has a fixed value depending only on the field. We see that if B is at a higher potential than A, *outside work* must be done in moving a positive charge from A to B. In other words, in going from A to B, forces of repulsion act on our charge tending to prevent motion. Thus, if a positive charge were placed at B, it would move from B to A under the influence of the field. In other words, *a positive charge tends to move from points of higher potential to points of lower potential.* Obviously, for negative charges, our results are reversed. *A negative charge tends to move from points of lower potential to points of higher potential.* Since at a point of lower potential, a negative charge has a *potential energy* which is greater than that at a point of higher potential (the product $V\times q$ is negative), a negative charge tends to move from points of higher potential *energy* to points of lower potential *energy*. This is so, of course, since the force exerted on a negative charge by the field is opposite to that exerted on a positive charge.

e. A difference of potential exists between the terminals of a battery. (This will be discussed in chapter 5.) For this reason, if a battery were connected as shown in figure 38, electrons in the wire would be repelled from the negative terminal and attracted to the positive terminal. As a result, a movement or flow of electrons through the wire would take place. In practice, it is customary to use the words *potential* and *voltage* interchangeably. So, for the two plates A and B, one can speak of either the voltage rise in going from A to

B or the voltage drop in going from *B* to *A*. Consider the following example: Plate *A* is at a potential of 2 volts and plate *B* is at a potential of 3 volts (the volt is the unit by which potential is measured, as will be explained later). The rise in potential in going from *A* to *B* is $3-2=1$ volt. The drop in potential in going from *A* to *B* is $2-3=-1$ volt. (The minus sign indicates that *B* is at a higher potential than *A*.) The potential rise in going from *B* to *A* is $2-3=-1$ volt. (There is actually a drop in potential in going from *B* to *A*.) The potential drop in going from *B* to *A* is $3-2=1$ volt. Thus we see that for any two points *A* and *B*, the potential rise in going from *A* to *B* is the same as the drop of potential in going from *B* to *A*. Either of these quantities may be negative, as shown by the example.

f. Wherever the word *potential* is used, the term *voltage* can be substituted. As mentioned previously, potential or voltage is measured in *volts*. Thus a volt is essentially work per unit charge. When we say that the voltage of a battery is 100 volts, we mean that the voltage or potential rise in going from the negative to the positive terminal is 100 volts. The negative sign that is used in battery diagrams does not mean that the negative plate is actually at a *negative potential* but merely that with respect to *any* reference point, it is at a *lower potential* than the plate marked *plus*. The term *ground* is very often used in electricity to mean a *reference point*. Thus, in a radio set, component parts are connected to the metal chassis which, in turn, is connected to ground. In this case, the chassis is the reference point. In trolley systems with overhead wires, the rails are connected to the earth or gound to prevent any injury to a person stepping on the rails.

37. Summary

a. Coulomb's law, $F=\pm\frac{q_1q_2}{\epsilon d^2}$, expresses the fact that the force of attraction or repulsion between *two point* charges varies directly as the product of the magnitudes of the charges and inversely as the square of the distance between the charges. The constant ϵ characterizes the medium in which the charges are located.

b. Surrounding any system of charges, there is an electric field.

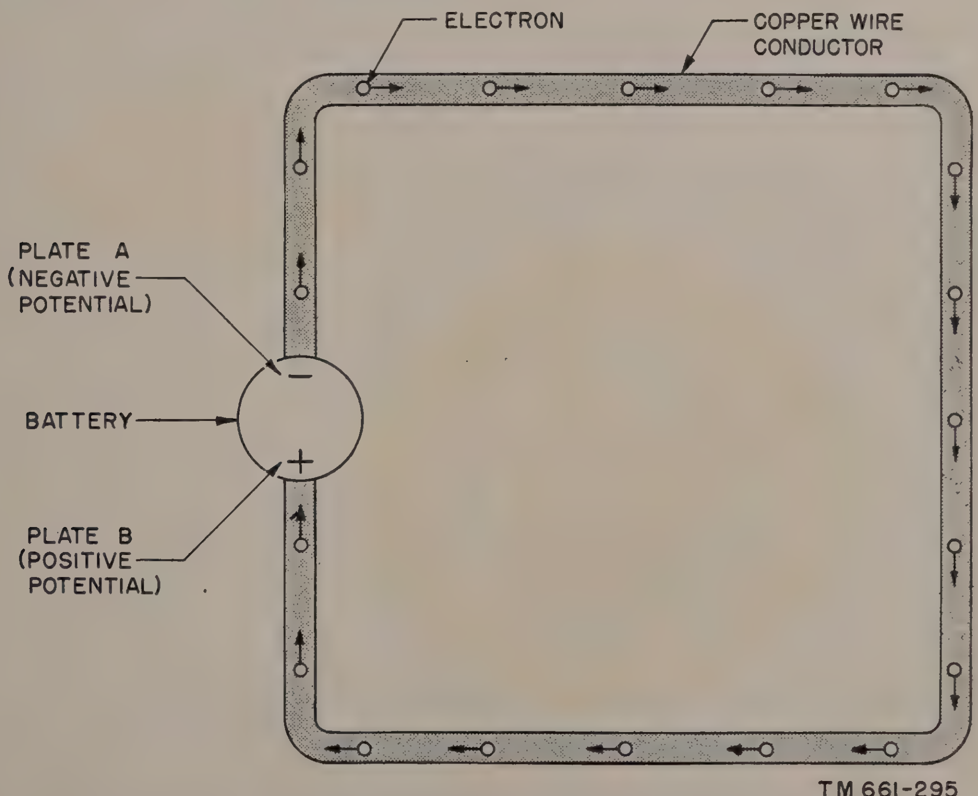


Figure 38. As a result of chemical action in the battery, plate B is higher in potential than plate A, and electrons flow through the wire from A to B.

c. The electric field intensity at any point in space is defined as a force exerted on a unit positive point charge placed at this point by the system of charges producing the field. A line of force is a line which at every one of its points shows the direction of the electric field intensity.

d. The potential at any point in an electric field is defined to be the work done in moving a unit positive point charge from any fixed reference point to this point.

e. The potential rise in going from any point A to any point B is the work done in taking a unit positive point charge from A to B. This work is the potential of B minus the potential of A regardless of what reference point is used to evaluate the potentials of A and B.

f. The amount of work that is done in bringing a charge to a certain point in the field can be regained by allowing the charge to move freely under the natural action of the field. The amount of work returned is exactly equal to the amount of work expended.

g. "Potential" and "voltage" are terms which are used interchangeably. Potential has the dimension work-per-unit charge and is measured in volts.

h. Ground is a term used to designate a point of reference and can be arbitrary. However, practical considerations are necessary in choosing it.

38. Review Questions

a. What is an electric field?

b. How is it produced?

c. Is Coulomb's law valid for any two charged bodies?

d. How is electric field intensity defined?

e. What is a line of force?

f. Can two lines of force cross themselves? Why?

g. What does the word potential imply?

h. What is the definition of the potential at a point in a field?

i. What in the definition of potential is arbitrary?

j. What is meant by the rise in potential from any point A to any point B?

k. Is there anything arbitrary in the definition of potential difference? Explain.

l. What is the dimension of potential and what is the practical unit used to measure it?

m. Draw an analogy between the electric field and the gravitational field of the earth.

CHAPTER 4

CONDUCTORS AND INSULATORS

39. Free Electrons

a. General. The electron theory (ch. 2) states that all matter is composed of atoms, and atoms are composed of subatomic particles called protons, electrons, and neutrons. In addition, all the moving electrons are tightly bound to the nucleus except those that revolve in the outer orbit. Figure 27 shows how the electrons in the outer orbits of the aluminum atoms may move constantly from one atom to another in a haphazard manner. Electrons that are able to move in this fashion are known as *free electrons*. Furthermore all matter is made up of positive and negative charges of electricity, and the atomic structure of a material determines whether or not the material will have many or few free electrons.

b. Electron Flow or Current. If the haphazard flow of free electrons in a material is controlled so that they move in the same direction simultaneously, we have an electron flow. This electron flow is known as an electric current and will be discussed more fully in chapter 5.

40. Conductors and Insulators

In general, all materials may be divided into two major categories, conductors and insulators. These categories are based on their ability to allow an electric current to flow. This, in turn, depends on their atomic structure.

a. Conductors. A good conductor is a material that has a *large number of free electrons*. All metals are conductors of electricity to some extent, but some are much better conductors than others. Examples of these conductors are silver, copper, and aluminum. Silver is a better conductor than copper, but copper is more widely used because it is less expensive than silver. Aluminum is used as a conductor where weight is a major consideration, such as cross-country high-tension lines with long spans between supports. Usually these are stranded cables and have a small steel wire core to provide the necessary tensile strength. The ability of a material to conduct electricity also depends on its dimensions. Conductors may be in the form of bars, tubes, or sheets; but the most common conductors are in the form of wire. We are all familiar with the telephone and power lines which cover the countryside. These lines are made of wire and conduct electricity from one place to another. They bring the electric power into our homes to operate the lights and other necessary electrical equipment. Many sizes of wire, from the fine hair-like wire used in the coils of sensitive measuring instruments, to the large bus-bar sizes used for carrying high currents in electric power-generating plants, are in use. Usually the ability of a conductor to carry electricity will vary directly with the area of its cross section. This is due to the fact that more atoms are present and therefore more free electrons are available. Stranded wire is used when flexibility is necessary, such as on the cords of lamps, electric irons, and toasters. In order to make wire easier to handle and also less subject to changes in weather and other external conditions, it is often covered with some other material such as rubber, cotton, plastic, or enamel. These coverings provide protection against short circuits and leakage and are known as insulators. Insulators will be discussed at length in *b* below.

b. Insulators and Dielectrics. An insulator is a material, or combination of materials, the atomic structure of which is such as to preclude practically any movement of electrons from atom to atom. In other words *an insulator is a material that has few free electrons*. No material known is a perfect insulator through which electricity cannot be *forced*, but there are materials which are such poor conductors that for all practical purposes they are classed as insulators. Porcelain, glass, air, dry wood, rubber, and oil also are insulating materials. One of the first things that we learn about electricity is that unless we are very careful when we handle it, we may have the unpleasant and sometimes dangerous experience of receiving a shock. In-

sulating materials are used so that electrical equipment can be handled safely. Also, insulators are used to prevent leakage and power losses Everyone has handled a common electric light bulb (incandescent lamp) and knows what it looks like (fig. 39). Conducting and insulating materials are used in their construction. The electric current which heats the fine wire, called the filament, must flow through a series of conductors connected to the generating plant. If we actually touched the filament while it is carrying current, that is when the bulb is lighted, we would get a shock or a burn. But we know we can insert and remove the bulb from the socket without injury by touching only the glass; therefore the glass, even though *it* is directly in contact with the current-carrying filament, is not carrying an electric current; hence it must be an insulator. The brass threads and other metal parts on the base of the bulb are the contact points with the line and therefore must be conductors.

41. Summary

a. The positive charges of electricity are called protons.

b. The negative charges of electricity are called electrons.

c. All matter is made up of positive and negative charges of electricity.

d. Electrons capable of movement from one atom to another are known as free electrons.

e. A conductor is a material that has many free electrons.

f. An insulator is a material that has few free electrons.

g. Examples of good conductors are silver, copper, and aluminum.

h. Examples of good insulators are rubber, glass, porcelain, and oil.

i. There is no such thing as a *perfect* conductor or a *perfect* insulator.

42. Review Questions

a. What is an electron? Define a *free* electron.

b. Define an electric current.

c. Define a conductor. Give examples of good conductors.

d. Define an insulator. Give examples of good insulators.

e. Why is copper used for most conductors?

f. Give two reasons for using insulation.

g. What is an atom?

h. What is the electric charge on a proton? On a neutron?

i. When is aluminum used as a conductor?

j. Conductors are used in various forms. Name several. What is the most common form?

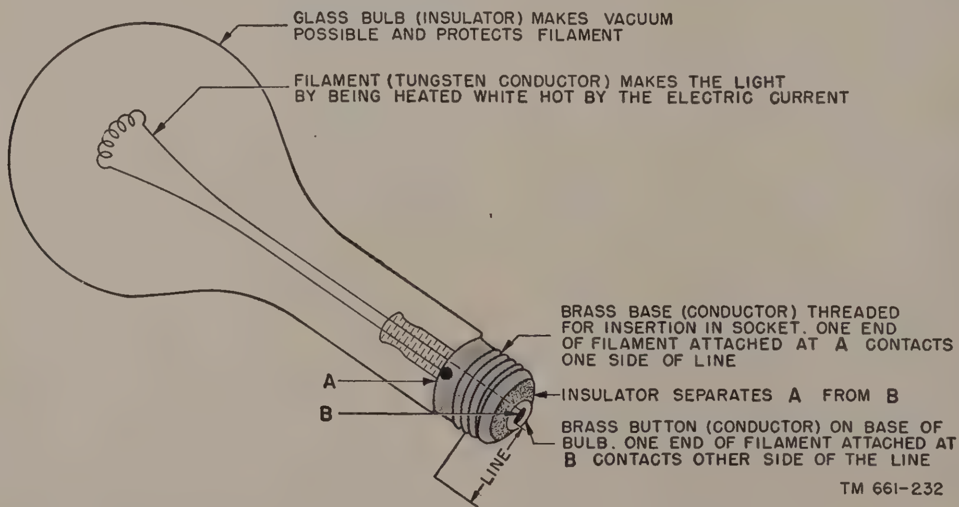


Figure 39. Incandescent light bulb showing insulators and conductors.

CHAPTER 5

CURRENT, VOLTAGE, AND RESISTANCE

43. Electric Circuit

There are three fundamental factors present in every *electric circuit; current, voltage,* and *resistance.* Thus, it is important that a precise explanation of each of these factors be given.

44. Current

The word current means running or flowing, and an electric current means a flow of electrons. As explained in chapter 3, negative charges tend to move from points of lower potential to points of higher potential. Also, because the charge on the electron is negative, *electrons tend to move from points of lower potential to points of higher potential.* To illustrate this principle, consider the following example:

a. In figure 40, *E* represents a 6-volt battery which is a device for creating a potential difference. Plate *B* has, because of the chemical action inside the battery, a surplus of electrons which makes it negatively charged, and plate *A* has a deficiency of electrons which makes it positively charged. As a result of these charges, a potential difference of 6 volts exists between plates *A* and *B*. Thus, if a positive test charge were to be moved from *B* to *A*, work would have to be done to overcome the repelling force on plate *A* (like charges repel each other).

b. When the switch shown in figure 40 is closed, point *C* is connected to plate *A* and point *D* is connected to plate *B*. Therefore, there will be a potential rise of 6 volts in going from *D* to *C*. Electrons will flow through the wire *DFC* in the direction shown (the wire is a source of free electrons). It might be thought that the positive charge on plate *A* would be neutralized as electrons arrived at this plate and that the negative charge on plate *B* would be neutralized as electrons left this plate. However, this does not happen because the battery continues to remove electrons from plate *A* and deposit them on plate *B* as long as the chemical action of the battery continues. As a result, electrons continue to flow in the wire or external circuit.

c. The example given above shows that *a closed circuit is required for a flow of electrons.* (Before the switch was closed, *A* and *B* were at different potentials but there was no flow of electrons.) Summarizing, we can say that a steady flow of electrons in a circuit requires a closed path, but that a difference of potential can exist without electron flow.

45. Definition of Current

In order to derive an exact definition of current, it is desirable to first understand the definition of current density.

a. In figure 41, the battery *E* maintains the faces (*MN*) of the conductor at a potential difference. Through the conductor, electron flow will

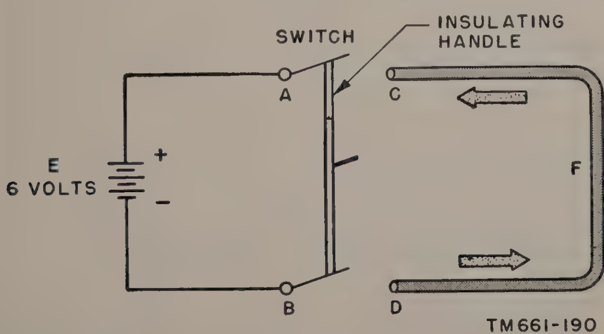


Figure 40. Current flows through circuit when switch is closed.

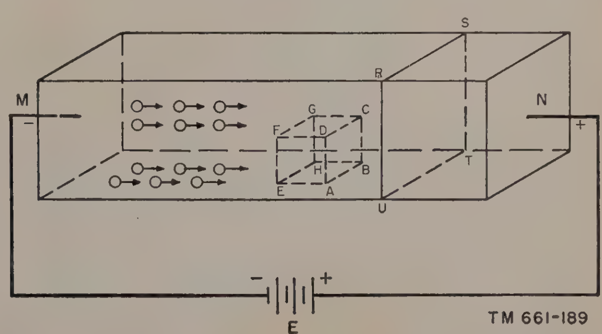


Figure 41. A total charge of one coulomb flowing past a cross section of a conductor in one second is said to be a current of one ampere.

be from face M to face N. (Electrons are assumed to be distributed throughout the conductor although only a few are shown.)

(1) Suppose a cube having faces of 1 square meter is chosen inside this conductor. The face $ABCD$ is perpendicular to the direction of electron flow, and is equal to a square meter cross section of the conductor. Let us compute the number of electrons which cross this section $ABCD$ in 1 second, assuming the electrons move at a drift velocity of 1 meter (39.37 inches) per second in the direction shown. It is obvious that all the free electrons inside the cube will cross the area $ABCD$ in 1 second, since each will have traveled a distance of 1 meter. Note that an electron in the conductor more than 1 meter distant from the face $ABCD$ cannot reach this face in 1 second.

(2) If the drift velocity of the electrons is 2 meters per second, the edges FD, GC, HB, and EA will have to be lengthened to 2 meters, and if the drift velocity is v meters, these edges will become v meters long. Since the area of $ABCD$ is 1 $(\text{meter})^2$ and the longitudinal side is v meters (where v is the drift velocity of the electrons), the volume of the figure is $v(\text{meters}) \times l(\text{meter})^? = v(\text{meters})^3$. Thus all the electrons inside this chosen volume will cross area $ABCD$ in 1 second. To find the number of these electrons, a knowledge of the number of electrons in a cubic meter is necessary; the volume of a cubic meter is 1 $(\text{meter})^3$. Suppose the number of electrons in a cubic meter is ρ. Then the number of electrons in v cubic meters is:

$$\rho \times v = \frac{\textit{number of electrons}}{1\ (\textit{meter})^3} \times v(\textit{meter})^3$$

$$= \text{electrons in } v(\textit{meter})^3.$$

Because each electron carries a charge of e coulombs, where $e = 16 \times 10^{-20}$, the amount of charge crossing area $ABCD$ in one second is equal to the number of electrons in $(ABCDEFGH) \times$ charge per electron. As a formula, this can be written: $i = \rho ve$

where i = *current density* = charge in coulombs crossing area $1(\textit{meter})^2$ in one second,
ρ = number of electrons in a cubic meter = charge density,
v = drift velocity of electrons in meters per second, and
e = charge of the electron in coulombs = 16×10^{-20} coulombs.

A coulomb per second is, by definition, an ampere. Thus the current density equals

$$\frac{\textit{coulombs}}{\textit{second} \times (1\ \textit{meter})^2} = \frac{\textit{amperes}}{(\textit{meter})^2}$$

b. The current density of a conductor is said to be 1 ampere per meter² when one coulomb of electricity crosses an area of 1 square meter (placed at right angles to the direction of electron flow) in one second. To find the total charge crossing an area such as $RSTU$, the current density at this face must be multiplied by the area of the face ($RSTU$). The total charge moving across any cross section of the conductor in one second is said to be the current flowing through the conductor. *A total charge of one coulomb moving past a cross section of the conductor in one second is said to be a current of 1 ampere.*

c. It might appear, at first, that if the cross section of a conductor varies, the current should also vary from point to point. This condition is shown in figure 42. Since the cross section at A is less than that at B, it might be concluded that the total charge crossing A in one second is different from that at B. But since the ammeter

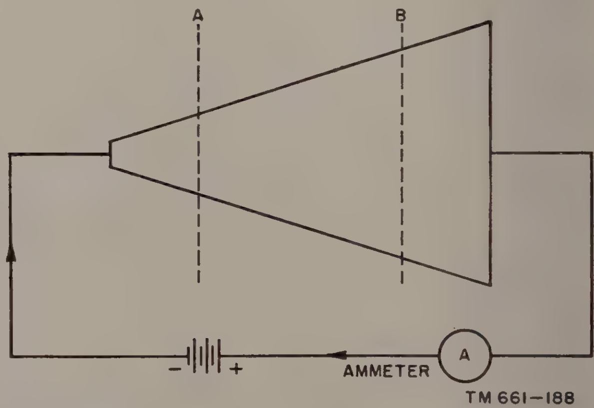


Figure 42. The same amount of current flows through cross sections A and B of this conical-shaped conductor.

A (current-indicating device) indicates a constant current, this cannot be true. The explanation is simple. The *current density* at *A* is larger than that at *B* so that, since current is the product of area and current density, the two results are the same.

46. Electron Theory of Conduction

As has already been explained, the modern view of electricity regards a current as a flow of electric charges. In a solid, some of the electrons are supposed to be permanently bound to particular atoms or molecules, while others, spoken of as free electrons, move about the interior of the solid, continually having their courses changed by collisions with other electrons and with molecules. It is very probable that the unrestricted motion of the free electrons explains the phenomenon of electric conduction.

a. Even when no emf (electromotive force) is applied to a conductor, the free electrons move about through the conductor; but they move at random in all directions. Each electron produces an electric field, and when no emf is applied, the fields produced by the random electron motions cancel, no net electron motion results, and no current flows. If an emf is applied to a conductor (by connecting a battery to the ends of the conductor), the motion of each electron is the resultant of its random motion and the motion produced by the applied emf. As a result, the electrons as a whole are driven through the conductor by the continued action of the applied emf. If it were not for their collisions with the molecules of the conductor, the electrons could gain indefinitely in velocity under the action of the applied electric force, but the effect of collisions is continually to check this growth of velocity. It is known that the random velocity of an electron is approximately 5.93×10^{7} cm/sec, or about 370 miles/sec. Despite this tremendous velocity, the *drift velocity* of the electrons corresponding to a current of 1 ampere is only $\frac{\text{cm}}{\text{sec}}$, or .428 inch/sec, or 6.76×10^{-6} miles/sec.

b. Now, if the electrons have such a relatively small velocity in the direction of the applied force, how can electrical energy supplied at the generating stations arrive at the receiving plants located many miles away in practically negligible time? For example, if the receiving plant is located twenty miles away, then, from the above figures, an electron would take 341 days to arrive at the receiving end from the sending end. Clearly, it cannot be the electron which moves down the wire at the speed of light, which is the speed at which electric transmission takes place. As a first step toward an explanation of this paradox, consider the following experiment:

(1) Suppose that the test charge of figure 43 is at a distance from an initially un-

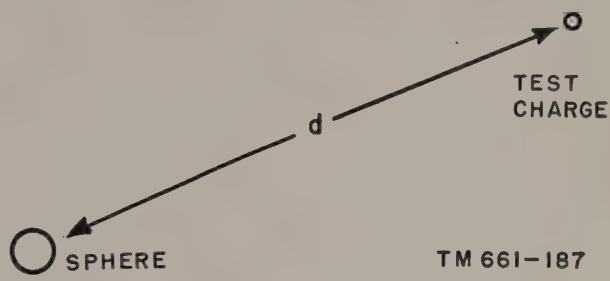


Figure 43. Test charge located at a distance d from a charged sphere.

charged sphere, and that no other charges are present, Under these conditions, the test charge will experience no force.

(2) If, by some means, the sphere is then charged to a value q, we might think that the test charge would feel a force the instant the sphere is charged. However, this is not found to be the case. The test charge will experience a force after a certain interval of time has elapsed; *this interval is proportional to the distance of the test charge from the sphere.* For a distance of 100 miles, the time lag is found to be 5.37×10^{-4} seconds, which is extremely small and consequently very difficult to detect in an experiment. *Thus it appears that the force in an electric field takes a definite amount of time to reach any point in space.*

c. Now consider the example of applying a voltage difference across a conductor:
In figure 44, a battery of *E* volts is connected to a DPST (double-pole, single-throw) switch. As was stated previously, as soon as the switch is closed points *C* and *D* assume the potentials of points *A* and *B*, respectively. However, in order for this to happen, point *C* must become positively charged and point *D* negatively charged. This means that electrons must flow from *C* to the plus terminal giving a deficit of electrons at *C* and electrons must flow into point *D* from the

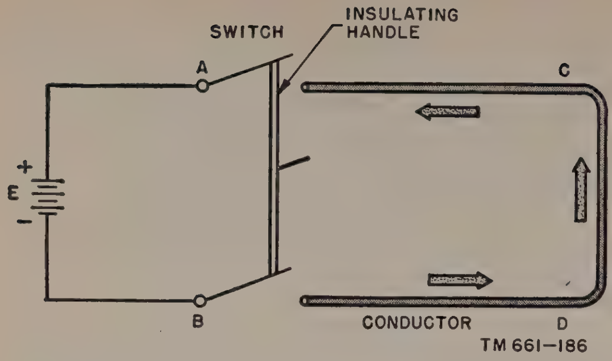


Figure 44. When switch is closed, point C *becomes positively charged and point* B *becomes negatively charged.*

negative terminal giving a surplus of electrons at *D*. This amounts to saying that the free electrons in the conductor must move in the direction *DC*. *But a charge will only move if it experiences a force.* To understand how this force arrives at the conductor terminals, refer to figure 45. In this diagram, the battery is shown disconnected (switch open). Suppose the switch is closed; the electrons at *A* being closer to the positively charged face move from right to left. Momentarily this leaves a deficit of negative charge at *A*, and electrons from *B* move in the direction shown to neutralize this charge. However, as soon as this occurs, point *B* becomes positively charged and electrons to the right of *B* rush to the left. This effect is propagated down the wire until electrons at *C* experience a force to the left. Obviously the same reaction occurs in the bottom conductor except that electrons are repelled from section *A′*, etc., until electrons at *C′* experience a force moving them from left to right. It is in this way that the field of the battery is felt at the load (receiving conductor). *Thus, despite the fact that the electrons themselves move very slowly, the effect of the changes in positions of the electrons is propagated down the wires connecting the source to the load almost instantaneously.* Actually, the effect of changes in the electric field is propagated at approximately the speed of light, which is 186,000 miles/second.

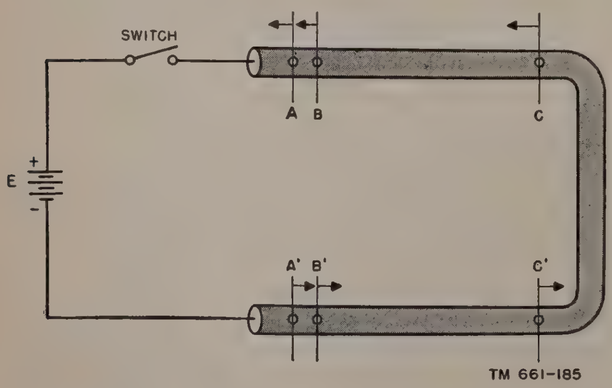


Figure 45. Although movement of electrons in a conductor is relatively slow, effects of change in electric field travels down conductor at approximately the speed of light.

47. Electric Current

Before the discovery of the electron, current flow was thought to be a motion of positive charge from points of higher potential to points of lower potential. However, with the discovery of the electron, this supposition has been discarded. *Current flow is a movement of free electrons from points of lower potential to points of higher potential.*

a. Types of Currents.

(1) A dc (*direct current*) is one whose magnitude and direction remains the same with time. There are some uses for which dc only is suitable, such as battery charging, electroplating, operation of dc motors, and certain parts of radio, telephone, and telegraph systems.

(2) A *unidirectional current* is one whose direction is fixed with time but whose magnitude may vary.

(3) A current which may reverse its direction with time is said to be *nonunidirectional.* Ac (alternating current) is one form of nonunidirectional current.

b. Evidence of Electric Current. The flow of electrons (electric current) makes itself evident to the average person in one of four ways; *the transmission of power, the production of heat, magnetism,* and *chemical action.*

(1) Current flowing through a transmission line carries electrical energy from distant power plants to consumers instantly, silently, and efficiently, in any quantity desired. It is the most economical method for transporting power ever devised, and without it we could not utilize the vast amounts of power produced at our hydroelectric dams.

(2) Current always produces heat when it flows through a conductor. The amount of heat produced depends on the material of the conductor and on the amount of current flowing. For example, electric irons and toasters must have heating elements that will produce enough heat

to be practical. The light produced by an electric bulb is caused by the current flowing through the thread-like conductor called the filament. This filament must be heated so that it glows. However, the conducting wires that carry the current to the filament must not become hot enough to glow.

(3) Current produces magnetism when it flows through a wire. This is a very important effect, for it is the basis for millions of electric machines such as generators, motors, and electromagnets (ch. 10). Without this effect, there is no known way to generate electricity cheaply or to convert it into mechanical energy for the purpose of performing work.

(4) Current produces chemical action when it flows through a liquid. Examples of this effect are the charging of a storage battery, the electroplating process, and the separation of precious metals from their ores. Electric shock is the unpleasant and sometimes dangerous sensation of a direct application of voltage to the human body. The effect of current flow on the body cells is chemical. We often speak of voltage as the cause of shock but the fact is that current really does the damage. The pain and violent muscular contraction are due to the effect of current on the nerve centers and on the nerves themselves, which are the best conductors in the body and also the parts most seriously injured.

c. EXAMPLES OF CURRENT FLOW. An idea of the value of current in terms of an ampere can be gained if we think of it in connection with some of the electrical devices with which we are familiar. Roughly speaking, a 100-watt electric light bulb requires about 1 ampere of current. Naturally, a 50-watt bulb would require about ½ ampere. A 10-horsepower motor requires about 75 amperes. An electric flat iron requires about 5 amperes. The current flowing in a radio circuit may be even less than 1/1000 of an ampere, or over an ampere. In starting the motor of an automobile, we may use 200 to 300 amperes, which is high compared with other devices. When we speak of small or low current, we are using a relative term. A home which uses the average number of electric lights and small motors to run electrical devices, such as washing machines and sweepers, usually has wires that will safely carry about 15 amperes.

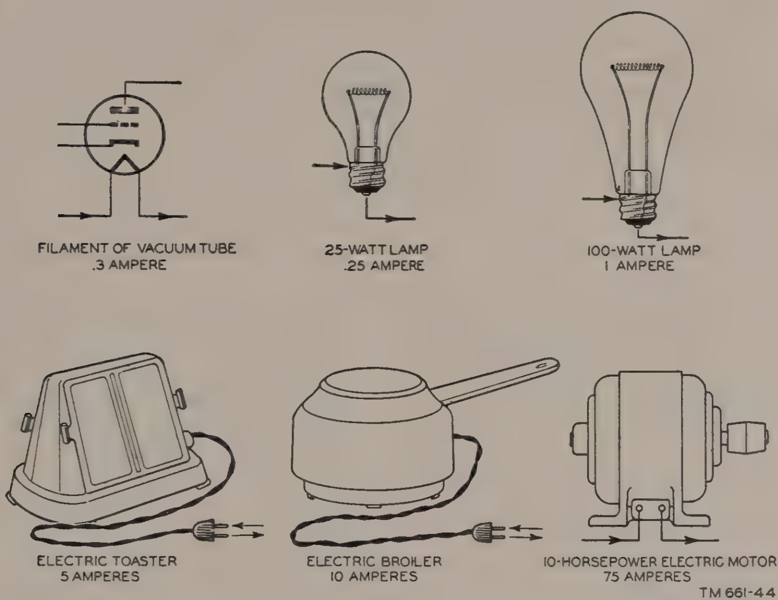


Figure 46. Examples of current requirements for common electrical devices.

48. Measurement of Current

a. Current is measured with an *ammeter*. Since the construction of this instrument is beyond the scope of this manual, only the method of using the ammeter in a circuit will be given (fig. 47).

b. A of figure 47 shows the proper way of connecting the ammeter. The instrument is put in series with the load, so that the total load current flows through it. Notice that the positive terminal of the ammeter is connected to the positive terminal of the battery. If the connections were reversed, the meter would deflect downscale, and

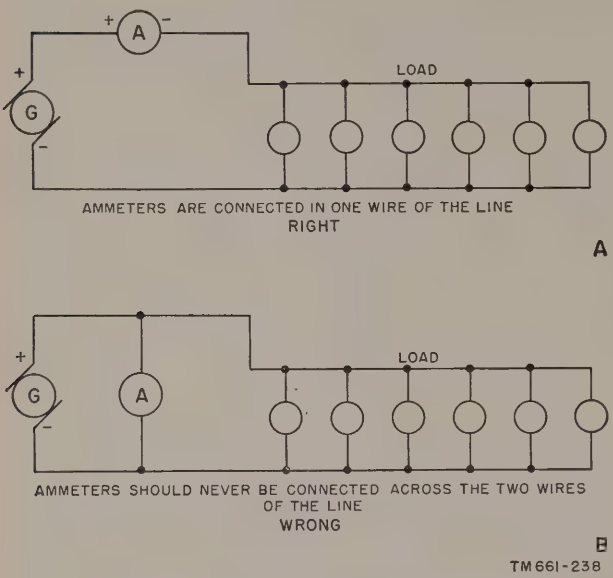


Figure 47. Correct and incorrect usage of an ammeter.

the initial impact of the needle with the lower side of the case might bend it considerably.

c. B of figure 47 shows the wrong way of using the ammeter. Notice that with this circuit the ammeter is directly across the battery terminals. *The meter is not designed to withstand even moderately high voltages. Consequently connecting an ammeter across the line will usually burn out the meter movement.*

49. Measurement of Emf (Voltage)

As we have seen, in order to set an electric charge into motion, an electric field is required. In other words, some method of creating a potential difference between the ends of a conductor is required. The most common methods of attaining potential differences are by the use of batteries and generators. The voltage difference which exists between the two plates of a battery when no current is being delivered is called the *emf of the battery*. It will be explained in chapters 7 and 9 that when a battery is connected into a circuit to produce current, the voltage difference between its output terminals will decrease. Therefore, the full battery voltage will not be available to force current through the load. An instrument for measuring voltage difference is the *voltmeter*.

a. A of figure 48 illustrates the correct way of using the voltmeter to read the load voltage. Note that the voltmeter is connected directly across the load, so that the full load voltage is impressed across it.

b. B of figure 48 shows the incorrect way of using the voltmeter. In this hook-up, the full load current flows through the instrument. *The voltmeter coil is not designed to stand appreciable current.* As a matter of fact, most voltmeters used in power work have coils which are designed to take currents of the order of ma (milliamperes). *A voltmeter is connected directly across the load in order to measure its voltage.* Proper polarity must be used when connecting a voltmeter.

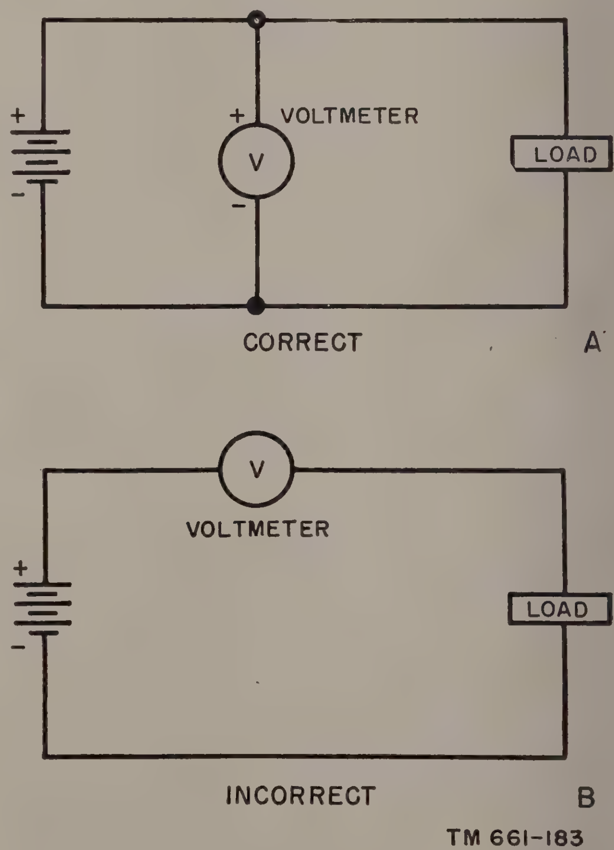


Figure 48. Correct and incorrect usage of a voltmeter.

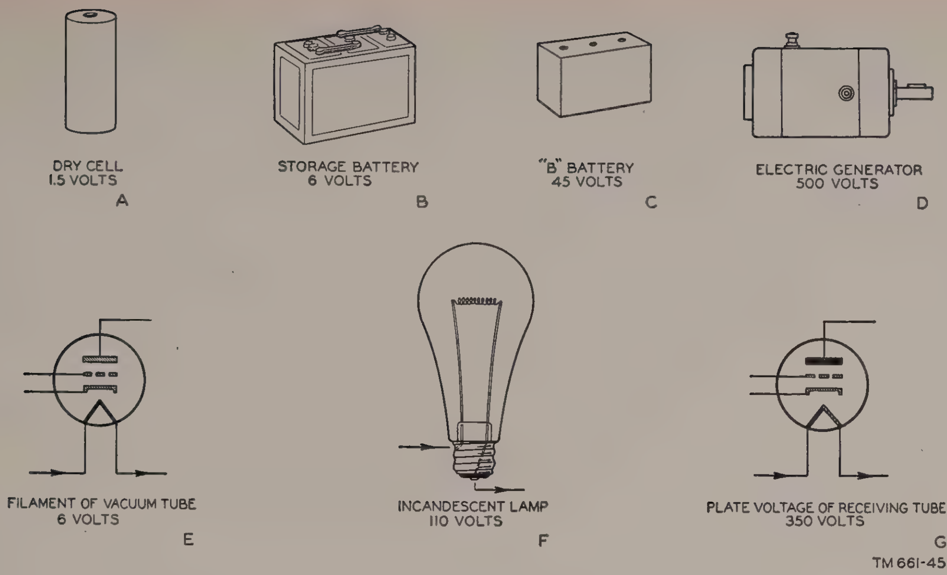


Figure 49. Voltage requirements for common electrical devices.

50. Electrical Resistance

It is found by experiment that the same voltage applied to specimens of different materials but having exactly the same physical dimensions will produce unequal currents. Similarly, if a fixed voltage is applied to specimens of the same material, but having different physical dimensions, the currents are again unequal. From these experimental facts, we can conclude that *for a fixed voltage, the amount of current flowing through any material depends on the type of material and the physical dimensions of the material.*

a. It is also found that the ratio of voltage and current for any type of conductor is a constant which depends on the material of which the conductor is made and its dimensions. This law was first discovered by Georg Simon Ohm, a German physicist, and the results of his experiments can be written mathematically in the following form:

$$E=I\times R$$

where E=voltage applied across the conductor,
I=current through the conductor, and
R=resistance of the conductor and depends on the material of which the conductor is made and its physical dimensions.

b. The manner in which the resistance depends on the conductor will now be explained (fig. 50).

Note that in the diagram, l is the length of the conductor *parallel to the direction of electron flow,* and A is the cross-sectional area of the conductor *perpendicular to the direction of current flow.* It is found that the resistance of this specimen is—

$$R=\rho\times\frac{l}{A}$$

where R=resistance of specimen,
ρ=*resistivity* and is a *constant* depending on the material of which the specimen is made,
l=length of specimen *parallel* to electron flow, and
A=cross-sectional area of specimen *perpendicular* to direction of current flow.

In the equation $E=I\times R$, E is usually expressed in volts, I in amperes, and R in ohms. Letting E=1 volt and R=1 ohm, we see that I=1 ampere. Thus, *a conductor has a resistance of one ohm when*

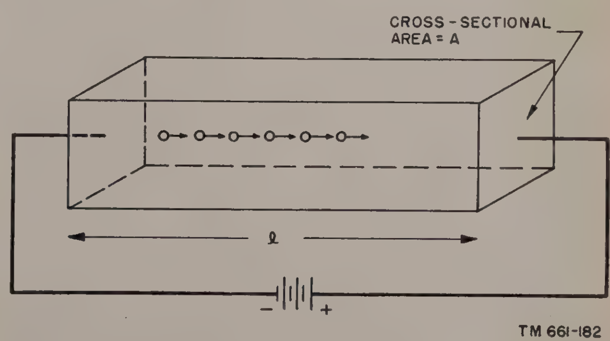


Figure 50. The resistance of a conductor varies directly with its length and inversely with its cross-sectional area.

an applied voltage of one volt gives rise to a current of one ampere. Returning to the equation $R=\rho\times\frac{l}{A}$, and letting $l=1$ *cm* (centimeter) and $A=1_{(cm)}{}^2$, we see that $R=\rho$. Therefore, the resistivity of any material is the resistance of a cube of the material 1 square cm. on each side. This is shown in figure 51.

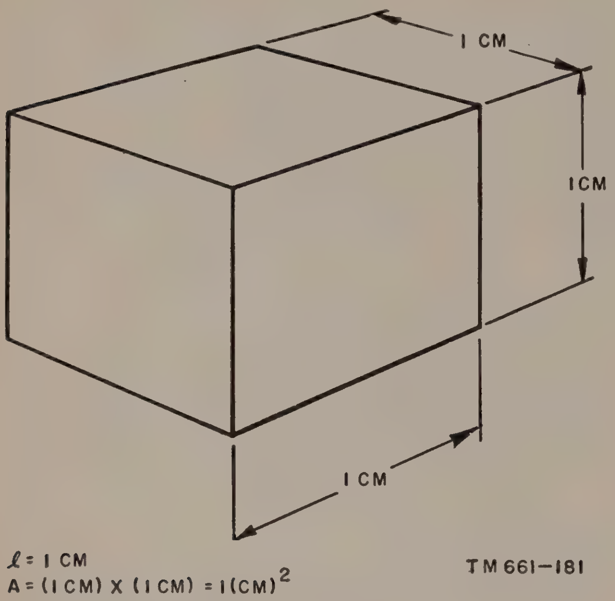


Figure 51. One cm. cube.

Since R is expressed in ohms, l in *cm*, and A in $(cm)^2$, from the equation $R=\rho\times\frac{l}{A}$, we see that ρ has the dimensions of

$$\frac{(resistance)\times(area)}{length}=\frac{(ohm)\times(centimeter)^2}{centimeter}$$
$$=ohm\times(centimeter).$$

However, it is customary to speak of ρ as the resistance of one cubic centimeter of the material.

c. Obviously, the more free electrons a conductor has the greater will be the current for any value of voltage. *Thus, the greater the number of free electrons a conductor has the smaller will be its resistance.* Since the drift velocity of the electrons depends on the number of collisions with other electrons and molecules, the greater the *length* of the conductor the greater will be the number of collisions, and consequently the smaller the drift velocity. However, the *current is proportional to the drift velocity and therefore its value will be decreased with an increase in length of conductor.*

d. Also the greater the cross section of a conductor the smaller the number of collisions due to the decreased density of electrons and molecules. Thus, *the greater the cross-sectional area of the conductor, the smaller its resistance.* These conclusions are expressed in the equation $R=\rho\times\frac{l}{A}$ *which shows that resistance varies directly with the length and inversely with the area of a conductor.*

e. In the previous discussion, the effect of temperature on resistance was not mentioned. However, the hotter the conductor becomes, the greater will be its resistance. The fundamental resistivity used in the computation of the resistance of *copper* conductors is the International Annealed Copper Standard which assigns a resistance, at 20° C, of .15328 ohms to an annealed copper conductor of 1/889 square cm. cross section, 1 meter long and weighing 1 gram, the copper having a density of 8.89. Using the equation for resistance we get

$$R=\rho\times\frac{100\ cm}{\frac{1}{889}\ (cm)^2}=.15328\ ohm.$$

Thus $\rho=1.7241\times10^{-6}$ ohm-cm, which is the resistance, at 20° C. of a conductor 1 cm long.

f. The variation of the resistance of *copper* with temperature has been found by experiment to be given by the following relationship:

$$R_t=R_o(1+at),$$

where a=coefficient of resistance at 0° C.= .00427,

R_t=resistance at temperature to centigrade, and

R_o=resistance at 0° C.

As an example, the resistance of a certain specimen of copper at 0° C. is 100 ohms. What is its resistance at 50° C?

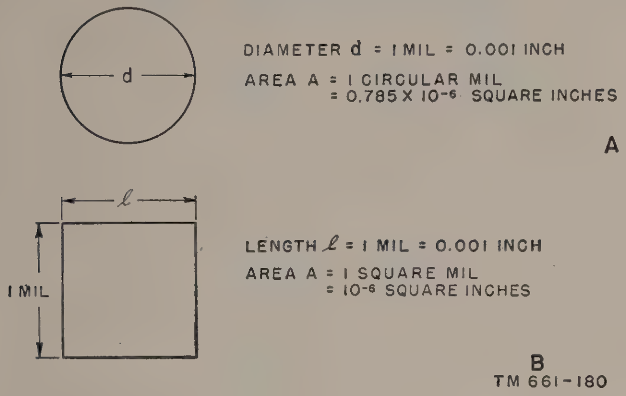
Solution: In this case, R_o=100 ohms and t=50° C. Therefore,

$$\begin{aligned}R_t&=100\ (1+(.00427)\ \times50)\\&=100\ (1+.21350)\\&=100\ (1.21350)\\&=121.35\text{ ohm (Answer.)}\end{aligned}$$

51. Measurement of Wire Conductors

In American practice, it is customary to express lengths of conductors in feet (occasionally in inches or in miles) and cross sections in circular mils (occasionally in square mils).

a. The mil is defined as the 1/1000 part of an inch (.001 in.), the circular mil (cir. mil) as the area of a circle having a diameter of 1 mil, and the square mil as the area of a square whose sides are 1 mil in length. This is illustrated in figure 52



A. Circular mil.
B. Square mil.

Figure 52.

b. In general, if a wire has a diameter of d mils, its area is d^2 circular mils. *In other words when the area of a circle is expressed in circular mils, the area is simply the square of its diameter expressed in mils.*

c. If the length of a conductor is expressed in centimeters and the cross-sectional area in square centimeters, the resistance for copper at 20° c. can be computed from the formula,

$$R \text{ (ohms)} = 1.7241 \times 10^{-6} \times \frac{l \text{ (centimeters)}}{A \text{(square centimeters)}}.$$

If the length is expressed in feet and the area in circular mils, the resistance for copper at 20° C. is given by the expression,

$$R \text{ (ohms)} = 10.37 \times \frac{l \text{ (feet)}}{A \text{ (circular mils)}}.$$

Consider the following example: a round copper conductor having a diameter of 2 inches and a length of 5 feet is at a temperature of 20° C. What is its resistance in ohms?

Solution: 2 inches $= 2 \times 1{,}000 = 2{,}000$ mils. Therefore, the area in circular mils $= (2{,}000)^2 = 4 \times 10^6$. Substituting values in the formula gives:

$$R = 10.37 \times \frac{5}{4 \times 10^6} = 12.96 \times 10^{-6} \text{ ohms.}$$

Though the formula for the variation of resistance with temperature has been given only for copper, it is true that the resistance of all metals changes with a change in temperature.

52. Conductance

Conductance is defined as the reciprocal of resistance. Obviously, the smaller the resistance, the greater the conductance and vice versa. The unit of conductance is the *mho* which is merely the world "ohm" spelled backwards. The formula for conductance is obtained from the equation $R = \rho \times \frac{l}{A}$, and is—

$$G = r \times \frac{A}{l}$$

where A = cross section of conductor,
l = length of conductor, and
$r = \frac{1}{\rho}$ = conductivity of material.

53. Common Conducting Materials

a. The fact that copper is used as a conducting material to a greater extent than any other material is accounted for not only by its high conductivity and comparatively low cost but also by the excellence of its physical characteristics in general. It has high tensile strength (49,000 to 67,000 pounds per square inch for hand-drawn copper), relative freedom from atmospheric corrosion, and is easily soldered.

b. Silver has the highest conductivity of all the metals but, because of its high cost, is only used when peak performance is more important than cost. Silver is used in measuring instruments and low-loss transmission lines.

c. Aluminum is the principal competitor of copper in high-voltage transmission lines. It has a resistivity of 2.828 microhms (1 ohm$\times 10^{-6}$) per cm-cube at 20° C., as compared with 1.7241 microhms for copper, making its conductivity 61 percent of that of copper. The density of aluminum is 2.67, or only 30 percent of that of copper. From this data, it can be determined that aluminum has about twice the conductance of an equal length and weight of copper. For example, if two transmission lines, one made of copper and the other made of aluminum, are to transmit the same amount of power with the same loss in transmission, the aluminum line will weigh about half the copper line. The total cost of the two installations will be the same only if the price of aluminum per pound is twice that of copper.

d. Metallic alloys exhibit numerous interesting characteristics, expecially with respect to temperature coefficient and resistivity. *Manganin,* for example, which is an alloy of 84 percent copper, 12 percent manganese, and 4 percent nickel, has a very low temperature coefficient (.000006) which makes it very useful in the construction of measuring instruments and their accessories, in which constancy of resistance, independent of the heating effects of current, is important.

54. Resistors

There is a group of materials which, although they are conductors, have such high resistivity that their principal function is to convert electrical energy into heat energy, as the rheostats and heating elements for various purposes. Such appliances are called *resistors.* For example, cast iron grids are used for the starting rheostats of industrial and railway motors. Rheostats are of two general types: two-terminal rheostats and three-terminal rheostats, or potentiometers. Figure 53 shows circuits using the two types.

a. One of the main uses of rheostats is to provide some means of varying the current through the fields of d-c generators and motors. In A of

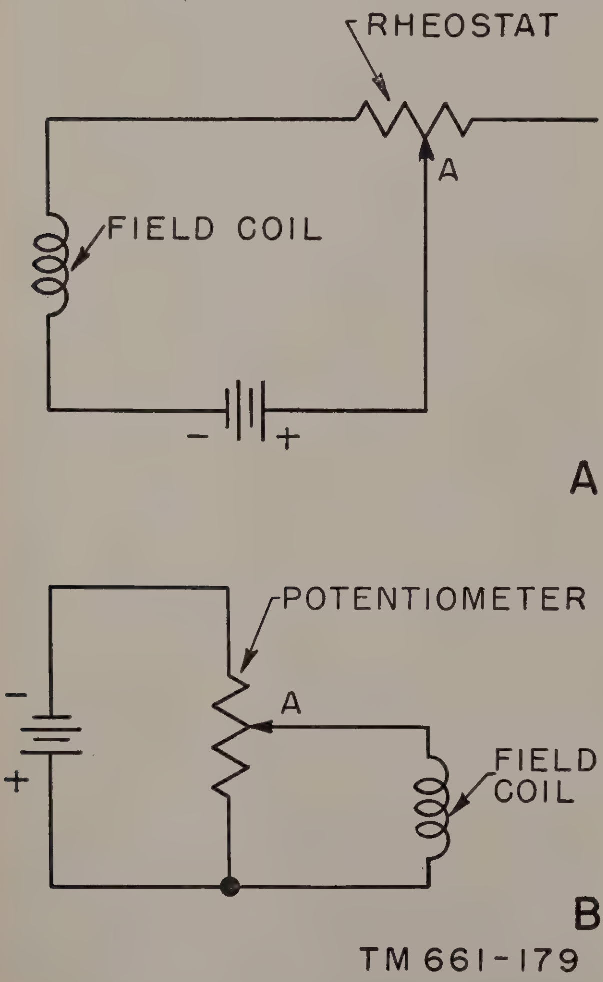


Figure 53. Circuits showing use of a rheostat and a potentiometer.

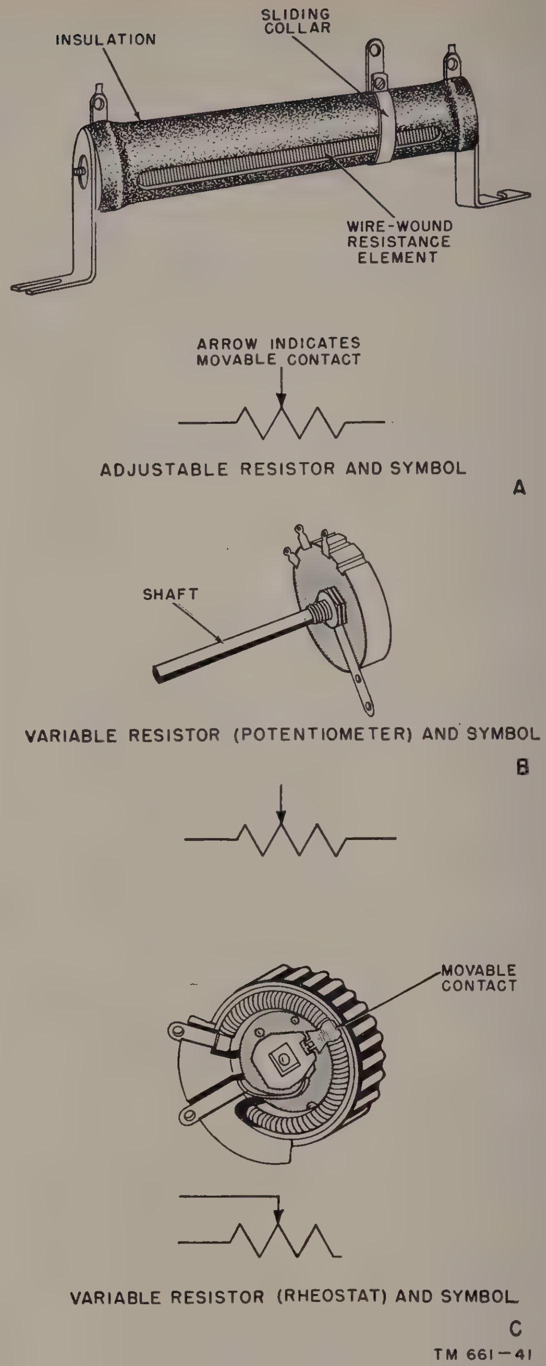


Figure 54. Types of variable resistances commonly used radio equipment.

figure 53, the variable terminal A (arrow) is an arm which can slide along the length of the resistor. This changes the total amount of resistance in the circuit and, by Ohm's law, changes the current. In B of figure 53, the field battery voltage appears across the potentiometer. By moving the variable arm A (arrow) along the resistance, the voltage across the field can be made to change and therefore the current through it.

b. Among the materials commonly used in manufacturing metallic resistors is *german silver*, comprised of copper, nickel, and zinc in varying proportions, with the resistivity increasing with the percentage of nickel. German silver has a high temperature coefficient. Copper-nickel alloys and iron-nickel alloys have high resistivity, that of the former running from 10 to 30 times that of copper. Nickel-chromium alloys (nichrome) have from 60 to 70 times the resistivity of copper. These alloys are especially useful in high-temperature devices such as electric furnaces.

c. A few substances have very remarkable resistance characteristics.

(1) Carbon, for example, has a resistivity ranging from 400 to 2,400 times that of copper, depending on the physical state of the material. Carbon is extensively used as brush material for generators and in resistors for radio and electronic equipment.

(2) Selenium, one of the nonmetallic elements, possesses the property of decreasing its resistance when light falls upon it.

(3) The resistance of bismuth is dependent upon the intensity of the magnetic field in which it lies.

55. American Wire Gage

The American Wire Gage (AWG), formerly called the Brown and Sharpe (B & S) Gage, is so constructed that the ratio of any one diameter to the next in order is a constant. The largest size (No. 0000 or 4/0) is assigned an arbitrary diameter of 460 mils, and No. 36 a diameter of 5 mils; between these two sizes there are 39 other sizes. The ratio of diameters taken in ascending order of size (descending order of gage numbers) is 1.123. Since cross sections vary as the squares of diameters, the ratio of cross sections of successive sizes, in ascending order of dimensions is

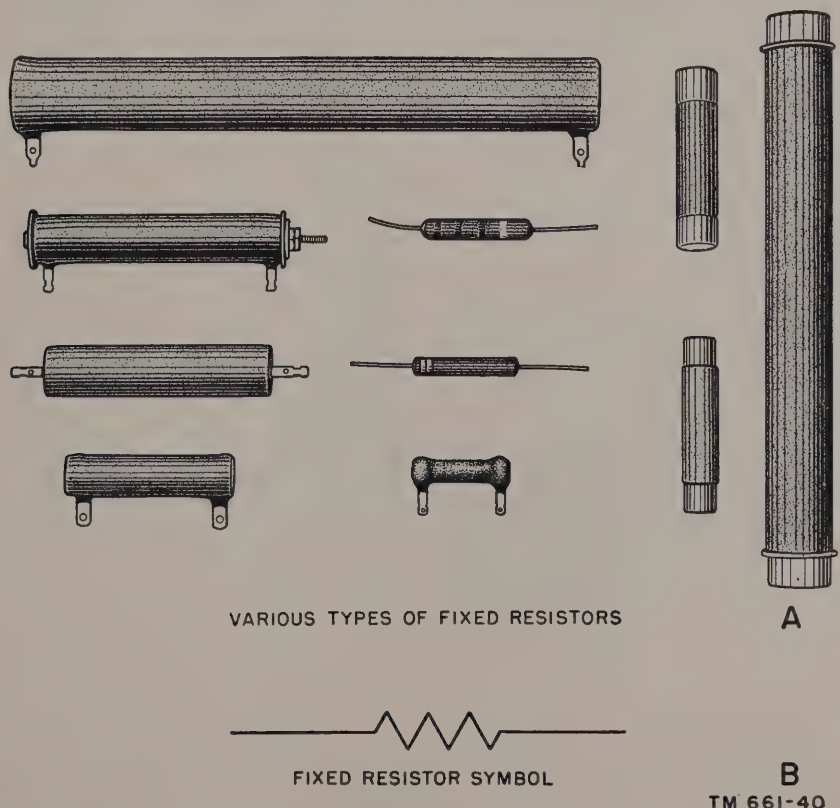


Figure 55. Types of fixed resistors commonly used in radio equipment.

$(1.123)^2=1.261$. Sizes that are 2 gage numbers apart will be in the ratio of $(1.261)^2=1.590$; sizes that are 3 gage numbers apart will be in the ratio of $(1.261)^3=2.005$, or practically 2. Sizes that are 10 gage numbers apart will be in the ratio of $(1.261)^{10}=10.164$, or very nearly 10.

a. If calculations are to be made in the absence of a wire table, the following rules will give a fair approximation:

(1) Number 10 wire has a diameter of 100 mils (actually 102 mils), a cross section of 10,000 circular mils, and a resistance of 1 ohm per 1,000 feet at 25° C.

(2) The cross section doubles every three numbers in the direction of increasing size of wire, and halves every 3 numbers in the opposite direction.

(3) The cross section increases tenfold every 10 numbers in the direction of increasing size.

b. The largest solid wire of circular section commonly used is No. 0000 wire, having a diameter of nearly ½ inch, and a cross section of nearly ⅙ square inch. Solid copper conductors of larger cross sections and greater current-carrying capacity, such as are required for buss bars, are usually of rectangular cross section, because of the greater ease with which they may be bent, bolted, and soldered. The greater ratio of lateral surface area to the cross-sectional area of each rectangular conductor is a distinct advantage in radiating the heat which develops as a consequence of the current flowing through it. In this way, the temperature is held down.

c. When circular conductors of larger size than No. 0000 are required, stranded cables are always used because of their greater flexibility. In such cases, the size is expressed directly in terms of circular mils. Stranded cables of smaller cross section than No. 0000 are also available for purposes where flexibility is essential, as in the case of a lamp cord. On the following pages, tables of resistance, conductance, and resistivities, together with standard Brown and Sharpe tables are given.

56. Table of Relative Resistances and Conductances of Some Common Materials as Compared with Annealed Copper

(*Value of copper=1*)

Material	Relative resistance	Relative conductance
Silver	0. 92	1. 08
Copper	1. 00	1. 00
Gold	1. 38	. 725
Aluminum	1. 59	. 629
Tungsten	3. 20	. 312
Zinc	3. 62	. 275
Brass	4. 40	. 227
Platinum	5. 80	. 172
Iron	6. 67	. 149
Nickel	7. 73	. 129
Tin	8. 20	. 121
Steel	8. 62	. 116
Lead	12. 76	. 081
Mercury	54. 60	. 018
Nichrome	60. 00	. 0166
Carbon	2, 030. 00	. 0004

57. Table of Specific Resistivities

Resistivities of various materials in ohms per circular mil-foot at 20°C.

Material	Resistivity
Silver	9. 56
Copper (annealed)	10. 4
Aluminum	17.
Tungsten	34.
Brass	42.
Nickel	60.
Platinum	60.
Iron	61.
Manganin	264.
Constantan	294.
Cast iron	435.
Nichrome	675.
Carbon	22, 000.

58. Wire Table

Approximate Dimensions and Resistance of Commercial Copper Wire American Standard (Brown & Sharpe) Wire Gage

B & S gage No.	Diameter bare wire		Area in cir mil (square of mils)	Safe carrying capacity, amperes		Ohms per 1,000 ft	
	In.	Mils		Rubber insulation	Other insulation	70° F.	167° F.
4/0	0. 460	460	211, 600	160—248	193—510	0. 050	0. 060
3/0	. 410	410	167, 800	138—215	166—429	. 062	. 075
2/0	. 365	365	133, 100	120—185	145—372	. 080	. 095
0	. 325	325	105, 600	105—160	127—325	. 100	. 119
1	. 289	289	83, 690	91—136	110—280	. 127	. 150
2	. 258	258	66, 560	80—118	96—241	. 159	. 190
3	. 229	229	52, 441	69—101	83—211	. 202	. 240
4	. 204	204	41, 620	60—87	72—180	. 254	. 302
5	. 182	182	33, 120	52—76	63—158	. 319	. 381
6	. 162	162	26, 240	45—65	54—134	. 403	. 480
7	. 144	144	20, 740			. 510	. 606
8	. 128	128	16, 380	35—48	41—100	. 645	. 764
9	. 114	114	13, 000			. 813	. 963
10	. 102	102	10, 400	25—35	31—75	1. 02	1. 216
11	. 091	91	8, 230			1. 29	1. 532
12	. 081	81	6, 530	20—26	23—57	1. 62	1. 931
13	. 072	72	5, 180			2. 04	2. 436
14	. 064	64	4, 110	15—20	18—43	2. 57	3. 071
15	. 057	57	3, 260			3. 24	3. 873
16	. 051	51	2, 580	6	10	4. 10	4. 884
17	. 045	45	2, 060			5. 15	6. 158
18	. 040	40	1, 620	3	6	6. 51	7. 765
19	. 036	36	1, 290			8. 21	9. 792
20	. 032	32	1, 020			10. 3	12. 35
21	. 028	28	812			13. 0	15. 57
22	. 025	25	640			16. 5	19. 63
23	. 024	24	511			20. 7	24. 76
24	. 020	20	404			26. 2	31. 22
25	. 018	18	320			33. 0	39. 36
26	. 016	16	253			41. 8	49. 64
27	. 014	14	202			52. 4	62. 59
28	. 013	13	159			66. 6	78. 93
29	. 011	11	128			82. 8	99. 52
30	. 010	10	100			106	125. 50
31	. 009	9	79			134	158. 20
32	. 008	8	64			165	199. 50
33	. 007	7	50			210	251. 60
34	. 006	6	40			266	317. 30
35	. 005	5. 6	31			337	400. 00
36	. 005	5	25			423	504. 50

Note.—This table is arranged according to American Standard (Brown & Sharpe) Wire Gage for copper wires. The first column carries the gage numbers, columns 2 and 3 are diameters in inches and mils, column 4 is area in square mils, columns 5 and 6 are the safe current-carrying capacities taken from National Electric Code. Columns 7 and 8 give resistance at two temperatures per 1,000 feet.

59. Heat Losses

a. It has been found experimentally that when a current of I amperes flows through a resistance of R ohms for t seconds, the amount of heat developed is—

$$H = RI^2t,$$

where H=heat developed in joules,
R=resistance in ohms,
I=current in amperes, and
t=time current flows in seconds.

The amount of heat generated in one second ($t=1$) is expressed by RI^2 joules. Now a *joule per second* is called a *watt* and is the unit which expresses the *rate* at which heat is produced. Obviously,

$$Watts = \frac{H}{t} = W = RI^2$$

W is commonly referred to as the I^2R loss. In most commercial equipment, it is necessary to keep this loss down to a minimum for two reasons:

(1) It represents energy wasted.
(2) It increases the temperature of the equipment.

b. Because a conductor heats when carrying current, it is important to choose a size wire which will not overheat when carrying the required current. For each size conductor, there is usually specified the maximum safe current that it can carry without overheating. Of course, in certain appliances such as lamps, irons, and toasters, the heating effect of electricity is desired. In many radio and electronic circuits, the current is so small that heating effects are negligible.

60. Power Rating of Resistors

Whenever current flows through any electrical equipment, a certain amount of power is consumed. The amount of power consumed in a resistor, in watts, is given by the formula $W=I^2R$, where W is the power in watts, I is the current in amperes, and R is the resistance in ohms. The power consumed by a resistor is dissipated completely in heat. Therefore, the larger the surface area of the resistor and the freer the circulation of air around it, the greater the ease with which the resistor can dissipate heat. In general, a large resistor (in physical size) will have a higher power rating than a small resistor. Manufacturers of resistors rate their product in two ways, ohms for resistance and watts for power. However, manufacturers specify that resistors must be mounted in the open a certain distance from any other object in order to give the rated service. Since resistors used in radio circuits and other communication equipment are always crowded together in confined spaces they must have a power rating higher than if they were to be used according to specifications. These resistors are frequently selected with from two to four times the power rating required for the same job under ordinary conditions.

61. Commercial Resistors

Resistors can be classified in two groups, depending on the materials of which they are made: wire-wound resistors for power ratings above the 2-watt size, and composition type resistors for power ratings of 2 watts and under.

a. Wire-Wound Resistors. There are two common types of wire-wound resistors:

(1) Fixed resistors having a definite value of resistance. These resistors have only two terminals, one at each end of the resistor.
(2) Variable resistors (rheostats and potentiometers) having values or resistance which can be varied from approximately zero to the maximum value. Rheostats are sometimes called ~~side~~ SLIDE wire resistors and have only two connections, one fixed and one movable. Potentiometers (a type of rheostat) have three terminals, two at the fixed ends of the coil and one on the movable contact. Types of variable resistors are shown in figure 54.

b. Composition Resistors. Wire resistors of large ohmic values require so much wire that their general use is prohibitive. By powdering carbon and mixing it with some binding material, high-value resistances can be made easily and economically. The proportion of carbon to binding material determines the resistance per inch, and resistors of any desired value may be made by this process. Composition resistors have wires (called pigtails) extending from the ends. These wires are used to connect the resistors into the circuit. Carbon resistors cannot be made as accurate as wire-wound resistors and are, therefore, usually marked with a colored band around the center to indicate the percentage of accuracy. This variation in actual value from the nominal

value is called the tolerance. If the actual value (determined by actual measurement) is within ± 5 percent of the nominal value a gold band is placed near the center of the resistor. If the measured value is withhin ± 10 percent, a silver band is put on the resistor. If there is no band, the indicated tolerance is within ± 20 percent of the nominal value. Figure 55 shows various types of fixed resistors.

62. Resistor Color Code

Modern radios and other communication equipment utilize many resistors of various sizes. In order to repair and service this equipment with minimum amount of effort and time, it is necessary that the technician be able to recognize the value of these resistors. After trying a number of different methods, the color code method of indicating size and tolerance was finally evolved. Colors were assigned definite values as follows:

Black	0	Green	5
Brown	1	Blue	6
Red	2	Violet	7
Orange	3	Gray	8
Yellow	4	White	9

a. Method *A* (fig. 56) is the most recent development and is used by JAN (Joint Army-Navy) and most manufacturers. In this method, four bands are placed on the resistor. Reading from the end toward the center, the first color band indicates the first digit, the second color band indicates the second digit, and the third color band indicates the number of zeros to be added in order to obtain the resistance value. The fourth band around the resistor (either gold or silver) indicates the tolerance. For example, a resistor of 10,000 ohms (± 10 percent) would have a brown band, a black band, and an orange band to indicate 10,000 ohms. The fourth band would be silver to indicate a tolerance of ± 10 percent. Thus, the value of the resistor would be between the limits of 9,000 and 11,000 ohms.

b. Method *B* (fig. 56) uses body, tip, and a band around the middle. The body indicates the first digit, the tip indicates the second digit, and the band around the center indicates the number of zeros to be added. Sometimes this method uses a dot in place of the band around the center to indicate the number of zeros or multiplying value. The JAN color code indicates tolerance on these resistors by a gold or silver band on the other end of the resistor. Color coding is not used for precision resistors.

63. Prefixes

In any system of measurements, a single set of units is usually not sufficient for convenient computations. For example, inches are usually used for small distances, while larger distances are given in feet, yards, or miles. If we were to express a large number of miles in inches, we would be entirely correct, but the number would be cumbersome and impractical. In electricity, especially in communications work, it is often necessary to use wide ranges of numbers representing the values of the different units such as volts, amperes, etc. A series of prefixes which appear with the name of the unit have been devised to be used for the various sizes of the units. There are six of these prefixes, also known as *conversion factors*, that are used extensively in electrical work. These are given as follows:

Mega means one million—1,000,000

Kilo means one thousand—1,000

Centi means one-hundredth— $\frac{1}{100}$

Milli means one-thousandth— $\frac{1}{1,000}$

Micro means one-millionth— $\frac{1}{1,000,000}$

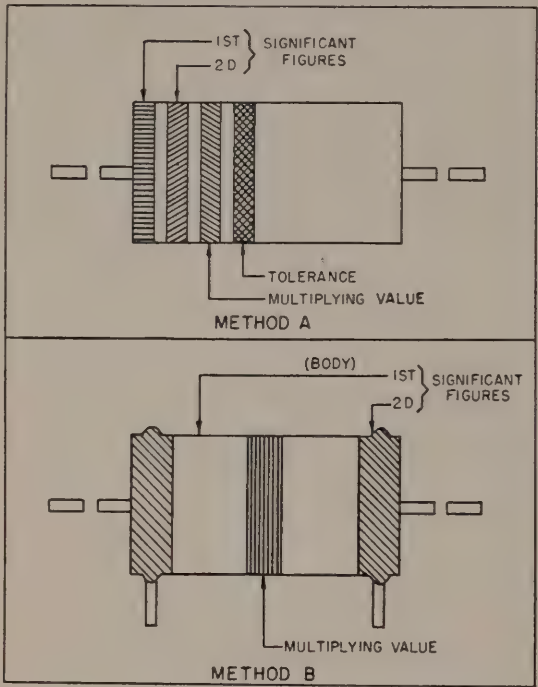
Micro micro means one-millionth-millionth—

$$\frac{1}{1,000,000,000,000}$$

For example, the word "kilo" means one thousand. Thus, 1 kv (kilovolt) equals 1,000 volts. Then 1 volt equals one-thousandth of a kv or $\frac{1}{1,000}$ kv. This may also be written 0.001 kv. The word "milli" means one-thousandth, 1 millivolt equals one-thousandth $\left(\frac{1}{1,000}\right)$ of a volt. Then 1 volt equals 1,000 mv (millivolts). These prefixes may be used with all electrical units, and provide a convenient method for writing extremely large or small values. Most electrical formulas require the use of values in the basic units, therefore, all values must usually be converted before computations can be made.

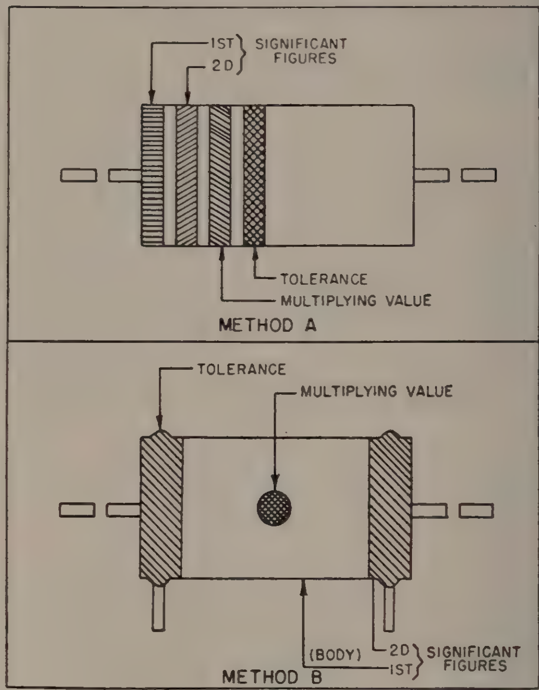
RESISTOR COLOR CODES

RMA COLOR CODE FOR FIXED COMPOSITION RESISTORS*



A

JAN COLOR CODE FOR FIXED COMPOSITION RESISTORS†



B

COLOR	SIGNIFICANT FIGURE	MULTIPLYING VALUE	TOLERANCE (%)
BLACK	0	1	
BROWN	1	10	
RED	2	100	
ORANGE	3	1,000	
YELLOW	4	10,000	
GREEN	5	100,000	
BLUE	6	1,000,000	
VIOLET	7	10,000,000	
GRAY	8	100,000,000	
WHITE	9	1,000,000,000	
GOLD	—	0.1	± 5
SILVER	—	0.01	± 10
NO COLOR	—	—	± 20

NOTES

* INSULATED FIXED COMPOSITION RESISTORS WITH AXIAL LEADS ARE DESIGNATED BY A NATURAL TAN BACKGROUND COLOR. NON-INSULATED FIXED COMPOSITION RESISTORS WITH AXIAL LEADS ARE DESIGNATED BY A BLACK BACKGROUND.

† RESISTORS WITH AXIAL LEADS ARE INSULATED. RESISTORS WITH RADIAL LEADS ARE NON-INSULATED.

RMA: RADIO MANUFACTURERS ASSOCIATION

JAN: JOINT ARMY-NAVY

THESE COLOR CODES GIVE ALL RESISTANCE VALUES IN OHMS.

TL32454S

Figure 56. Resistor color codes.

64. Conversion Table

1 ampere	equals	1,000,000	microamperes.
1 ampere	equals	1,000	milliamperes.
1 centimeter	equals	.01	meter.
1 cycle	equals	.000001	megacycle.
1 cycle	equals	.001	kilocycle.
1 farad	equals	1,000,000,000,000	micromicrofarads.
1 farad	equals	1,000,000	microfarads.
1 farad	equals	1,000	millifarads.
1 henry	equals	1,000,000	microhenrys.
1 henry	equals	1,000	millihenrys.
1 kilocycle	equals	1,000	cycles.
1 kilovolt	equals	1,000	volts.
1 kilowatt	equals	1,000	watts.
1 megacycle	equals	1,000,000	cycles.
1 megohm	equals	1,000,000	ohms.
1 mho	equals	1,000,000	micromhos.
1 mho	equals	1,000	millimhos.
1 microampere	equals	.000001	ampere.
1 microfarad	equals	.000001	farad.
1 microhenry	equals	.000001	henry.
1 micromho	equals	.000001	mho.
1 microhm	equals	.000001	ohm.
1 microvolt	equals	.000001	volt.
1 microwatt	equals	.000001	watt.
1 micromicrofarad	equals	.000000000001	farad.
1 micromicroohm	equals	.000000000001	ohm.
1 milliampere	equals	.001	ampere.
1 millihenry	equals	.001	henry.
1 millimho	equals	.001	mho.
1 milliohm	equals	.001	ohm.
1 millivolt	equals	.001	volt.
1 milliwatt	equals	.001	watt.
1 volt	equals	1,000,000	microvolts.
1 volt	equals	1,000	millivolts.
1 watt	equals	1,000,000	microwatts.
1 watt	equals	1,000	milliwatts.
1 watt	equals	.001	kilowatt.

65. Circuit Symbols

When an electrician or radio man draws a circuit of some equipment which contains resistors, batteries, and other electrical parts, he always makes use of circuit symbols. For example, figure 54 shows different types of variable resistors with their electrical symbols, and figure 57 shows the method of using symbols in a circuit diagram. Other symbols are used if other parts appear in a circuit. A variety of symbols commonly used in electrical work are shown in figure 58.

a. A circuit diagram which makes use of symbols is known as a *schematic diagram.* Actually, pictures of the parts in a circuit could be drawn instead of circuit symbols but this would be time-consuming and, in the case of circuits using hundreds of parts, would be difficult to read.

b. Figure 57 shows the simplicity of a schematic diagram as compared with a pictorial diagram. Note that the parts wired into the circuit of A of figure 57 are partially replaced by circuit symbols in the pictorial-schematic diagram of B of figure 57, and entirely replaced by circuit symbols in the schematic diagram of C of figure 57. Also note that the symbol for the tube (*V*1) actually contains more information than the picture of the tube. That is, the symbol shows various parts inside the tube which cannot be shown clearly in a picture diagram.

c. It is not expected that the reader will understand the electrical functioning of the circuit shown in figure 57. The theory of radio circuits is beyond the scope of this manual. However, the fundamental principles of electricity must be known before an understanding of radio can be obtained.

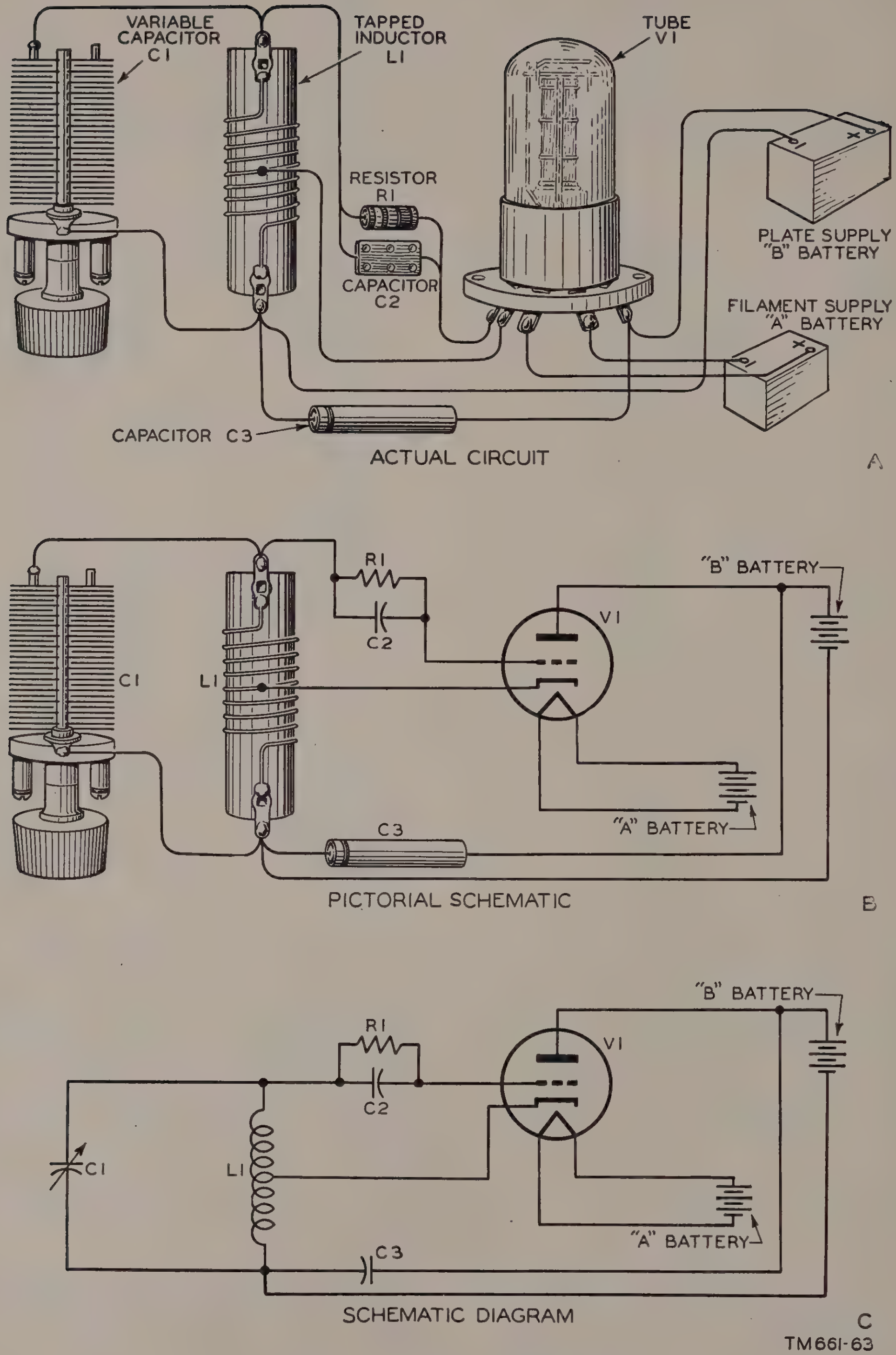


Figure 57. Use of circuit symbols.

66. Summary

a. The random velocity of electrons in a conductor is very great, being of the order of 400 miles/sec.

b. When a field is superimposed on the electrons in a conductor, the resultant drift velocity is 1 cm/sec, which is relatively slow.

c. When current flows through a resistor, heat is produced.

d. An ammeter is used to measure current. A voltmeter is used to measure voltage.

e. A coulomb per second is defined as an ampere.

f. The resistance of any material depends on its length, cross section, and temperature.

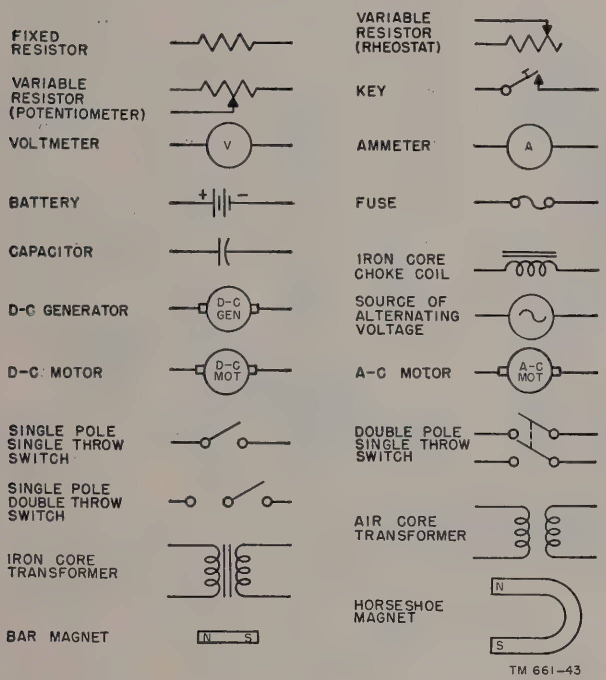


Figure 58. Circuit symbols.

g. Ohm's law states: the voltage drop across a resistor is equal to the resistance times the current through the resistor.

h. Copper and aluminum wire are most commonly used for the transmission of electrical power. Copper has a higher conductivity than aluminum; aluminum has a lower density than copper.

67. Review Questions

a. What is resistance? State some of the factors that influence resistance.

b. Define random velocity. What is the drift velocity of electrons in a conductor?

c. Why is the drift velocity low and the random velocity high?

d. Do electrons travel in the conductor at the speed of light?

e. How is electrical energy transmitted from one point to another at approximately the speed of light?

f. Give the physical factors affecting resistance.

g. What is the law giving the temperature variation of resistance?

h. State Ohm's law.

i. Define current density and current.

j. State the factors that determine current density.

k. What is an ammeter and how is it used? What is a voltmeter?

l. Why is a color code necessary? Describe and explain the color code.

m. What colors would be used to indicate a resistor of 10,000 ohms?

n. A resistor has, reading from end to center, the following colors: red, black, green, and silver. What is the value of the resistor?

CHAPTER 6
OHM'S LAW

68. General

The relationships of current, voltage, and resistance must be considered in every electric circuit. Direct current will flow only in a closed circuit (fig. 40) which provides a *continuous conducting path* from the negative to the positive terminal of the voltage source. As discussed in this chapter, Ohm's law shows the relationships of voltage, current, and resistance *in d-c circuits.*

TM 661-310

Figure 59. Georg Simon Ohm demonstrates the relationship of E, I, and R.

69. Ohm's Law

The relationships of current, voltage, and resistance were first proved by Georg Simon Ohm (1787–1854), a German physicist. Using very poor and deficient apparatus, but the best to be had at that time, he performed a series of experiments which completely settled the questions of (1) the way *emf* (electromotive force) is distributed throughout a circuit, and (2) the relationships of voltage, current, and resistance. He published the results of his experiments in 1827.

a. Statement of Ohm's Law. OHM'S LAW STATES THAT THE CURRENT IN AN ELECTRIC CIRCUIT IS PROPORTIONAL TO THE VOLTAGE AND INVERSELY PROPORTIONAL TO THE RESISTANCE.

b. Equations for Ohm's Law. This law may be expressed by any one of three equations:

(1) $I=E/R$, or current equals the voltage divided by the resistance.

(2) $R=E/I$, or resistance equals the voltage divided by the current.

(3) $E=IR$, or the voltage is equal to the product of the current times the resistance.

70. Practical Application

A schematic diagram of a simple circuit is shown in figure 60. The positive terminal of the 12-volt battery is connected by a wire conductor (called a lead) to the positive terminal of an ammeter. A second lead connects the negative terminal of the ammeter to a 3-ohm resistor, and a third lead connects the other end of the resistor to the negative terminal of the battery. This is a *series* connection, and it provides a continuous conducting path from the negative to the positive terminal of the battery. The leads have some resistance, and the battery has internal resistance (ch. 7), but for the purposes of this discussion, the internal resistance of the battery and the resistance of the lead wires will be neglected. In other words, we will consider only the resistance of each piece of equipment (the resistor, in this instance) external to the source of emf. Notice that the ammeter is connected so that all the current must pass through it. The voltmeter is connected across the battery; that is, the positive terminal of the meter is connected to the positive terminal of the battery, and the negative terminal of the meter is connected to the negative terminal of the battery.

a. PROBLEM. Find the current when the battery voltage (as indicated on the voltmeter) is 12 volts and the resistance is 3 ohms; proceed as follows:

(1) Express the known values in terms of the symbols representing them:

$E=12$ volts
$R=3$ ohms
$I=?$

(2) The Ohm's law formula for determining the current when E and R are known is—

$I=E/R$

(3) Substituting for symbols their known values,

$I=12/3$.

(4) Solving by dividing 12 by 3:

$I=4$ amperes. This is the current that the ammeter will indicate.

b. PROBLEM. Find the resistance when the voltage is 12 volts and the current is 4 amperes. The Ohm's law formula for resistance is,

$R=E/I$.

Substituting in this equation the known values and solving for R,

$R=12/4=3$ ohms.

c. PROBLEM. Find the voltage when the current is 4 amperes and the resistance is 3 ohms. The Ohm's law formula for voltage is

$E=IR$.

Substituting in this equation the known values and solving for E,

$E=4\times 3=12$ volts.

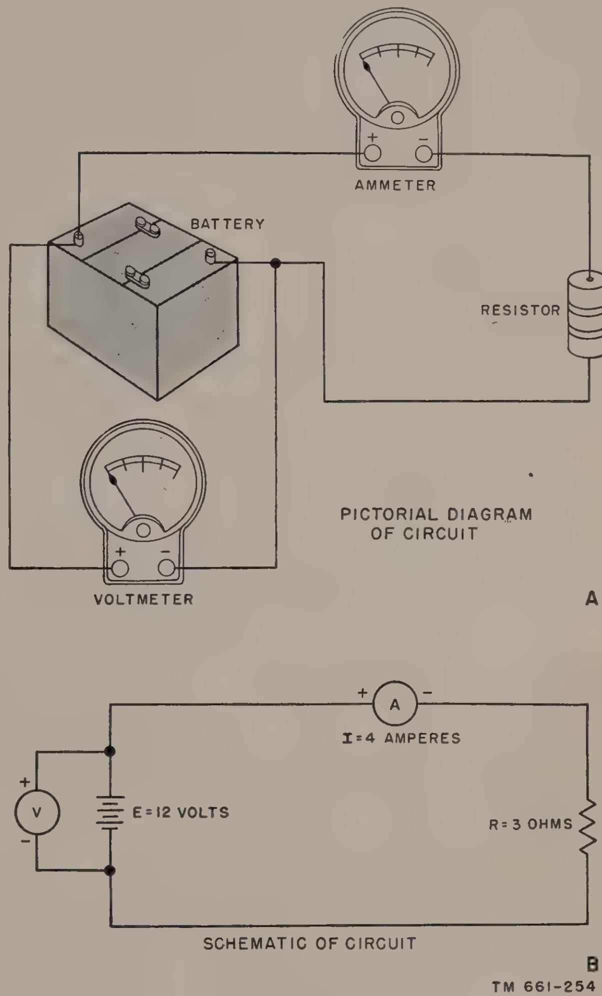


Figure 60. Simple circuit, parts A and B.

71. Memory Method

The formulas for solving problems involving Ohm's law must be learned. *Do not bypass these formulas, they must be understood and remembered.* A simple aid to memorizing the Ohm's law formulas is shown in figure 61. Part *A* of this figure shows a circle with the three symbols arranged in separate spaces. If any one of the symbols is covered, the arrangement of the other two symbols forms the right-hand side of the formula for determining the value of the covered symbol.

Thus, if a finger is placed over the I (B of fig. 61), E/R remains, and indicates that I is equal to E/R; if R is covered (C of fig. 61), E/I remains, and indicates that R equals E/I; and if E is covered (D of fig. 61), IR remains, and indicates that E equals IR.

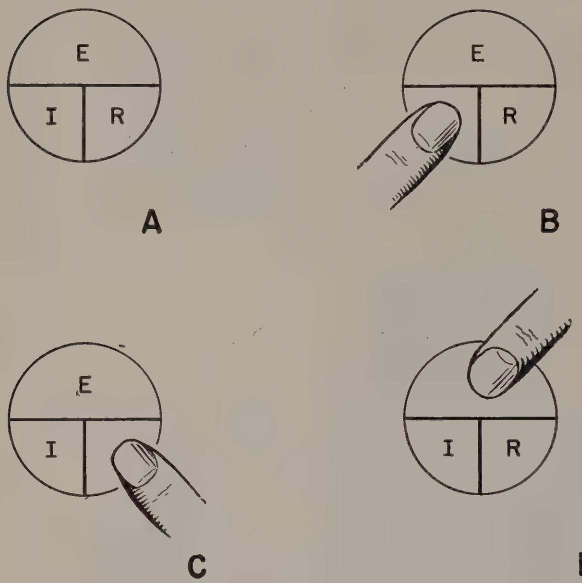


Figure 61. Memory aid for learning Ohm's law.

72. Units of Measurement

It is best, when solving problems with Ohm's law, to use the practical units of measurement; that is, in terms of volts, amperes, and ohms. Problems are frequently given with fractional or multiple values of these units, such as the milliampere, kilovolt, etc. Whenever such values are given in a problem, use the conversion table (par. 64) and convert *all* values to the basic units; namely, volts, amperes, and ohms.

73. Applications

If the voltage is increased while keeping the resistance constant, there will be an increase in the current. If the resistance is increased while keeping the voltage constant, there will be a decrease in the current.

a. Effect of Voltage Increase. Figure 62 shows four separate circuits A, B, C, and D. These circuits are alike in that they have a voltage source connected to a 6-ohm resistor and an ammeter. There is only one path for the current flow. The difference in the four circuits is in the value of the applied voltage which is obtained from dry cells, each of which furnishes 1.5 volts.

(1) In A of figure 62 there is only one cell in the circuit and the voltage is 1.5 volts as indicated on voltmeter V_1. The resistance in the circuit is 6 ohms. What is the value of the current?
By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{1.5}{6}$$

I=0.25 ampere.

(2) In B of figure 62, two cells are connected in the circuit. The voltage as indicated on voltmeter V_2 is 3 volts, or just double the voltage of one cell. The resistance is still 6 ohms. What current will flow in this circuit?
By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{3}{6}$$

I=0.5 ampere.

Notice that by doubling the voltage, we also doubled the current (increase from .25 ampere to .5 ampere).

(3) In C of figure 62, another cell has been added, making a total of three cells in the circuit. The voltage, as indicated on V_3, is now increased to 4.5 volts. The resistance has not changed. What is the current?
By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{4.5}{6}$$

I=.75 ampere.

The current has again increased.

(4) D of figure 62 shows a fourth cell added to the circuit. The voltage is now 6 volts, as indicated on voltmeter V_4. The re-

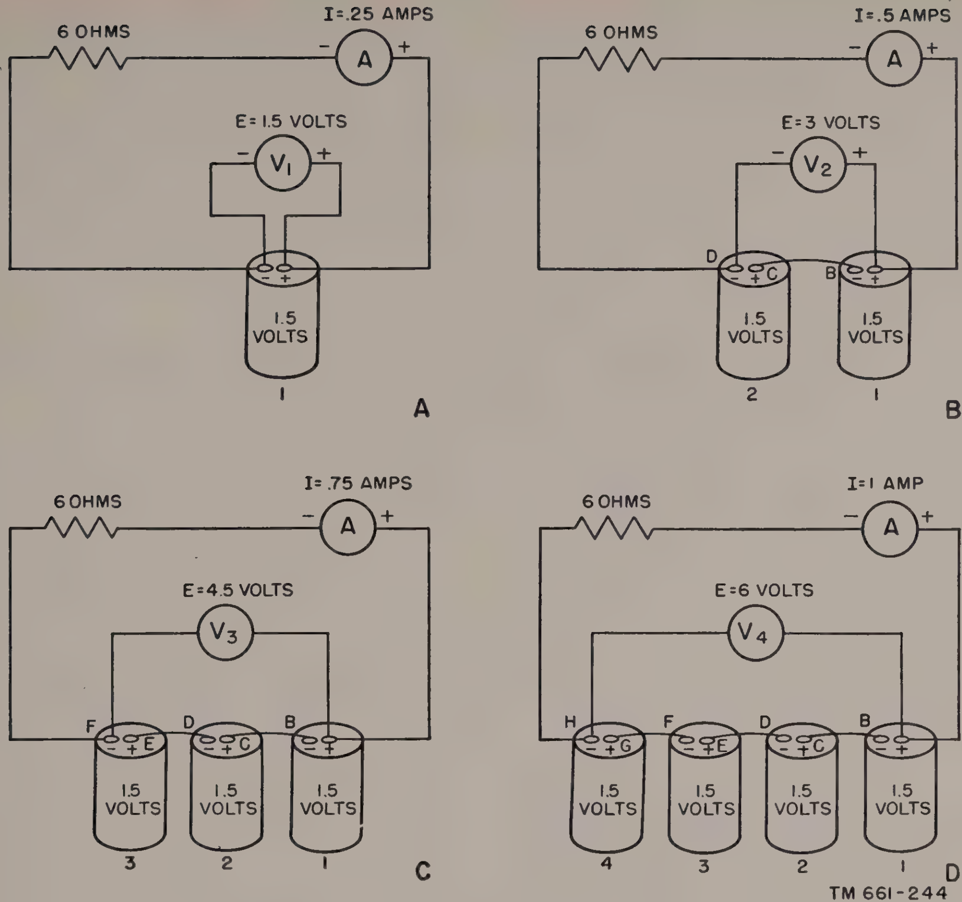


Figure 62. Effect of changing voltage.

sistance is 6 ohms. What is the current? By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{6}{6}$$

$I=1$ ampere.

The current in the circuit of D of figure 62 is four times greater than the current in A of figure 62. By this analysis, it was shown that *if the resistance is kept constant and the voltage is increased, the current will increase in proportion to the increase in voltage.*

b. Effect of Increasing Resistance.

(1) For the first example (A of fig. 63), the 2-ohm section R_1 is the only resistance in the circuit. The measured voltage V_1 is 6 volts. What is the current? By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{6}{2}$$

$I=3$ amperes.

(2) With two 2-ohm resistors in the circuit (B of fig. 63), the total circuit resistance is $2+2=4$ ohms. The voltage remains constant at 6 volts. What is the current? By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{6}{4}$$

$I=1\frac{1}{2}$ amperes.

(3) With three 2-ohm resistors in the circuit (C of fig. 63), the total circuit resistance is equal to 2+2+2=6 ohms. The voltage remains at 6 volts. What is the current?

By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{6}{6}$$

$I=1$ ampere.

(4) With four 2-ohm resistors in the circuit (D of fig. 63), the total circuit resistance is equal to 2+2+2+2=8 ohms. The voltage is still 6 volts. Find the current.

By Ohm's law,

$$I=\frac{E}{R}$$

$$I=\frac{6}{8}$$

$I=\frac{3}{4}$ ampere.

Notice that the current has decreased with the addition of resistance. Therefore, *if the voltage is kept constant and the resistance is increased, the current will decrease as the resistance is increased;* that is, the current is inversely proportional to the resistance.

c. Effect of Increasing Current. One of the forms of Ohm's law is $E=IR$.

(1) If the current through a resistor is 2 amperes, and the value of the resistance is 2 ohms, the voltage across the resistor is—

By Ohm's law,

$$E=IR$$

$$E=2\times2$$

$E=4$ volts.

(2) If the current through the same resistor is 1 ampere, the voltage across the resistor will be—

By Ohm's law,

$$E=IR$$

$$E=1\times2$$

$E=2$ volts.

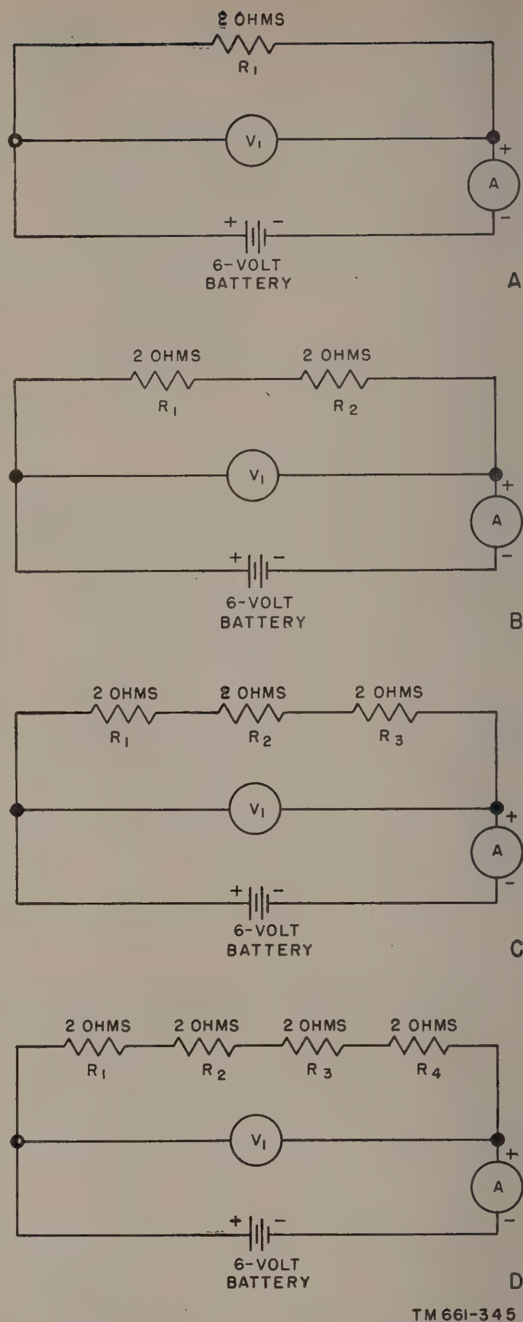


Figure 63. Effect of changing resistance.

In other words, it takes only one-half as much voltage to force 1 ampere of current through a resistance of 2 ohms as it does to force 2 amperes through the same resistance. *Therefore, it can be concluded that the greater the current through a resistor, the greater will be the voltage across the resistor.* The voltage across a resistor (or any other component) is called

a *voltage drop*. A voltage drop is the potential difference required to produce the current through the part of the circuit under consideration. For example, when the current through a 2-ohm resistor is 1 ampere, the voltage drop, or potential difference between the ends of the resistor is 2 volts.

74. Summary

a. Ohm's law states the relationships of voltage, current, and resistance.

b. The symbols: E means *voltage*, I means *current*, and R means *resistance*.

c. Ohm's law for current is $I=\frac{E}{R}$, or current equals voltage divided by resistance.

d. Ohm's law for resistance is $R=\frac{E}{I}$, or resistance equals voltage divided by current.

e. Ohm's law for voltage is $E=IR$, or voltage equals current times resistance.

f. The practical units of measurement are—

(1) *Amperes* for current.

(2) *Ohms* for resistance.

(3) *Volts* for voltage.

g. If we increase voltage and keep resistance constant, current will increase.

h. If we decrease voltage and keep resistance constant, current will decrease.

i. If we increase resistance and keep voltage constant, current will decrease.

j. If we decrease resistance and keep voltage constant, current will increase.

k. The formula $E=IR$ may be used to compute the voltage (voltage drop) across any part of a circuit.

l. The following table summarizes some of the above information:

Summary table

Factor	Practical unit	Symbol	Measure with—	Ohm's law formula
Current	Ampere	I	Ammeter	$I=\frac{E}{R}$.
Resistance	Ohm	R	Ohmmeter	$R=\frac{E}{I}$.
Emf / Voltage	Volt	E	Voltmeter	$E=IR$.

75. Review Questions

a. Write the Ohm's law formulas for finding—

(1) Voltage.

(2) Current.

(3) Resistance.

b. What current will flow through a coil having a resistance of 6 ohms if the applied voltage is 48 volts?

c. What is the resistance of an electric light bulb through which 1 ampere flows while 110 volts are applied?

d. How many volts will be required to produce a current of 1.5 amperes through an electric bell having a resistance of 6 ohms?

e. Name the units that must be used when solving problems by Ohm's law.

f. In a given circuit the voltage is doubled and the resistance is held constant. What happens to the current?

g. An electric iron requires 5 amperes to heat it properly. The resistance of the heating element in the iron is 22 ohms. What voltage will produce a current of 5 amperes through the iron?

h. The resistance of a certain resistor is 100 ohms. It is connected across the terminals of a 4-volt battery. How much current will flow through the resistor?

i. What are the symbols for:

(1) Voltage?

(2) Resistance?

(3) Current?

j. In a given circuit the resistance is increased and the voltage is held constant. What change occurs in the value of the current?

CHAPTER 7

PRIMARY CELLS

76. Early Experiments and Discoveries

Although our early knowledge of electricity and magnetism dates back thousands of years, little progress in the science of electricity was made until late in the eighteenth century when the electric cell, or battery, was discovered by Luigi Galvani, an Italian physicist. The discovery occurred while Galvani was preparing an experiment for a class in anatomy. For the experiment, Galvani had removed frog legs from a salt solution and had suspended them by means of a copper wire. He then noticed that each time an iron scalpel was brought in contact with a frog leg, a reaction occurred, that is, the leg jerked. Galvani concluded incorrectly that electricity was produced by the muscles of the frog. Just three years later, Allessandro Volta, another Italian scientist, found that the frog's leg was not responsible for the electricity but rather, the electricity was the result of chemical action between the copper wire, iron scalpel, and salt solution. Pursuing these findings still further, Volta built the first electric battery, which he called a voltaic pile.

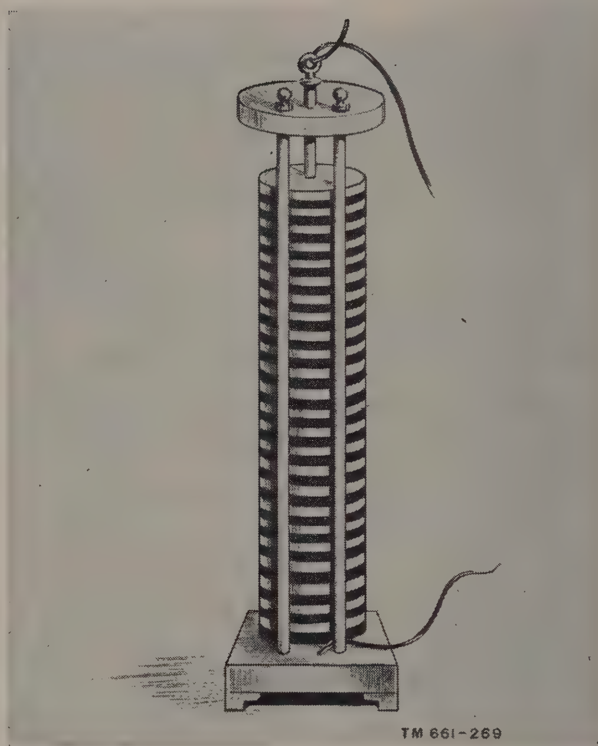


Figure 64. Voltaic pile.

77. Voltaic Pile

The voltaic pile (fig. 64) consists of a stack of alternate silver and zinc plates separated from each other by pieces of cloth staurated with salt solution. Using this pile, Volta found that he could produce an electric current through a wire connecting the silver and zinc plates. Thus, the voltaic pile was the first practical method of producing electricity by chemical action. Later, Volta made a single cell by placing copper and zinc plates in a glass filled with a lye solution and again found that he could cause an electric current to flow through a wire when it was connected between the copper and zinc plates (fig. 65). In recognition of his work, this cell is called the voltaic cell.

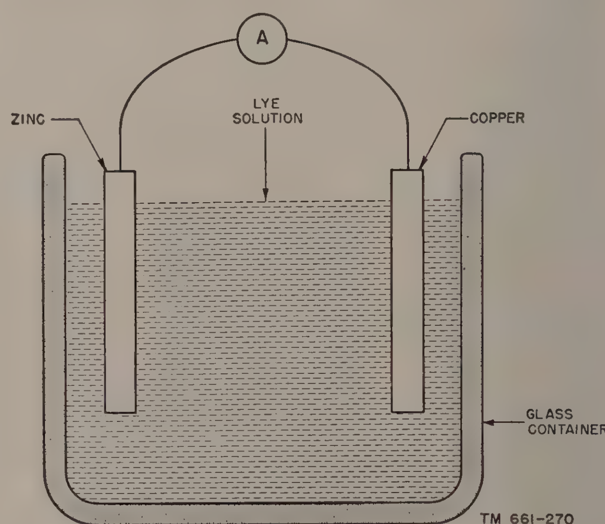


Figure 65. Simple voltaic cell.

78. Simple Voltaic Cell and its Parts

Although the voltaic cell described in paragraph 77 was comprised of copper and a zinc plate plus a lye solution, it is not necessary that the cell be so constructed. Actually, a voltaic cell can be built by using either a salt solution or an acid solution, and almost any metals can be used, provided the two metals are different from each other and the solution will act chemically on at least one of the metals used.

a. The solution used in the cell is commonly called an *electrolyte.* Solutions of hydrochloric and sulphuric acids or ammonium and sodium chlorides make good electrolytes.

b. The metals in the cell are called *electrodes.* Various metals can be used as electrodes because the cell action depends on the electrolyte acting more on one of the metals than on the other. No chemical action occurs if both electrodes are made of the same metal. Zinc, iron, silver, copper, carbon, and platinum are suitable metals.

79. Chemical Explanation of Cell Operation

When two electrodes are placed in a solution in order to make a voltaic cell, a chemical action takes place. *The most important result of this chemical action is that one of the electrodes is given a positive charge and the other electrode is given a negative charge.* Thus, there is a difference of potential, or emf, between the two electrodes and a source of electricity is available. An understanding of how the chemical action produces an emf requires some knowledge of chemistry.

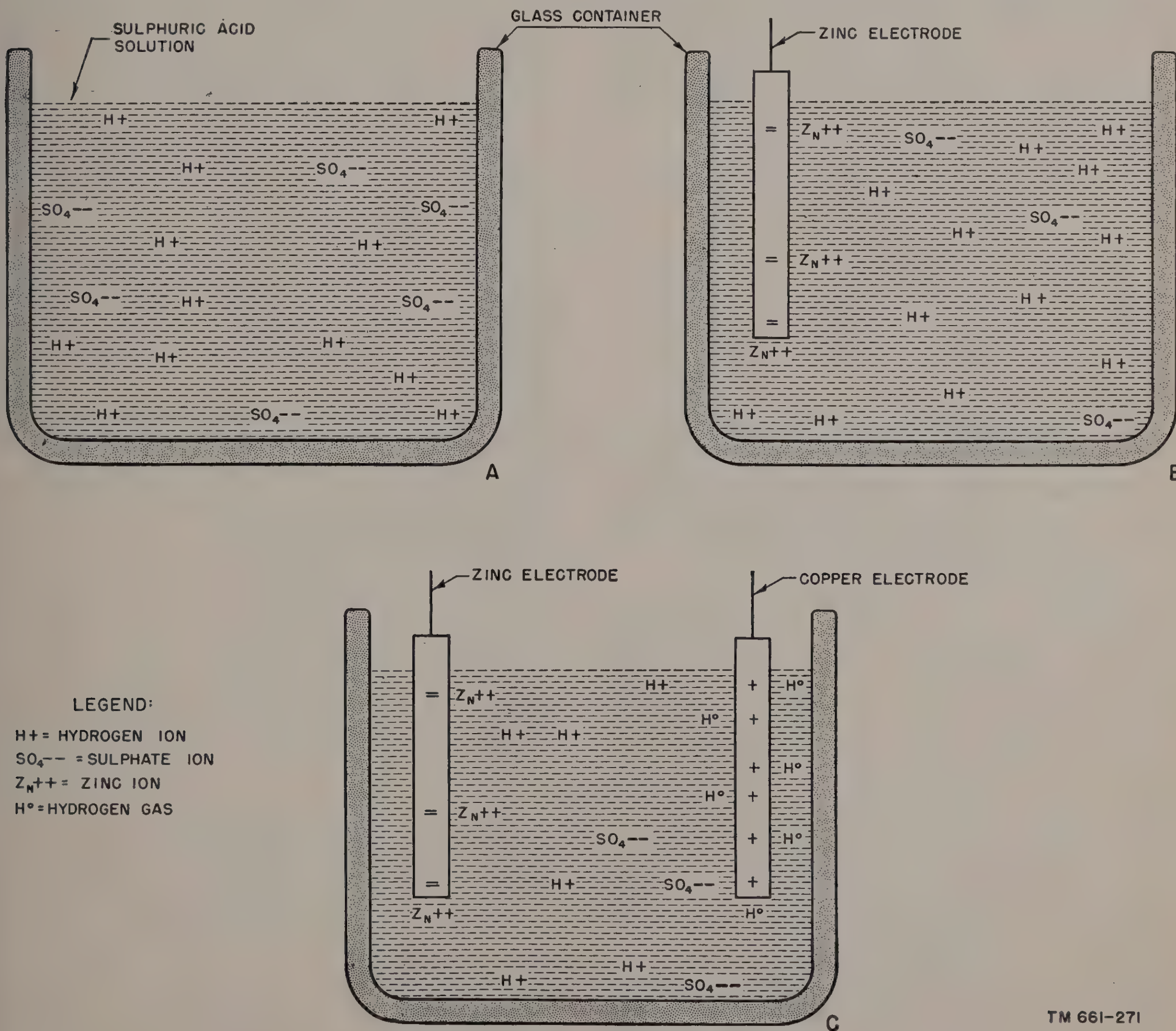


Figure 66. Chemical action of the voltaic cell.

a. Whenever an acid or salt is mixed with water to make a solution or electrolyte, two actions occur. First, the acid or salt is dissolved in the water and, second, a chemical process called *ionization* takes place. That is, some of the substance which has dissolved in the water breaks up into tiny parties which carry electrical charges. These particles are called ions. *An ion is defined as a particlc of subatomic, atomic, or molecular dimensions which carries either a positive or negative electrical charge.* The ion is positive if it has a deficiency of electrons and is negative if it has an excess of electrons. For example, when sulphuric acid (H_2SO_4) is dissolved in water, some of the particles break down and form hydrogen ions (H^+) (each of which has a single positive charge) and sulphate ions (SO_4^{--}) (each of which has two negative charges). A of figure 66 shows how some of these ions are distributed throughout the solution. The entire solution is electrically neutral, that is, it contains equal numbers of these positive and negative charges.

b. Other electrolytes break down as follows:

(1) $CuSO_4$ (copper sulphate). Upon ionization, this becomes:
Cu^{++} (copper ion) and
SO_4^{--} (sulphate ion).

(2) NH_4Cl (ammonium chloride). Upon ionization, this becomes:
NH_4^+ (ammonium ion) and
Cl^- (chloride ion).

c. It has been shown in *a* above that ionization occurs when sulphuric acid is placed in water, but that the solution has no external electrical effect because it contains equal numbers of positive and negative charges. That is, the sulphuric acid solution does not produce electricity.

d. When a dilute sulphuric acid solution is placed in a glass container and a zinc electrode is immersed in the solution (B of fig. 66) some of the zinc dissolves in the solution and produces positively charged zinc ions, with each zinc ion so produced leaving two electrons on the zinc electrode. In B of figure 66, the zinc ions are designated as Zn^{++}, indicating two positive charges; the electrons on the electrode are designated as negative charges ($^{--}$). The excess positive ions in the solution cause the solution to become *positively charged*. The excess electrons on the zinc electrode, on the other hand, cause the electrode to become *negatively charged*. A difference of potential then exists between the zinc electrode and the solution of sulphuric acid. Also, the positive zinc ions are attracted by the negative zinc electrode, and they accumulate around it.

e. When a copper electrode is inserted in the solution (C of fig. 66), some of the positive hydrogen ions (H^+) leave the solution and go to the copper electrode. Each of these hydrogen ions which reaches the copper electrode combines with an electron in a copper atom to form hydrogen gas. The loss of electrons from the copper causes the copper electrode to become positively charged with respect to the solution. In C of figure 66, the hydrogen gas is designated as H° and the positive charges on the copper are designated by plus signs ($^+$).

f. Because the zinc electrode is negative with respect to the solution and the copper electrode is positive with respect to the solution, it follows that *the zinc electrode is negative with respect to the copper electrode*. Actually, the potential difference between the electrodes is approximately 1.08 volts for this particular type of cell.

g. If these two electrodes are connected by a conductor, electrons will flow through the conductor from the negative zinc electrode to the positive copper electrode. When the electrons leave the zinc electrode, more zinc turns into zinc ions, thus replenishing the electrons on the zinc electrode. The newly formed positive zinc ions also repel the positively charged hydrogen ions and cause some of them to be deposited on the copper electrode. Here they combine with the electrons arriving from the zinc electrode through the conductor and turn into hydrogen gas. In this manner the charge on each electrode is kept almost constant and their potential difference remains practically the same.

h. When the zinc ions enter the solution, they attract and combine with the sulphate ions to form zinc sulphate, which remains dissolved in the solution. If the electrodes remain connected by a conductor, current will flow in the conductor until all the zinc turns into ions (dissolves in the solution of sulphuric acid), at which time current will cease.

i. It is not expected that a student will long remember the details of the chemical action in the voltaic cell. However, it is expected that a student will remember that chemical reactions can be used to furnish a source of electricity or emf.

80. Local Action

When chemically pure zinc is used in the electrode of the voltaic cell described in paragraph

79, the zinc is not consumed by chemical action when no external circuit is connected between the positive and negative electrodes. In short, the cell does not wear out when not in use. However, in most cases, zinc electrodes are not chemically pure zinc. Instead, tiny impurities such as particles of iron and carbon are imbedded in the zinc. These impurities enter into chemical action with the electrolyte to produce small electrical currents around the zinc plate even when no external circuit is completed between the positive and negative electrodes. As a result, chemical energy is consumed and the life of the cell is decreased. This condition is called *local action*.

a. To prevent local action, zinc electrodes are usually amalgamated with mercury. When this is done, the zinc dissolves in the mercury but the impurities do not. As a result, the zinc is free to enter chemical action with the electrolyte during normal operation of the cell (when an external circuit is connected) but the impurities are covered with mercury and do not enter into chemical action with the electrolyte to produce local action.

b. In well-designed cells and batteries, local action is minimized.

Note. An example of the detrimental effects of local action can be observed by trying to remove the dry cells from a flashlight which has not been used for a long time. In this case, local action causes the zinc container (negative electrode) to be eaten away, the electrolyte spills out of the cell, and the cell becomes damp and swells, making removal of the cell from the flashlight difficult.

81. Internal Resistance

a. If a voltmeter is connected between the electrodes of the voltaic cell shown in A of figure 67, a reading of 1.08 volts, the open-circuit voltage of the cell, will be obtained. That is, 1.08 volts is the emf produced between the two electrodes when the cell is not delivering current to an external circuit.

b. If a resistor is also connected across the ter-

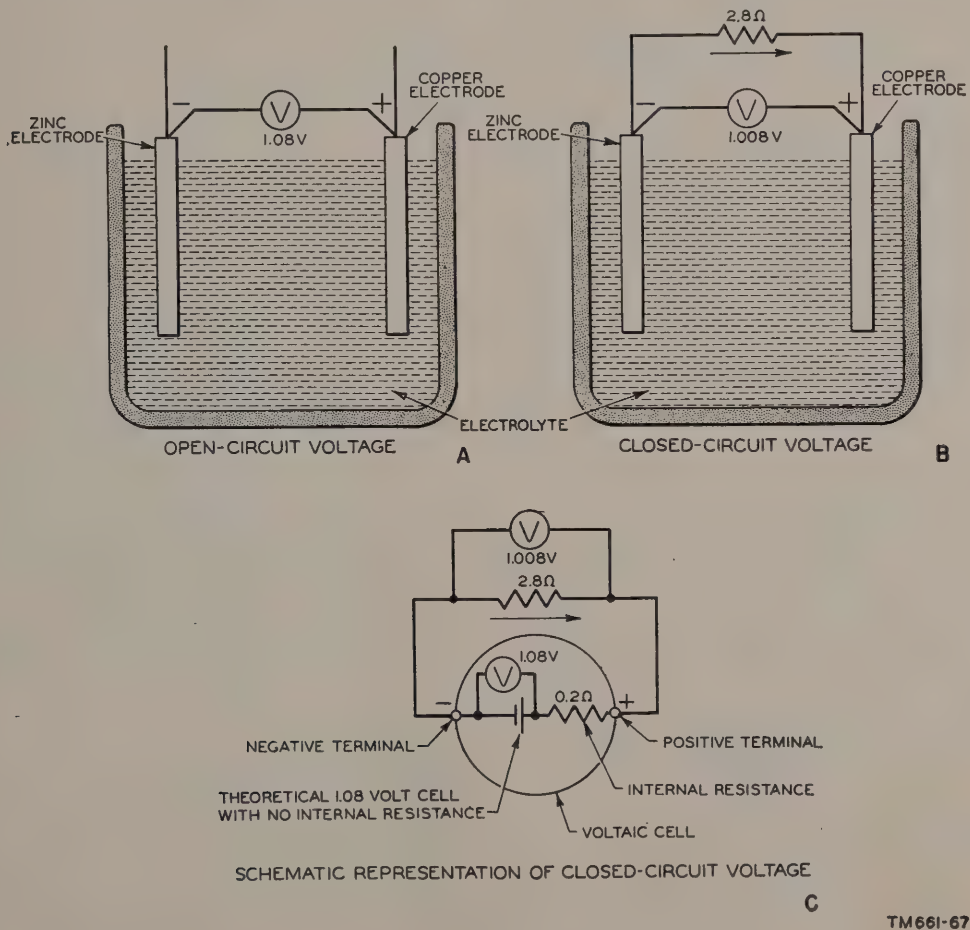


Figure 67. Effect of internal resistance on all voltage.

minals of the electrodes, for example, the 2.8-ohm resistor in B of figure 67, current will flow from the negative to the positive electrode, as indicated by the arrow. The current which leaves the cell at the negative electrode and the current which enters the cell at the positive electrode must flow through these electrodes, which have a small amount of resistance. Furthermore, zinc ions have to be pushed into the solution at the zinc electrode, and hydrogen ions have to be pushed out of the solution at the copper electrode. The electrolytic solution presents resistance to the motion of the ions. Therefore, it takes force to make these ions move. A part of the total emf of the cell is expended in making the ions move inside the cell.

c. Current can be considered as the motion of charged particles. Since the ions are charged particles, their motion can be considered to be current. The positive zinc ions liberated around the negative zinc electrode (fig. 66) attract all the negative sulphate ions (SO_4^{--}) in the cell and cause them to migrate toward that part of the cell. This constitutes a flow of negative or electron current. The movement of the positively charged hydrogen ions toward the copper electrode is a flow of positive electricity. Thus the previous statement that a part of the emf of the cell is expended in making the ions move inside the cell can be restated as follows: "A part of the total emf of the cell is expended in making the current flow inside the cell."

d. This behavior of the cell is precisely that of resistance, so that the cell is said to have *internal resistance*. Suppose that the internal resistance of the cell in B of figure 67 is .2 ohm. The cell can then be represented as a theoretical 1.08-volt cell which has no internal resistance in series with a resistance of .2 ohm (C of fig. 67). Since the external resistance in the circuit is 2.8 ohms, the total resistance in the series circuit is .2 plus 2.8, or 3 ohms.

1.08

e. The total current in the circuit is $\frac{E}{R}=\frac{10.8}{3}=$ .36 ampere.

f. The voltage drop across the internal resistance of the cell is—

$$IR=.36\times.2=.072 \text{ volt}$$

g. The voltage or potential difference across the 2.8-ohm resistor is—

$$IR=.36\times2.8=1.008 \text{ volts}$$

This is the voltage at the terminals of the voltaic cell.

h. By definition, the *open-circuit voltage* of a cell is the voltage between the terminals when no appreciable current is flowing. This voltage is measured by placing a voltmeter across the terminals of the cell when it is not delivering current. The voltmeter, because of its high resistance, draws so small a current to deflect its needle that there is said to be practically an open circuit between the terminals.

i. The *closed-circuit voltage* of a cell is the voltage between its terminals when current is flowing through an external circuit (B of fig. 67). The *terminal voltage* of a cell is merely the voltage between its terminals (electrodes) and can therefore be either the open-circuit or closed-circuit voltage.

j. In *a* above, the open-circuit voltage is 1.08 volts; the closed-circuit voltage is 1.008 volts. The cause of the difference between the open-circuit and closed-circuit voltage is the internal resistance. *The emf of the cell remains the same,* but a voltage drop, equal to the current drawn times the internal resistance of the cell, occurs within the cell. This internal voltage drop must be subtracted from the emf of the cell and results in a lowered terminal voltage when the cell is delivering current. The closed-circuit voltage is also referred to as the *load* or *working* voltage. The open-circuit voltage is referred to as the *emf* or *nominal* voltage.

82. Polarization

a. When a primary cell is furnishing current, positive hydrogen ions combine with electrons at the positive electrode and turn into hydrogen gas. This gas appears in the form of bubbles which do not rise to the surface as rapidly as they are formed. Some of the bubbles cling to the positive electrode and form a layer of nonconducting gas about it. This interferes with the action of the cell in two ways: first, the internal resistance of the cell is increased since the nonconducting bubbles of hydrogen gas on the positive electrode diminishes the area through which the positive hydrogen ions can reach the positive electrode, combine with electrons, and turn into hydrogen gas. Second, the positive hydrogen ions moving in the direction of the positive electrode, collect on or in the region of this gas, and repel other hydrogen ions moving toward that electrode. This is equivalent to an emf which *opposes* the emf of the cell.

b. A cell whose internal resistance is increased in this manner and in which an opposing or counter emf exists is said to be *polarized*. The net result of polarization is to decrease rapidly the closed-circuit voltage of the cell. Consequently, the current furnished by the cell rapidly diminishes in strength.

c. One method commonly used to counteract polarization is by the addition of manganese dioxide, or some other oxidizing agent, to the electrolyte. The manganese dioxide combines with the hydrogen gas on the plate to form water, thus preventing polarization. Another method which can be used on some wet primary cells is to remove the positive pole from the cell and wipe off the layer of gas.

83. Cells and Batteries

At one time, a cell was considered to be a single unit, such as a voltaic cell, which furnished a source of electricity. Also, a battery was defined as a combination of two or more cells connected together in order to obtain higher voltages or longer operating life. However, this distinction between the cell and battery is no longer valid. Instead, the terms *cell* and *battery* are now used interchangeably.

Figure 68. Dry-cell battery.

a. Primary Cells. All cells or batteries are classified under two general headings, primary cells and secondary cells. The voltaic cell already described is a primary cell, a name given to any cell in which an electrode is consumed gradually during normal use, and cannot be restored to its original useful state by recharging electrically. For example, in the voltaic cell described in paragraph 79, the zinc electrode disintegrated gradually with use as zinc ions were produced and entered the solution of sulphuric acid. However, most primary cells in general use are not of this type. Instead, the use of these *wet* primary cells is confined to laboratory, experimental, and special-purpose work. Most primary cells are of the so-called *dry* type which are described in following paragraphs. Common examples of dry primary cells are those used in flashlights, portable radios, etc. A dry cell can be used for a considerable period of time but it must be discarded when one of its electrodes is worn out or consumed. It is not practical to rebuild a dry cell. Dry cells are used extensively in military communication equipment and furnish an emf for a variety of circuits. See figure 72.

b. Secondary Cells. In a secondary cell, an electrode is not destroyed during normal use. The secondary cell may be renewed or recharged electrically when it becomes *run down*. Common types of secondary cells are the *storage batteries* used to supply energy for electrical parts in automobiles. Chapter 8 of this manual describes the secondary cell.

84. The Dry Cell (LeClanche Cell)

a. To overcome the defects of local action and polarization, many other types of primary cells using different chemicals have been made. Of these, the most widely used commercial type is the LeClanche *dry cell*. A typical 1.5-volt dry cell, the kind used in flashlights, is shown in figure 68. The component parts of this cell are shown in figure 69.

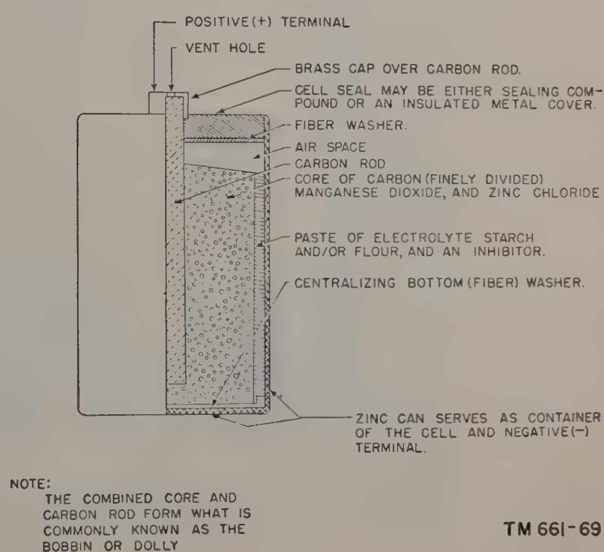


Figure 69. Cross-sectional view of a dry cell showing parts.

945343°—51——6

b. The zinc container is the negative electrode and the carbon rod is the positive electrode. A brass cap or terminal is forced on the end of the carbon rod to insure good electrical connections with the apparatus in which the cell is used. Instead of a liquid electrolyte, the electrolyte takes the form of a paste. For this reason, the cell is called a dry cell although it is *not really dry*. The electrolyte is a concentrated solution of ammonium chloride (an inhibitor to prevent zinc corrosion) and sufficient cornstarch or flour to form a stiff paste. A mixture of finely divided carbon, manganese dioxide, and zinc chloride is packed around the carbon electrode. The manganese dioxide, called a depolarizer, is used to counteract polarization.

c. The chemical action that takes place within a dry cell is similar to that of the simple voltaic cell previously explained. In the dry cell, the zinc container acts as the negative electrode and the carbon rod is the positive electrode. The solution of ammonium chloride (sal ammoniac) is the electrolyte.

(1) When the cell is first assembled, the electrolyte breaks down into positive ammonia ions (NH_4^+) and negative chloride ions (Cl^-). When the zinc contacts the electrolyte, positive zinc ions enter the solution and drive positive hydrogen and ammonia ions toward the carbon rod. This action produces a negative charge on the zinc and a positive charge on the carbon. That is, a potential difference exists between the two electrodes.

(2) When an external circuit is connected between the terminals of the electrodes, electrons flow from zinc to carbon through the external circuit. As this happens, further chemical action within the cell maintains the difference of potential between the electrodes until the zinc is consumed or the electrolyte loses its useful chemical properties.

d. In addition to the commercial type LeClanche dry cell, three other types of dry cells are used by the Army, namely: RM (Rubin-Mallory), magnesium-silver-silver chloride, and zinc-silver chloride. These types are discussed in TM 11–415 and are considered only briefly here.

(1) The RM dry cell (fig. 70) has different chemicals than the LeClanche dry cell and can furnish the same amount of current for a longer period of time when the current drain is *continuous*. For most military applications, the current drain is intermittent and for such service the LeClanche cell is still used.

(2) The magnesium-silver-silver chloride dry cell is manufactured without any electrolyte and requires the addition of tap or salt water to activate it. However, once activated, this type of cell must be used immediately since it dissipates its electrical energy whether it is connected to a load or not. The cell is used to deliver a large current for a short period of time.

(3) The zinc-silver chloride dry cell (fig. 71) is used only when very small current is required (10 ma or less).

(4) There is also a reserve type of LeClanche dry cell that requires water to activate it.

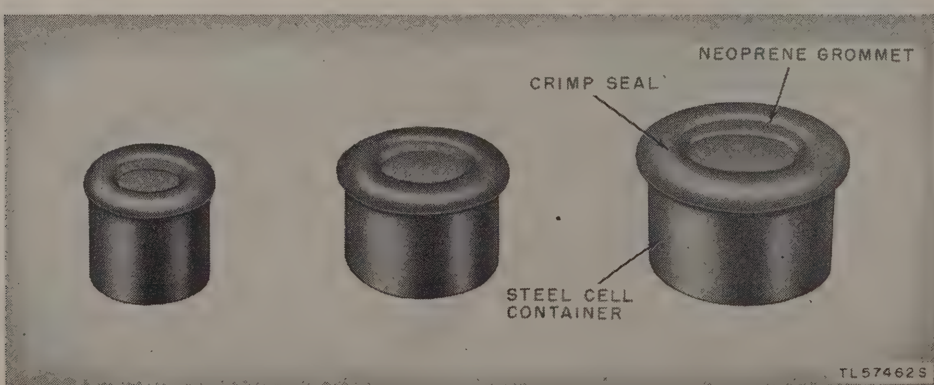


Figure 70. RM cells.

Figure 71. Zinc-silver chloride cells.

85. Terminals

It is not usual to mark the polarity of the terminals of the LeClanche dry cells, although it is the practice to do so invariably in the case of secondary or storage cells. Probably, the reason for this is that the terminals of a primary cell are so obviously different that they can be recognized immediately; in a storage cell there may be no apparent difference in appearance between the terminals. For any of the dry cells shown in figure 72, the center or carbon terminal is positive, and the outside or zinc terminal is negative. However, when several dry cells are combined to increase either the voltage or current capacity and sealed as a unit in a case or container, the polarity usually is marked by plus and minus signs. Sometimes, polarity is indicated by the use of a red wire for the positive terminal and a black wire for the negative terminal.

86. Voltage and Current of Dry Cells

a. As previously stated, the chemical action in a cell sets up a difference of potential or voltage between the two electrodes of the cell. The magnitude or amount of this potential difference depends on only two things:

(1) The materials of which the electrodes are made.

(2) The type of electrolyte used in the cell. Note that the size of the electrodes or the amount of electrolyte does not affect the potential difference developed by the cell.

Figure 72. Cylindrical dry cells (BA numbers are military nomenclatures).

b. As an example, the open-circuit voltage of the copper-zinc cell shown in figure 67 is approximately 1.08 volts. A cell that has electrodes of different materials such as carbon, zinc, and an electrolyte of chromic acid would have a voltage of approximately 2 volts. The LeClanche dry cell has an open-circuit voltage of approximately 1.5 volts. Other types of dry cells have open-circuit voltages of 1.3 volts or less.

c. Increasing the size of the electrodes of a particular cell will not increase the voltage of that cell; but the larger the electrodes, the greater the amount of current the cell will deliver at its rated closed-circuit voltage.

d. When dry cells are new, and are not delivering a greater amount of current than that for which they are designed, the closed-circuit voltage is very nearly the same as the open-circuit voltage and is only about .1 volt less per cell when the cell is first used. Toward the end of the useful life of the cell, the closed-circuit voltage drops to about 1 volt per cell instead of 1.5 volts.

e. In the first few months after a dry cell has been made, the local action which occurs is negligible. After that the local action proceeds more rapidly, increasing the internal resistance and therefore decreasing the closed-circuit voltage. Local action will eventually render useless the commercial type of dry cell even though the cell may have been sitting idle on a shelf. Thus the commercial (LeClanche) dry cell has a shelf life of only a little over a year. The date of manufacture is stamped on the covers of these batteries.

f. When a dry cell is used, the chemical action in it increases its internal resistance due to polarization, so that after a period of time the closed-circuit voltage will decrease and in time drop to practically nothing.

g. The amount of current that a cell can furnish without polarization depends on the size of the cell; the bigger the cell, the larger the electrodes, the less the internal resistance, and therefore the greater the current the cell can furnish without too great a difference between its open-circuit and closed-circuit voltages. In general, a large cell can supply a larger current than can a small cell. Also, a large cell can supply a given current for a longer period of time than can a small cell. A dry cell such as Battery BA–23 may be used to ring doorbells or to operate other devices requiring up to 5 amperes intermittently, or it may be used to furnish a steady current of up to 1 ampere. In the case of the intermittent load and larger current, the depolarizer has time to reduce the polarized cell back to normal while the cell is idle. In the case of the steady load and smaller amount of current drain, the depolarizer can reduce the polarization as fast as it occurs, thus maintaining the voltage and current of the cell.

h. The effective life of a dry battery can be greatly prolonged by keeping all terminals and contacts clean and free from corrosion. This is especially true of spring contacts. To remove the dull white surface formed by oxidization of the zinc electrode, scrape the electrode terminal lightly with a knife, being careful not to gouge the metal, and polish it with sandpaper until the terminal is bright. Batteries, battery compartments, and spring contacts should be kept clean and dry.

i. Dry cells swell up when worn out, and they should be removed from equipment when the equipment is stored. Otherwise, the cell or battery will become exhausted by local action, swell up in the equipment, and be very hard to remove. A common example of this is a swollen flashlight battery.

87. Current Capacity Rating of Dry Cells

a. Cells are rated from the point of view of their capacity or ability to furnish a certain amount of current for a certain length of time. The unit in which this capacity is expressed is the *ampere-hour*. To complete the number of ampere-hours, multiply the current flowing in amperes by the length of time it flows in hours. Thus, if a certain cell can furnish ½ ampere continuously for 1 week, its ampere-hour capacity is .5 ampere $\times$ 24 hours per day $\times$ 7 days per week, or 84 ampere-hours.

b. The capacity or ampere-hour rating of a cell is based on a specified *rate* of current flow usually expressed in amperes. If the rate of current flow exceeds that specified, the capacity of the cell may be decreased, and conversely, if the current drain is less, the capacity may exceed the manufacturer's rating. Intermittent use also increases the capacity, as the cell is given the opportunity to rejuvenate when no current is being drawn.

88. Testing Batteries

a. From the preceding paragraphs it is evident that when a load is placed on a cell or battery the terminal voltage decreases. This terminal voltage decrease is relatively small when the battery

is new (not discharged), but the voltage is greater as the battery becomes discharged. Therefore, it is evident that to determine the ability of a battery to perform a specific job for a reasonable length of time, it *must be tested under load*. This method is known as the closed-circuit voltage test.

b. The open-circuit voltage of a dry cell is not a true indication of its condition. A nearly exhausted dry cell may indicate about 1.5 volts on open circuit.

c. Many communication failures are due to the fact that maintenance personnel or operating personnel have failed to test properly the batteries used in a specific instrument before putting the instrument in service. Most communication apparatus used in the field is operated either by batteries or motor generators, hence it is of primary importance that the student learn and *remember* how to test batteries. Never take equipment into the field without checking the closed-circuit voltage of spare batteries and the batteries in use.

89. Methods of Testing Batteries

a. Testing a Battery While Connected to Its Equipment. An excellent method of testing a battery under load is to measure the voltage 15 seconds after it is actually connected and delivering current to the apparatus with which it is used normally. However, this method presupposes that the apparatus is free of trouble and presents its normal load to the battery.

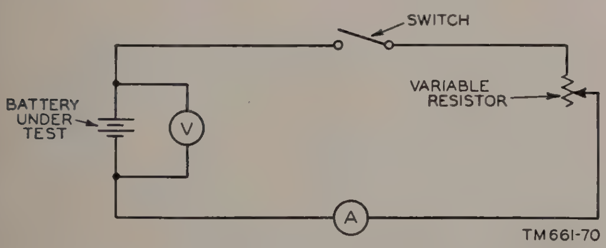


Figure 73. Circuit used for testing a dry cell.

b. Testing a Battery While Not Connected to Its Equipment.

(1) To test a battery which is not connected to its equipment, it is necessary to place its normal load on it before testing its closed-circuit voltage. Battery test sets may be available for this test. If not, place the battery in a circuit such as shown in figure 73. If the normal current drain is known, adjust the variable resistor until the ammeter indicates the proper current. If the normal current drain is not given in the technical manual on the equipment, it may be possible to figure out the resistance in the circuit by referring to the schematic diagram. The variable resistor can then be set to this value by means of an ohmmeter and the proper value of current will flow when the switch shown in figure 73 is closed. After 15 seconds, the closed-circuit battery voltage indicated by the voltmeter will show, by comparison with the required voltage, whether or not the battery is serviceable.

(2) If the current drain or load resistance cannot be ascertained, refer to Department of the Army Supply Bulletin SB 11–30, Shipment and Shelf Life Information—Testing and Disposition of Dry Batteries. This supply bulletin presents a dry-battery testing procedure similar to that shown in figure 73, and furnishes the exact value of load resistance for the various types of batteries used by the Signal Corps.

90. Connecting Dry Cells

The arrangement of the cells in a battery depends on the requirements of voltage and current. If the voltage must be high, cells are connected in series. If the current requirement is high, cells are connected in parallel. If both the voltage and current requirements are high, cells are connected in series-parallel.

91. Connecting Cells in Series

a. When the voltage required exceed 1.5 volts, it is necessary to use more than one cell and the cells must be connected in series. A of figure 74 shows four 1.5-volt commercial type dry cells connected in series. The negative terminal of the first cell is connected to the positive terminal of the second cell, the negative terminal of the second cell is connected to the positive terminal of the third cell, and the negative terminal of the third cell is connected to the positive terminal of the fourth cell. The positive terminal of the first cell and the negative terminal of the fourth cell are free and become the output terminals of the battery. B of figure 74 is a schematic representa tion of the four cells connected in series. The

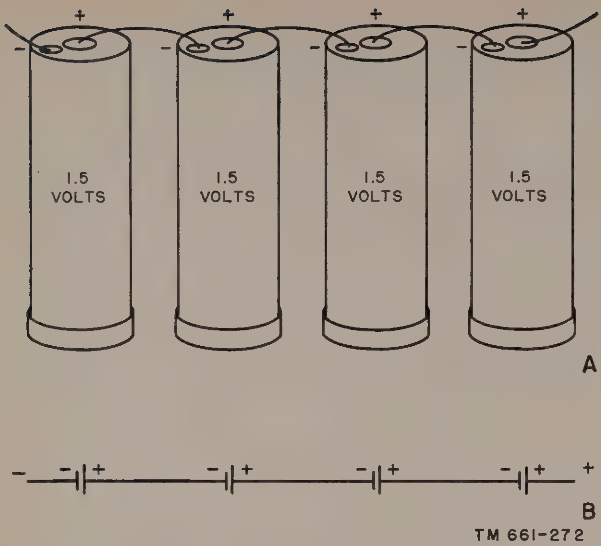


Figure 74. Series connection.

long line represents the positive terminal of each cell and the short line represents the negative terminal of each cell.

b. When cells are connected in series, the same amount of current flows through each cell.

c. When cells are connected in series, the total voltage is equal to the sum of the voltages of the cells. In figure 74, the four 1.5-volt cells furnish 6 volts:

$$1.5+1.5+1.5+1.5=6 \text{ volts.}$$

This is the open-circuit voltage and is obtained only when no current flows or if only a very small current flows. If an appreciable current flows, the closed-circuit voltage is less than the open-circuit voltage due to the combined internal resistance of all the batteries. In the battery described, if the current is .5 ampere, and the internal resistance of each cell is .2 ohm, the voltage drop in each cell is $E=IR$ or $.2\times.5$ or .1 volt. For the four cells, the amount is $4\times.1$ or .4 volt. Subtracting the voltage drop, which due to internal resistance from the open-circuit voltages, gives 6 —.4 or 5.6 volts, which is the closed-circuit voltage of the battery when the current drain is .5 ampere.

d. The ampere-hour capacity of a battery made up of cells in series is the same as that of a single cell, since the same current flows through all cells. The whole battery deteriorates at the rate of a single cell.

92. Connecting Cells in Parallel

a. The normal current drain of a type BA–23, 1.5-volt cell is ¼ to ⅛ ampere. When the current required is more than this, it is necessary to connect two or more cells in parallel. Figure 75 shows four 1.5-volt cells connected in parallel. Note that all the positive terminals are connected together and one terminal is the positive terminal of the battery. Likewise, all the negative terminals are connected together and one terminal is the negative terminal of the battery.

b. When cells are connected in parallel, the total current obtained from the battery is equal to the sum of the currents drawn from the individual cells. Also, since their internal resistances are in parallel, the total internal resistance of the battery is less than the internal resistance of any individual cell, and the ampere-hour capacity of the battery is then slightly greater than the *sum* of the ampere-hour capacities of the individual cells. If the current drain on the battery shown in figure 75 is .5 ampere, each cell furnishes ¼ or .5 ampere, or .125 ampere.

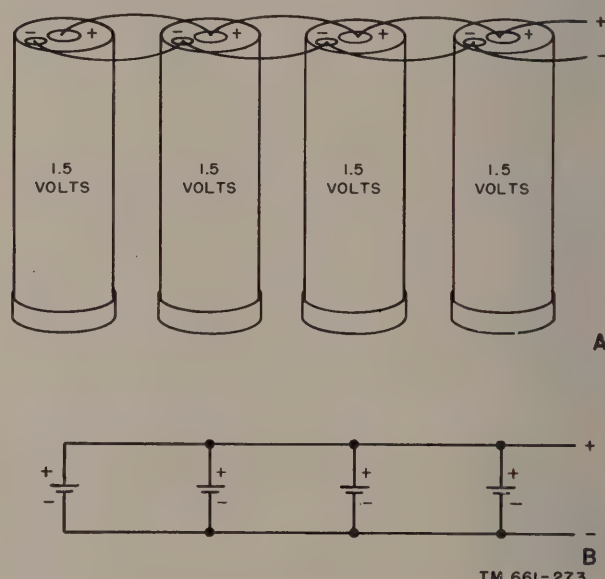


Figure 75. Parallel connection.

c. When cells are connected in parallel, the voltage of the battery is the same as that of one cell. The internal resistance of the battery will be decreased, however, since the internal resistances are in parallel. Thus, if the internal resistance of each cell in figure 75 is .2 ohm, the internal resistance of the battery is .2 divided by 4, or .05 ohm. If the current is .5 ampere, the voltage drop across the internal resistance of the battery is $I\times R$ or $0.5\times.05$, which is .025 volt. By comparison with the series connection of the same four 1.5-volt cells shown in A of figure 74 it can be seen that

the voltage drop due to internal resistance in the parallel combination is considerably less than for a series combination. In both instances the current drain is .5 ampere. In the series connection, the drop in voltage due to internal resistance is .4 volt, while in the parallel circuit the internal voltage drop is .025.

d. The ampere-hour capacity of a parallel combination is equal to the ampere-hour capacity of one cell multiplied by the number of cells.

93. Improper Connections of Cells in Parallel

a. A of figure 76 shows what happens when cells are incorrectly connected in parallel. The voltage of both cells acts in the same direction to send a high current through the complete circuit formed by the two cells. This is equivalent to short-circuiting both cells, and they will be quickly ruined.

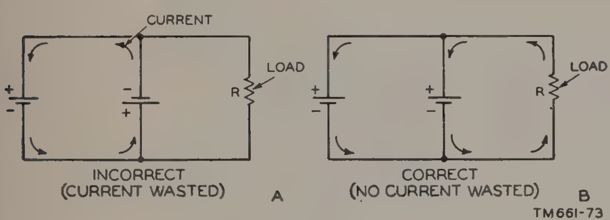


Figure 76. Circuit action when cells are incorrectly connected.

b. B of figure 76 shows the correct method of connecting the two cells in parallel; all the current flows through the external resistance R which represents the load.

94. Effects of Unequal Cell Voltages in Parallel

Only cells of *equal* voltage should be connected in parallel. Figure 77 shows what happens when cells having different voltages are connected in parallel. The cell with the higher voltage, in this instance 2 volts, discharges into the lower-voltage, 1.5-volt cell. Before long, both cells have the same lower voltage.

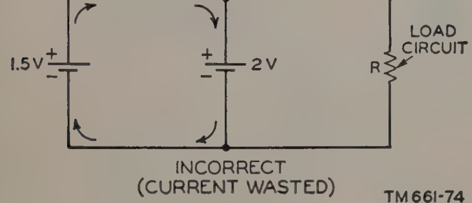


Figure 77. Circuit action when cells of unequal voltages are connected in parallel.

95. Connecting Cells in Series-Parallel

a. If both the voltage and current requirements are higher than the rated voltage and current of a single cell, it is necessary to use two or more cells connected in a series-parallel combination. To obtain the higher voltage, a number of cells are connected in series, and to get the higher current capacity, a number of such series-connected groups must be connected in parallel.

b. Figure 78 illustrates the external connections for four 45-volt batteries arranged to furnish an emf of 90 volts; figure 78 also shows the schematic diagram of such an arrangement. This combination is a series-parallel circuit, with two 45-volt batteries in series to furnish 90 volts, and an additional series-connected group or bank of two 45-volt batteries connected in parallel to furnish increased current capacity.

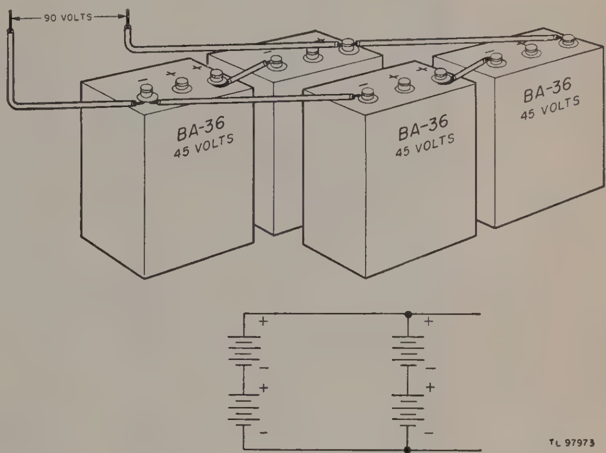


Figure 78. Series-parallel connection.

c. If the current drain from this battery is 100 milliamperes, each series-connected group or bank of cells furnishes 50 ma. Also, since the same current flows through all the cells in a series-connected group, 50 ma flows through each cell. The current drawn from this battery is therefore twice the current drawn from any individual cell.

d. To further increase the battery voltage, it is necessary to increase the number of cells in each bank To further increase the ampere-hour capacity of the battery, it is necessary to connect additional banks in parallel.

e. For cells connected in series-parallel, the following rules apply:

(1) The voltage of the combination is equal to the sum of the voltages of one series-connected bank.

(2) The current in the external circuit is supplied equally by each parallel bank.
(3) The ampere-hour capacity is equal to the ampere-hour capacity of one cell multiplied by the number of parallel banks.

96. Classification of Dry Batteries in Accordance with Use

Dry batteries are classified in accordance with use as follows: "A" batteries, "B" batteries, "C" batteries, and pack batteries

a. "A" batteries (fig. 79) supply high currents at low voltages, and are used in flashlights, in vacuum-tube filament circuits, and in the transmission circuits of local battery telephone systems.

b. "B" batteries (fig. 80) supply small currents at high voltages, and are used to supply plate and screen voltages to vacuum tubes used in radio and radar equipment. An example of a "B" battery is the 22½-volt Battery BA–2 which consists of 15 series-connected 1½-volt cells.

c. "C" batteries (fig. 81) are used to supply minute currents at medium voltages, and are used to furnish bias voltage in grid circuits of vacuum tubes.

d. A pack battery (fig. 82) consists of different types of batteries in one container, such as an "A" and "B" battery.

97. Summary

a. Chemicals can be used to furnish a source of emf.

b. The difference between the open-and closed-

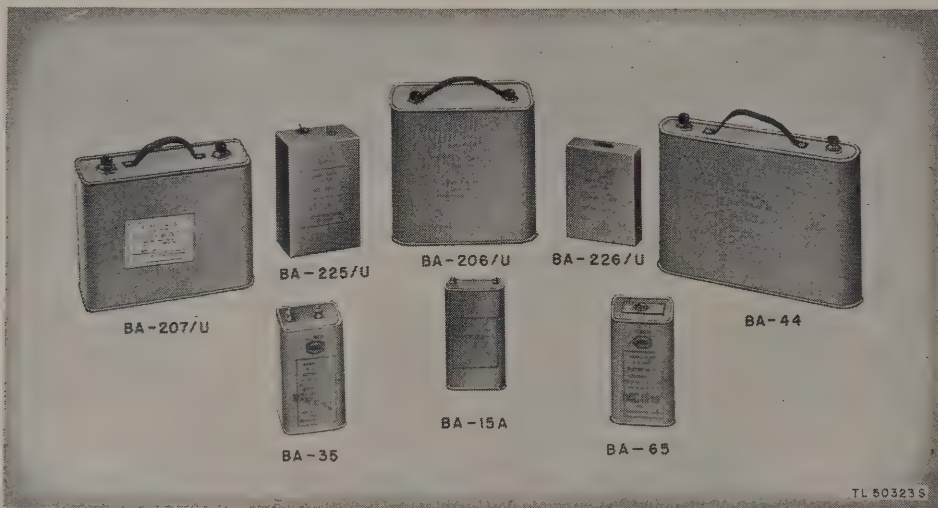


Figure 79. Typical "A" batteries.

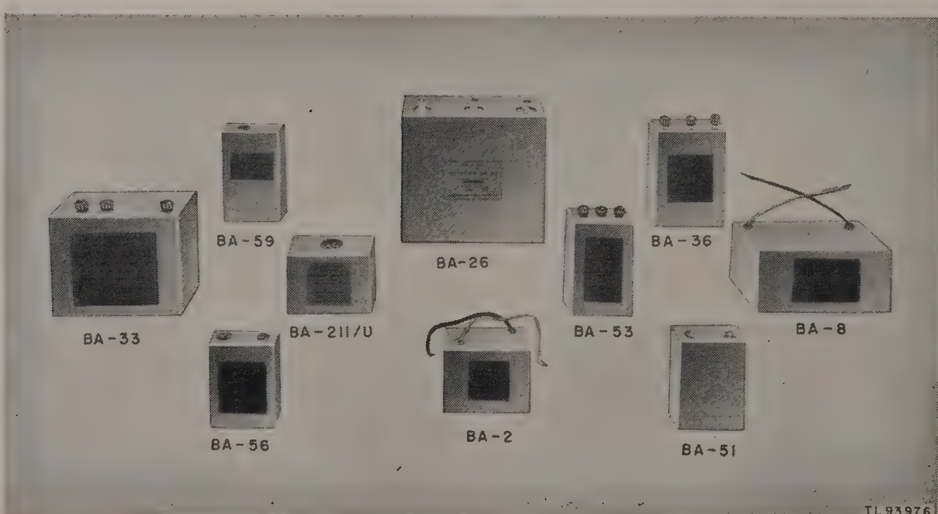


Figure 80. Typical "B" batteries.

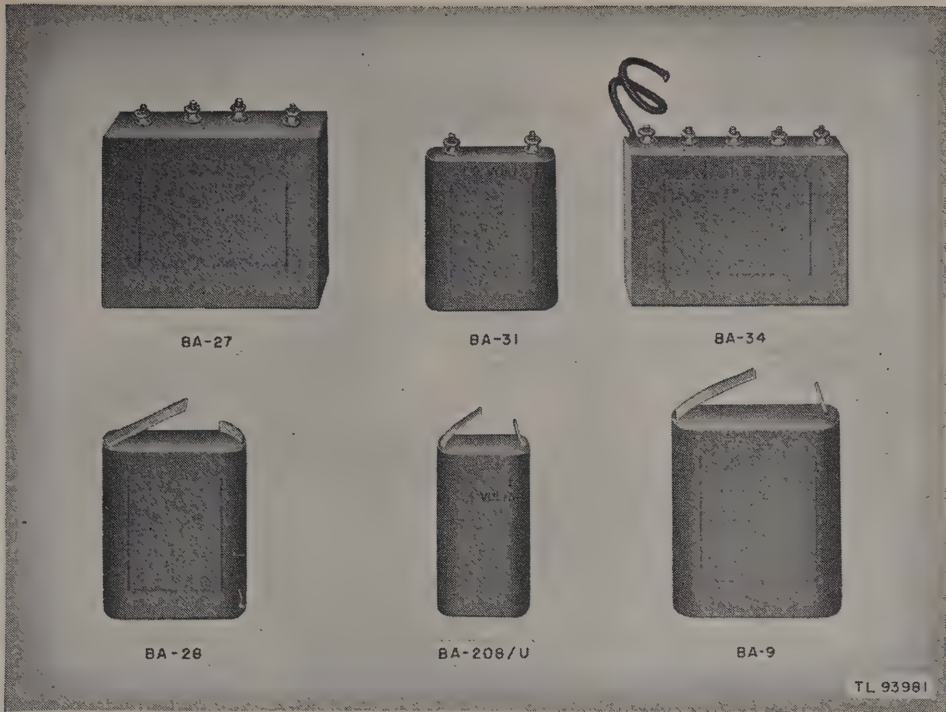


Figure 81. Typical "C" batteries.

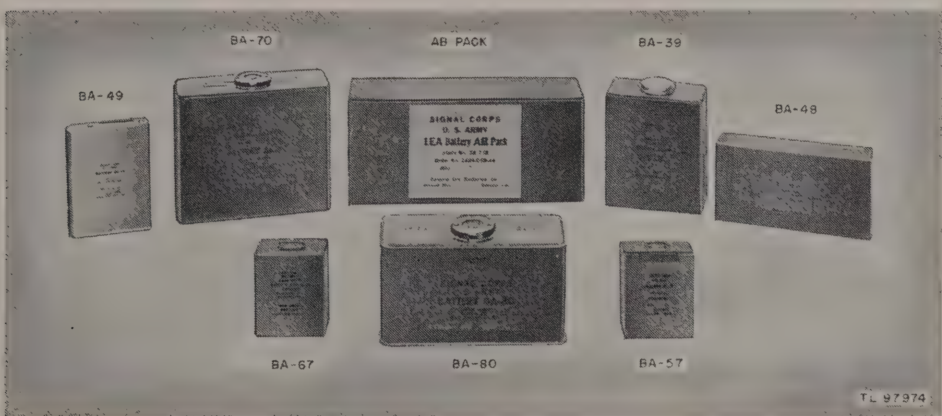


Figure 82. Typical pack batteries.

circuit voltage of a cell is due to the *IR* drop across internal resistance of the cell.

c. The open-circuit or nominal voltage of a cell is dependent on its chemical make-up.

d. The ampere-hour capacity is dependent on the size of the electrodes.

e. Batteries wear out in storage due to local action. Store dry batteries in a cool dry place to decrease local action, but allow them to return to normal temperature before use.

f. Remove dry batteries from equipment when equipment is stored.

g. Red wire indicates the positive terminal.

h. Dry cells are rated in ampere-hours at a specified rate of current flow.

i. Batteries should be tested under load.

j. For cells connected in series:

(1) The voltage of the battery is the sum of the voltages of the individual cells.

(2) The total internal resistance is the sum of the internal resistances of the individual cells.

(3) The same current flows through each cell.

(4) The ampere-hour capacity is the same as the ampere-hour capacity of one cell.

k. For cells connected in parallel:

(1) The voltage of the battery is the same as the voltage of one cell.

(2) The internal resistance is the internal resistance of one cell divided by the number of cells.

(3) The total current obtained from the battery is the sum of the equal currents drawn from each cell.

(4) The ampere-hour capacity of the battery is slightly greater than the sum of the ampere-hour capacities of the individual cells.

(5) When cells are improperly connected in parallel, they are quickly ruined.

l. For cells connected in series-parallel.

(1) The voltage of the battery is the sum of the voltages of the cells connected in series in one bank.

(2) The internal resistance is the sum of the resistances of the cells in one bank divided by the number of banks.

(3) The total current obtained from the battery is the sum of the equal currents drawn from each bank.

(4) The ampere-hour capacity of the battery is equal to the ampere-hour capacity of one cell multiplied by the number of parallel banks.

m. *A* batteries supply high currents at low voltages; *B* batteries supply small currents at high voltages; *C* batteries supply very small currents at medium voltages; pack batteries consist of several types of batteries in one container.

98. Review Questions

a. Name the components of a simple voltaic cell.

b. Is one of the electrodes of the primary cell consumed during discharge of the cell?

c. What is the difference between a primary and secondary cell?

d. What is local action?

e. How is local action reduced?

f. Will local action discharge an unused cell?

g. Why does a cell have internal resistance?

h. What is meant by the open-circuit voltage of a cell?

i. What is meant by the closed-circuit voltage of a cell?

j. Why is the open-circuit voltage greater than the closed-circuit voltage?

k. What is meant by polarization?

l. What effect does polarization have on the internal resistance of a cell?

m. Why is manganese dioxide included in the LeClanche dry cell?

n. Are all types of dry cells used by the Army ready for immediate service? Explain.

o. How is polarity indicated on primary batteries?

p. Upon what does the voltage of a primary cell depend?

q. Upon what does the current furnishing ability of a dry cell depend?

r. Why are dry batteries removed from equipment when not in use?

s. Why should dry batteries be stored in a cool dry place?

t. Why is the date of manufacture stamped on dry batteries?

u. What is meant by the capacity of a cell? In what unit is the capacity expressed?

v. Why should a battery be tested under load?

w. How can the proper load current be determined?

x. What is the open-circuit voltage across the outside terminals of five cells of 1.5 volts each connected in series?

y. What is the open-circuit voltage across the outside terminals of five cells of 1.5 each connected in parallel?

z. Will connecting cells in series increase the length of time that an individual cell will furnish a given discharge current? Explain.

aa. Over a period of time does the voltage of a cell decrease as it discharges?

ab. Is the internal resistance of a combination of cells greater when connected in series or when connected in parallel?

ac. Show schematically:

(1) 5 cells connected in series.

(2) 4 cells in parallel.

(3) A battery of 30 cells which are connected 10 in series and have 3 banks in parallel.

ad. A battery of 24 cells, each cell having a resistance of .2 ohm and an emf of 1.5 volts, is connected 6 cells in series and 4 banks in parallel. What is the emf of the battery? What current will the battery drive through an external resistance of 1.97 ohms? What is the closed-circuit voltage?

ae. What is the difference between "A," "B," "C," and pack batteries?

CHAPTER 8
SECONDARY CELLS

99. General

a. Secondary cells, or storage batteries, as they are familiarly referred to, are well known to everyone because of their general use to provide electricity for automobile lighting, ignition, and starting systems. Secondary cells supply the electric power for telephone central offices, emergency operation of radio equipment at sea, and for a wide variety of mobile equipment designed for military and civilian use. Portable electric light plants make use of secondary cells for automatic starting.

b. The term *storage battery* is an unfortunate choice, since the battery does not store electricity. Chemical reactions take place when a direct current of proper polarity is passed through the battery, which is known as *charging*. The substances thus formed react chemically to produce electricity when the battery is in use, and this is referred to as *discharging*. Therefore, electrical energy is transformed into chemical energy during charging. When the battery is connected to a load, the chemical energy is reconverted to useful electrical energy. Therefore, it can be said that chemical energy can be stored, but electrical energy cannot be stored.

c. The operation of the secondary cell, like the primary cell, is based on chemistry. However, there is one important difference: The secondary cell can be recharged, restoring it approximately to its original chemical condition and thus extending its useful life. This is not true in the case of primary cells. We learned in chapter 7 that the negative electrode of the primary cell is consumed during use. Therefore, the dry cell destroys itself and must be replaced with a new cell, since it is not practical to replace its electrode and electrolyte. One class of primary cells, the wet cell type, can be restored to usefulness by replacing the negative electrode and the electrolyte. However, this replacement of the elements of the cell should not be confused with the recharging process. The chemistry of the charging and discharging of a secondary cell is explained in subsequent paragraphs of this chapter.

d. Two types of secondary cells are in universal use today: One is the lead-acid cell and the other is the Edison cell. The lead-acid battery is widely used by the military services. Consequently, the theory of operation and care of the lead-acid type is described in great detail and only general information covering the Edison cell is included.

100. Chemical Action of Lead-acid Secondary Cells

a. The action of a lead-acid secondary cell or storage battery is similar to the action of a primary cell, since a chemical reaction between the two electrodes and the electrolyte provides electrical energy to an external circuit. A fundamental type of a lead-acid cell is shown in figure 83. When the cell is fully charged (A of fig. 83), the *active* material of the negative electrode of plate is made of gray metallic lead in finely divided form commonly called spongy lead (Pb) and the *active* material of the positive electrode is made of a chocolate-brown colored lead peroxide (PbO_2). Lead peroxide is sometimes called lead dioxide. These materials are not hard enough to be made into plates that are mechanically rugged, therefore, frames or grids of a hard lead-antimony alloy are used, and the spongy lead and lead peroxide are pressed into the openings of the grids (fig. 85). The term *active material* is used to refer only to the spongy lead and the lead peroxide. The electrolyte is a dilute solution of sulphuric acid (H_2SO_4) and water (H_2O).

b. As shown in figure 83, the chemical actions that take place in the battery are as follows:

(1) For a fully charged battery:

(*a*) The negative plate is composed of spongy lead (Pb).

(*b*) The positive plate is composed of lead peroxide (PbO_2).

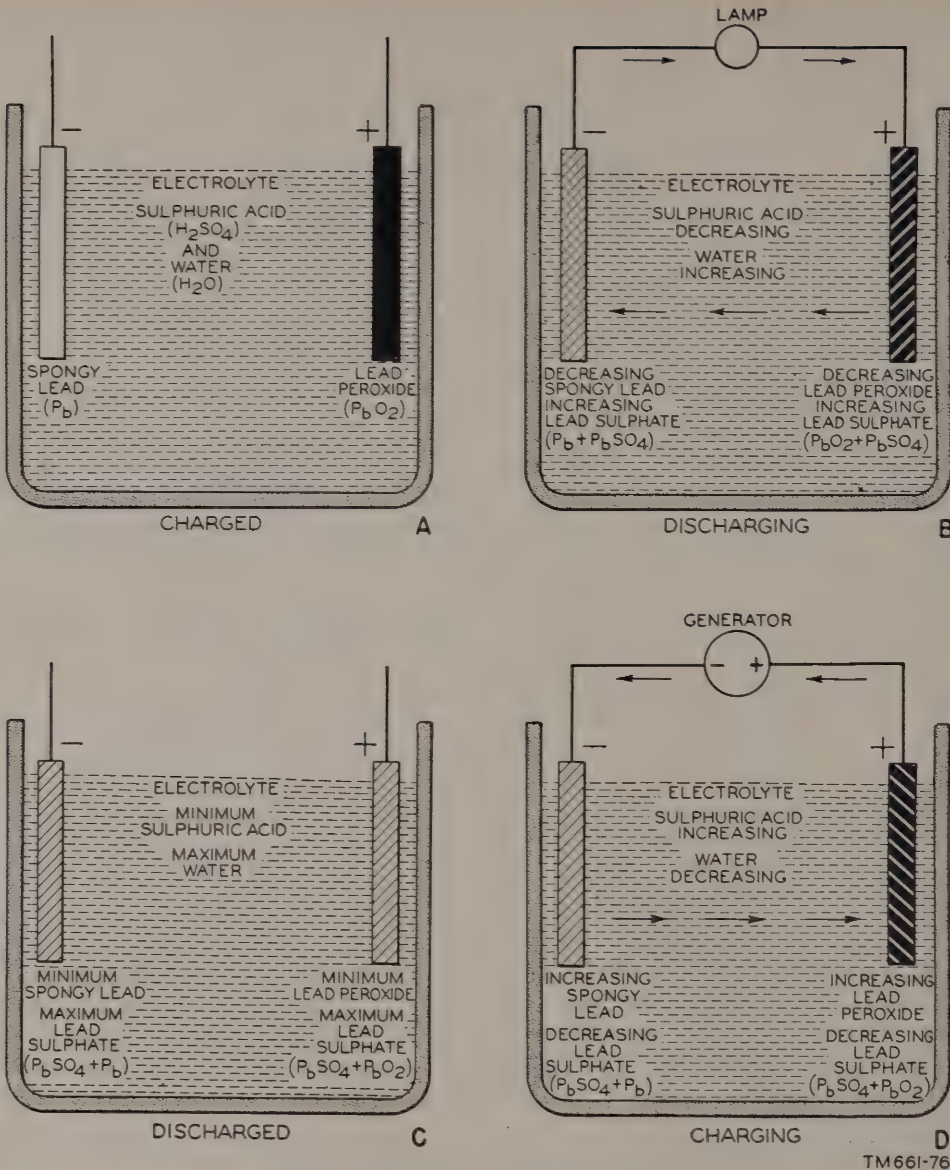


Figure 83. Chemical action of lead-acid cell.

(c) The electrolyte is a dilute solution of sulphuric acid (H_2SO_4) in water (H_2O).

(2) For a discharging battery:

(a) At the negative plate, the spongy lead (Pb) decreases and lead sulphate ($PbSO_4$) increases.

(b) At the positive plate, the lead perioxide (PbO_2) is decreasing and the lead sulphate ($PbSO_4$) is increasing.

(c) The sulphuric acid (H_2SO_4) in the electrolyte is decreasing and the water (H_2O) is increasing.

(3) For a discharged battery:

(a) The negative plate has changed to a minimum of spongy lead (Pb) and a maximum of lead sulphate ($PbSO_4$).

(b) The positive plate has changed to a minimum of lead peroxide (PbO_2) and a maximum of lead sulphate ($PbSO_4$).

(c) The electrolyte has changed to a solution of minimum sulphuric acid (H_2SO_4) and a maximum of water (H_2O).

(4) During charging of a battery:

(*a*) + terminal of the battery is connected to the + terminal of the battery charger and the − terminal of the battery to the − terminal of the charger.

(*b*) The negative plate is changing back to spongy lead (Pb) and the lead sulphate ($PbSO_4$) is decreasing.

(*c*) The positive plate is changing back to lead peroxide (PbO_2) and the lead sulphate ($PbSO_4$) is decreasing.

(*d*) The percentage of sulphuric acid (H_2SO_4) in the electrolyte is increasing and the percentage of water (H_2O) is decreasing.

(5) Hydrogen and oxygen gases, when combined, form an explosive mixture which can be ignited by a spark.

(6) Distilled water must be added during charging to replace the water lost.

c. The chemical formula for the reactions that take place in a lead-acid battery is:

$$\underset{\text{(spongy lead)}}{Pb} + \underset{\text{(lead peroxide)}}{PbO_2} + \underset{\text{(sulphuric acid)}}{2H_2SO_4} \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftarrows}} \underset{\text{(lead sulphate)}}{2PbSO_4} + \underset{\text{(water)}}{H_2O}$$

Refer to appendix II for a complete discussion of the chemistry of a lead-acid cell.

101. Construction of Lead-Acid Storage Batteries

a. The simplified cell shown in figure 83 is of little practical value because its capacity or flow of current is too small. To get large capacity, the electrodes or plates must have a large area of active material. In commercial batteries (fig. 84) this is accomplished by connecting several negative plates in parallel and sandwiching the positive plates between the negative plates. A plate (fig. 85) consists of a thin rectangular grid

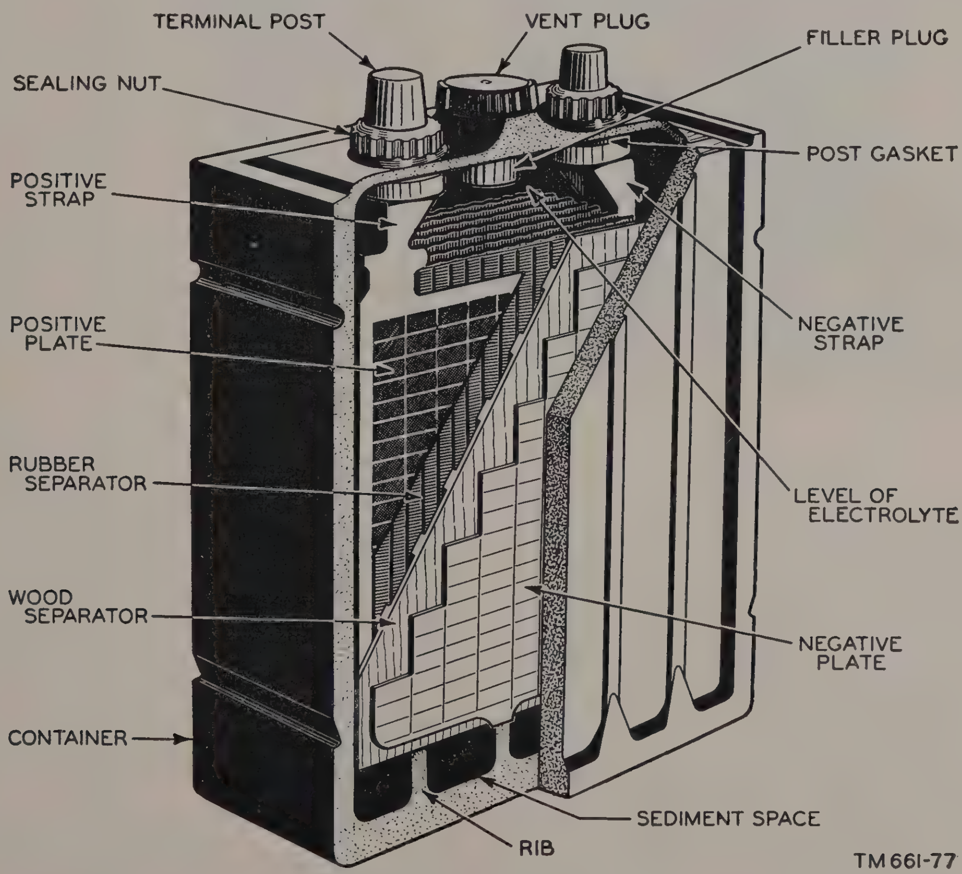


Figure 84. Cutaway view of single cell.

framework cast of a lead-antimony alloy. This grid is designed to give the plates mechanical strength and to provide a means for conducting electric current between the exterior terminals and the active material with which the spaces in the grid are filled. This active material is composed chiefly of oxides of lead when it is originally applied in paste form. When this paste has dried, the plates are given a forming charge to make them either positive or negative. The positive plate is made by a forming charge that causes the active material to become lead peroxide (PbO_2). It is very porous to permit the electrolyte to penetrate the plate freely. The negative plate is made by a forming charge that makes the plate spongy lead, and it is also porous so the electrolyte can penetrate it freely.

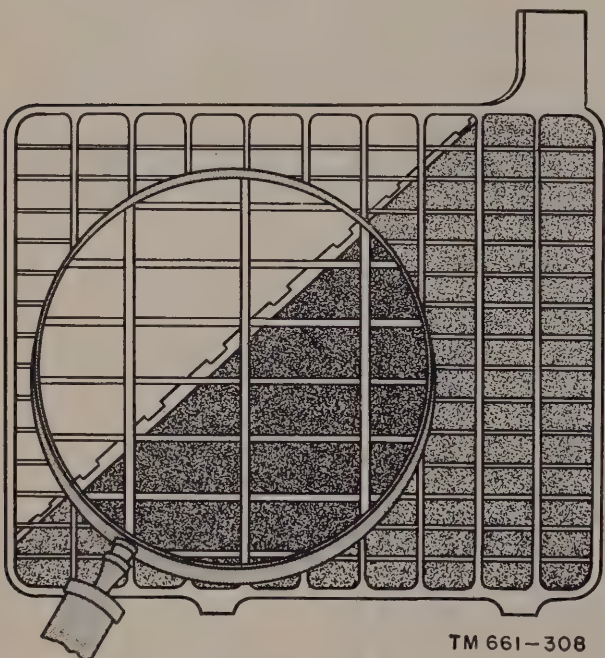


Figure 85. Plate showing grid framework.

b. If positive and negative plates touch, the battery will be short-circuited. To prevent this, thin sheets of nonconducting porous material called separators are inserted between the plates. The separators must be porous to permit the chemical action of the battery. Ordinarily, separators are made of wood, rubber, or fibrous glass. They are smooth on the side next to the negative plate and grooved vertically to allow the escape of the oxygen gas that is formed and to permit loosened active material to fall to the bottom on the side next to the positive plate. Wooden separators usually are made of cedar. This wood is chemically treated to make it more porous and to remove certain resins that would injure the battery. When batteries are designed to withstand excessive vibration, a mat of spun glass or a thin sheet of perforated rubber is placed between the separator and the positive plate. This helps hold the active material in the plate.

c. Each plate has a lug on one top corner. The positive plates are arranged with all their lugs on one side of the cell and the negative plates are arranged with all their lugs on the opposite side of the cell. Plates are combined into positive and negative groups by welding these lugs into slots in a plate strap, which is made of lead. The negative group usually has one more plate than the positive group. The plate strap has a cylindrical terminal post that forms one outside connection for the cell.

d. The positive and negative groups are assembled to form an *element*. Usually, each positive plate has a negative plate on either side of it. There may be an odd number of plates in an element, and they may be of any desired size. When separators are placed between the plates of an element and the whole assembly is immersed in electrolyte, it becomes a *cell*. The open-circuit voltage of a charged cell is always about 2.1 volts regardless of the size of the element. However, the total area of the plates in a cell determines the capacity of the cell.

e. The cells of a battery are contained in a single molded receptacle divided into compartments. Each compartment is the container for one cell. The container is made of hard rubber, wood, glass, or composition material that is resistant to acid and mechanical shock. In batteries that are not top supported, the bottom of each cell compartment usually has molded narrow rests or ribs which support the element. Each plate has projections or stub feet that fit in these rests. In batteries having element rests, the area between serves as sediment space. During the normal life of the battery, the positive plates gradually shed the active material which then falls down through the grooves of the separators and collects in the sediment space between the elements rests.

f. The top of each cell is fitted with a cell cover, molded from hard rubber or acid-resistant composition. The terminal posts from the positive and negative groups extend through openings in the cover. A third opening is fitted with a vent plug. This plug is removed to inspect the electrolyte

level of the cell or to add distilled water. When the plug is closed, a small hole in its center permits the escape of gases formed in the cell. The space between the edges of the cell covers and the container is sealed with an acid-resistant battery sealing compound. The cells in a battery are permanently connected together in series (sometimes in parallel) by connector straps. These straps, which are made of a lead-antimony alloy, fit over the terminal posts of adjacent cells and are welded tilled water. The electrolyte of a fully charged battery usually contains about 38 percent sulphuric acid by weight or about 27 percent by volume. New batteries usually arrive from the manufacturer filled with electrolyte and only require proper charging. However, sometimes batteries are shipped dry and must be filled with electrolyte having the recommended specific gravity. A more detailed study of what specific gravity means and how it is employed to test

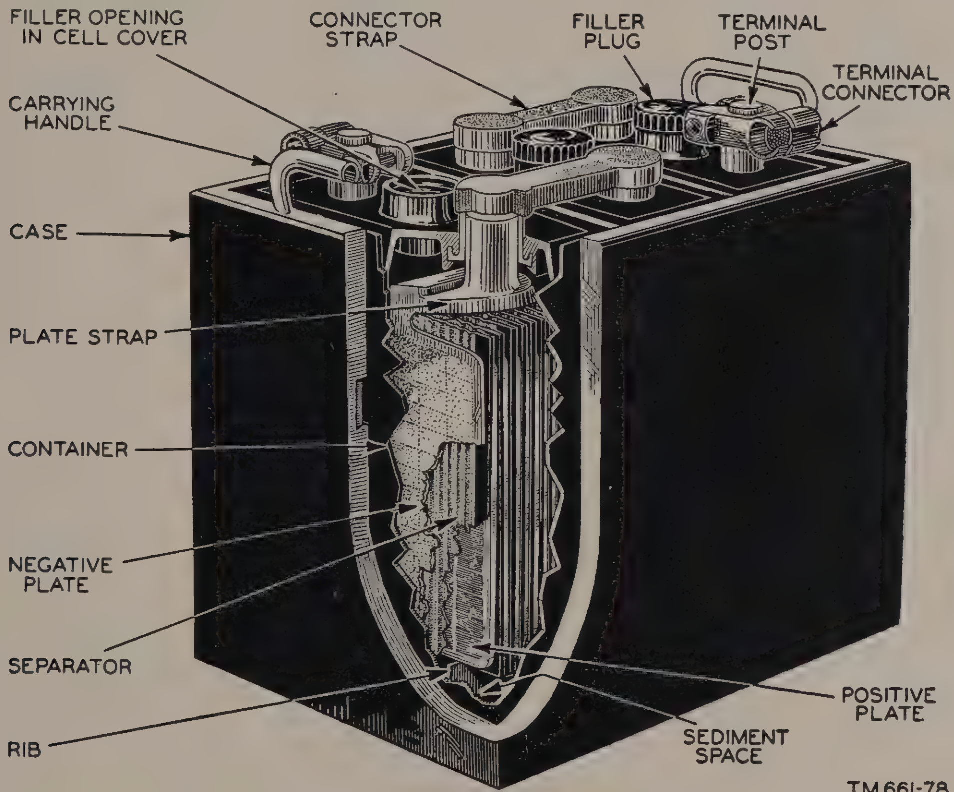


Figure 86. Cutaway of a 6-volt, 3-cell battery.

in place. The elements are so arranged that the negative terminal of one cell is next to the positive terminal of the next cell. The terminal posts may be marked + or —, or the positive terminal alone may be marked with a + or with red paint. The positive terminal is slightly larger than the negative terminal to lessen the possibility of connecting the battery in reverse. A sectional view of a typical 6-volt, 3-cell battery is shown in figure 86.

102. The Electrolyte

The electrolyte used in a lead-acid secondary cell is a dilute solution of sulphuric acid and dis-

batteries will be taken up later. New batteries may come with acid of specific gravity of 1.835 or electrolyte of 1.400 specific gravity. In these cases it will be necessary to dilute the acid with distilled water. The container used to dilute the acid should be made of glass, earthenware, or lead.

Caution: Never pour water into acid. Always pour acid into water. If water is added to sulphuric acid, it will splatter and may cause severe acid burns. Pour the acid slowly into the water, stirring gently with a clean wooden paddle. It is wise to wear goggles, a rubber apron, and rubber gloves to protect the body and clothing.

103. The Rating of Secondary Cells

a. A battery for a particular application must have the correct voltage and must be able to furnish sufficient current for a long enough time. Let us now consider what is meant by battery voltage, battery current, and ampere-hour rating.

b. Voltage can be described in various ways. It is said to be a potential difference. This means that the chemical action of the battery produces a rise in potential from the negative plate to the positive plate. This is because electrons move from points of lower potential to points of higher potential. This potential difference is an emf that forces electrons to flow from the negative to the positive plate, when an external load is connected between the plates. The emf or voltage causes an electric current to flow through the load and do work. The open-circuit voltage (voltage when not discharging) of a fully charged cell is about 2.1 volts. This is true regardless of the size of the cell. The normal voltage of a discharged cell is 1.75 volts. When supplying a moderate current to an external load, the voltage drops to 2.0 volts. This is due to the internal resistance of the cell. When additional voltage is required, more cells are connected in series. A 6-volt battery has three cells connected in series, and a 12-volt battery has six cells connected in series. Therefore, the voltage of a battery depends on the number of cells, and is independent of the size of the cells.

c. The current that a battery will deliver depends on the total area of the plates comprising a cell, the internal resistance of the cell and connections, and the rate at which the chemical reaction takes place. This means that the number of plates which are connected in parallel, and their condition, determine the current capacity of a cell. A cell that has become *sulphated* or has lost some of its active material due to *shedding* will deliver less current. A battery which has corroded connections will have more resistance and, consequently, will have less current capacity. Finally, the speed of the chemical reaction within a cell depends, to a great extent, on the strength of the electrolyte. This fact indicates that a fully charged battery will deliver more current than a discharged battery.

d. Battery capacity is a current-time rating and is measured in ampere-hours. This capacity is determined by multiplying the amperes of current the battery will deliver by the number of hours the battery will deliver it. Thus, a battery capable of delivering 5 amperes of current for 20 hours has a rating of 100 ampere-hours. This means we can expect to draw 5 amperes of current for 20 hours before the battery reaches its normal discharge condition (1.75 volts). Also, the same battery will deliver 2.5 amperes for 40 hours. The ampere-hour rating is normally based on a 20-hour discharge rate at 80° F. In general, the ampere-hour rating depends upon the total plate area multiplied by the number of cells in parallel. If a battery is discharged at a current higher than its 20-hour rate, its ampere-hour capacity will be much less than its rated ampere-hours; if discharged at a lower rate, it may deliver considerably more than its rated ampere-hours. Thus, a battery will deliver more ampere-hours at a long, low, or intermittent rate of discharge than at a short, high, or continuous rate. The ampere-hour rating divided by 20 is the normal rate of discharge for the battery. Temperature also affects the ampere-hour capacity. There are other battery ratings and the manufacturers specifications should be consulted for this information. Also, further battery ratings are given in TM 9–2857.

104. Charging of Lead-Acid Batteries

a. During the life of a battery, some device must be used to keep the battery in a charged condition. In the case of automobiles, a generator that is attached to the engine supplies electric current to the battery when the engine is running. However, in other applications, the electric current taken out must be replaced by recharging. It is necessary to place more energy in the battery on charge than is taken out on discharge. This is because batteries are not 100-percent efficient. It is desirable to keep batteries as nearly fully charged as possible. When a battery becomes discharged, it must be recharged immediately or permanent damage may result.

b. Only direct current can be used to charge a storage battery. If only alternating current is available, it must be converted to direct current by means of a rectifier or motor generator. Methods of charging can be divided into two broad classifications: constant-current charging and constant-voltage charging.

c. The constant-current method is used to charge single batteries or a number of batteries in series. A circuit for charging a battery is shown in figure 87. A d-c generator whose emf is greater

than the battery is used. The positive terminal of the charging source is connected to the positive terminal of the battery and the negative terminal of the generator is connected to the negative terminal of the battery. The rheostat and ammeter are included to adjust and observe the charging rate. The rheostat is adjusted to supply a voltage slightly more than the internal voltage of the battery with a charging current of from 5 to 10 amperes. As the voltage of the battery rises, the rheostat is set to an increased voltage to keep the charge of current constant. This is a slow method, requiring from 20 to 30 hours. If properly watched, this method has the advantage of not overheating the battery, thus reducing the danger of buckling the plates.

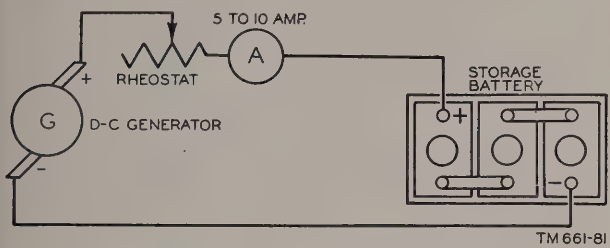


Figure 87. Constant-current charging circuit.

d. The constant-voltage type of charger consists of a motor generator or rectifier that provides a constant voltage which is slightly higher than the voltage of the battery. When more than one battery is to be charged, the batteries are connected in parallel. Figure 88 shows constant-voltage charging. When the battery is in a discharged condition, the source supplies a very high amperage, frequently as high as 30 to 50 amperes. As the battery becomes charged and its voltage increases, the amperage from the charger reduces. This is called the taper-charge method. This type of charging is relatively fast, taking 6 to 8 hours. One drawback to this method is that it may cause overheating and buckling of plates, if used on a completely discharged battery.

e. Two specialized types of charging are high-rate charging and trickle charging.

(1) A high-rate charger delivers an extremely high current (100 amperes or more) for a very short time and then drops to a normal value. During charging, the maximum battery temperature is 125° F. instead of 110° F. which is common to all other types of charging. This method is capable of destroying a battery if not controlled correctly and is not used at present by the Signal Corps.

(2) Trickle charging is the continous application of low-charging current and is usually a constant-voltage method. This maintains the electrolyte at maximum specific gravity and the batteries remain at full charge for long periods of time. This type of charging is generally used for stationary batteries.

f. It is important to remember that charging rates are specified by manufacturers. These rates should always be followed to avoid overcharging or charging at too high a rate. Either condition will result in the *shedding* of the active material of the plates, thus reducing the capacity of the battery and shortening its life. Violent gassing indicates that the charging rate is too high.

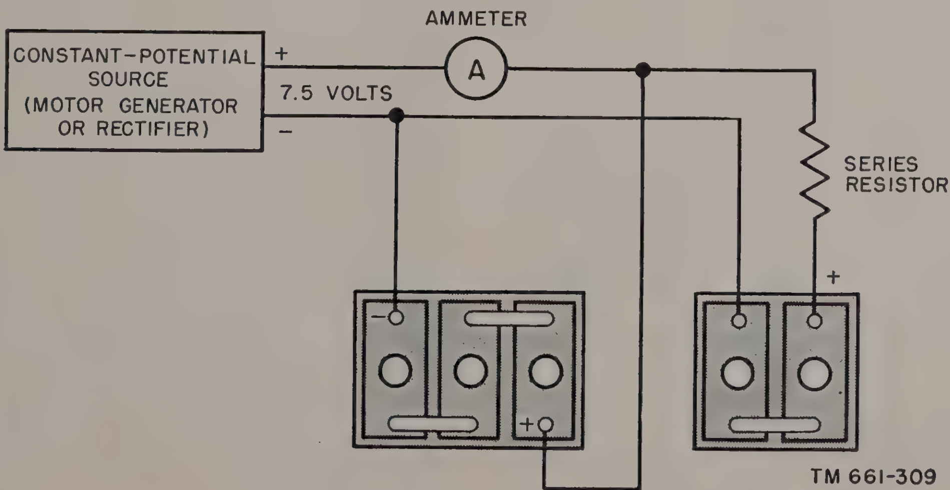


Figure 88. Constant-voltage charging circuit.

105. Testing of Lead-Acid Batteries

a. Previously in this chapter, it was learned that the amount of sulphuric acid decreases and the amount of water increases as the battery discharges (par. 100*b*). Also, the reverse is true as the battery charges. This means that the strength or concentration of acid in the electrolyte indicates the amount of charge in the battery. The specific gravity of the electrolyte is a measure of the amount of sulphuric acid in solution. Specific gravity is defined as the ratio of the weight of a given volume of any liquid to the weight of a given volume of water. Pure sulphuric acid has a specific gravity of 1.835 and the specific gravity of water is 1.000. Therefore, any solution of sulphuric acid and water will have a specific gravity greater than 1.000. A normal fully charged battery has a specific gravity of 1.270 to 1.300. Usually the decimal point is ignored and it is customary to say that a fully charged battery has a specific gravity of approximately 1280.

b. To test the charge of a lead-acid battery, we measure its specific gravity. A convenient instrument for making rapid measurements of specific gravity is the hydrometer (fig. 89). The hydrometer works on the principle discovered by Archimedes, that a floating body will sink deeper into a light liquid (one of low specific gravity) than into a heavy liquid. The hydrometer consists of a glass syringe containing a glass hydrometer float. The float is a long-necked bottle, weighted in the lower end, and having a long thin neck with graduations from 1100° to 1300°. The thin neck also carries the legends GOOD, FAIR, POOR, and DEAD with a zoning color scheme. The battery electrolyte is sucked up by the syringe until the glass float is free to float. The depth to which this float sinks is an indication of the specific gravity of the electrolyte. The electrolyte of a discharged battery is lighter than the electrolyte of a fully charged battery. The float will sink deeper in the case of a discharged battery. It is said to have lower specific gravity and the graduations on the float will indicate that fact. A fully charged lead-acid battery of the portable type should have a specific gravity of 1270 to 1300. A completely discharged cell has a specific gravity of 1150 to 1210. These figures are for a normal range of temperatures.

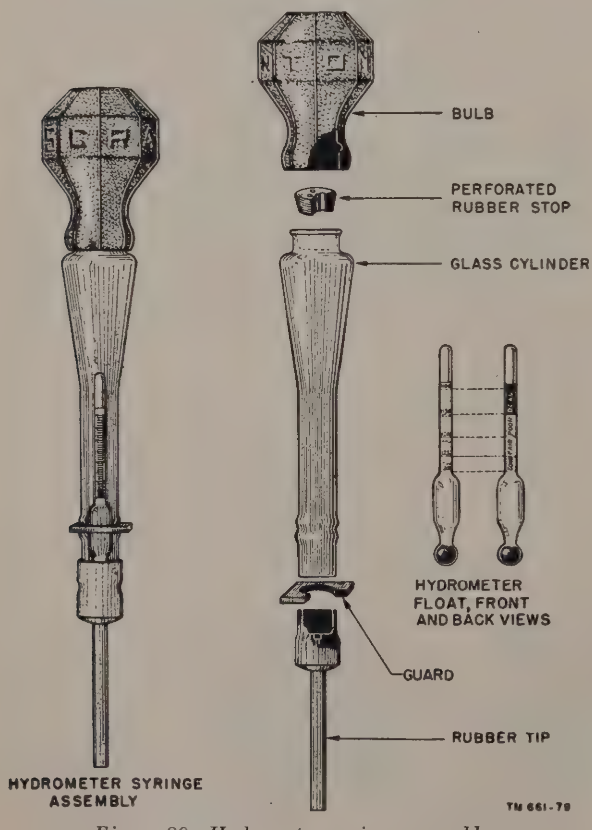


Figure 89. Hydrometer syringe assembly.

c. Care should be exercised in using the syringe hydrometer. The electrolyte should be drawn in slowly to prevent breakage of the float against the top of the syringe; similarly, the electrolyte should be expelled gradually to avoid breaking the float against the bottom of the syringe. The hydrometer must be held in a vertical position with just enough electrolyte drawn in so that the float moves freely. The float must not touch the sides or bottom of the syringe when readings are taken.

d. The specific gravity of the electrolyte is affected by its temperature, decreasing at high temperatures and increasing at low temperatures. To correct hydrometer readings for temperature, add 4 points to the reading for every 10° above 80° F. and subtract 4 points for every 10° below 80° F. A correction chart is shown in figure 90. To use this chart, add or subtract the figure opposite the temperature reading to or from the specific gravity reading. For example, a hydrometer reading of 1280 at 40° F. should be 1280 minus 16, or 1264; a reading of 1210 at 110° F. should be 1210 plus 12, or 1222. If a battery is to be operated at temperatures as low as −65° F., the specific gravity of the electrolyte should be increased according to specific instructions for the particular battery.

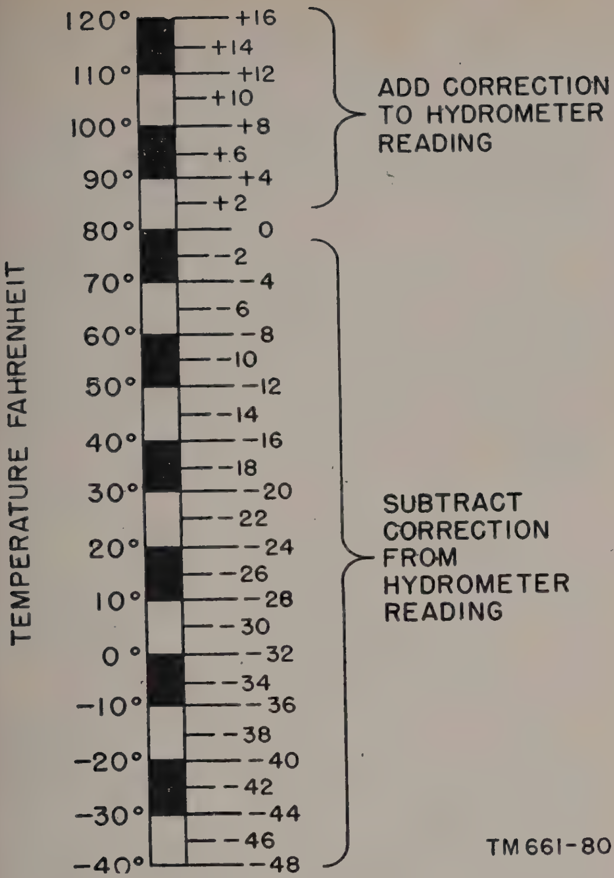


Figure 90. Hydrometer correction chart.

e. All storage cells do not have the same maximum and minimum specific gravity. For example, the open type of stationary storage battery used in common battery telephone applications, has a maximum specific gravity of 1210 and a minimum specific gravity of 1170. The manufacturer of a battery furnishes instructions with each battery and these instructions should be followed in battery maintenance. Also, the maximum specific gravity of the electrolyte is increased when used in cold climates, and conversely, decreased when used in hot climates. For more detailed information on this subject, see TM 9-2857.

106. Care of Lead-Acid Batteries

a. Batteries, like any other piece of equipment, need intelligent care. To one who has a thorough understanding of how a lead-acid battery works, care and maintenance procedures should have more meaning. If these procedures are followed faithfully, reliable and long-life service of the battery will result.

b. One of the important things to remember is that a battery should never be allowed to remain in a discharged condition for any appreciable time. If allowed to remain in a discharged condition, the lead sulphate will grow into a hard, white crystalline formation. This is known as *sulphation.* This condition closes the pores in the active material and destroys the plates.

c. The electrolyte should be maintained at its proper level, which is ⅜ inch above the top of the plates. Failure to do so, leaves the plates exposed to air and causes rapid sulphation. Regular checking of the electrolyte level is a "must" and, if low, should be filled with distilled water.

d. The reason that a sulphated battery may be damaged beyond repair is that, when placed on charge, some of the lead sulphate, instead of changing back to spongy lead or lead peroxide, is dislodged from the plates in small particles and drops to the bottom as a sediment. This material is lost forever for active use. We would like to point out that in normal operation all cells shed a small amount of active material. This is the reason they wear out. However, this process is quickened in the case of a sulphated cell, and it should be obvious that the life of the battery is greatly reduced.

e. Frequently, a greenish deposit of copper salts forms on terminals and connectors. This corrosion may be caused by spilled electrolyte or seepage through the terminal post seals. It should be scraped off with a dull knife or removed with a stiff brush. The corroded area should be washed with ammonia and water or baking soda and water to neutralize any remaining electrolyte. These areas should be coated with a thin film of grease or rust preventive compound to prevent corrosion.

f. The freezing point of the electrolyte depends on its specific gravity. As the specific gravity drops, the freezing point rises. If a battery freezes, the active material is forced from the plates, and if extensive the plates will buckle and the separators will break. Therefore, for cold weather operation, the battery should be kept as near to full charge as possible.

g. Never add electrolyte or sulphuric acid to a storage battery unless it is known that electroltye has been spilled. In this case, replace the electrolyte that has been spilled. Merely adding electrolyte, although it raises the specific gravity, does not recharge a battery, because only a direct current will chemically change the lead sulphate.

107. The Edison Storage Battery

a. The Edison storage cell or nickel-iron-alkaline battery is not so widely used as the lead-acid storage cell, but it will be found useful to know some general facts about it. This is one of the many developments of Thomas A. Edison, whose name is associated with the battery.

b. The positive plate consists of rows of nickel-coated tubes filled with alternate layers of nickel hydroxide and very thin flakes of pure nickel. The negative plate consists of a grid of cold-rolled nickeled steel having a number of rectangular pockets, which are filled with an iron oxide. The separators, which are used to separate the plates, are made of hard rubber. The electrolyte is a 21-percent solution of potassium hydroxide in distilled water to which is added a small amount of lithium hydrate. The electrolyte has a specific gravity of approximately 1.200 at 60° F and does not vary appreciably during the charge and discharge cycles. The container is made of nickeled sheet steel.

c. The active material of the positive plate is an oxide of nickel and that of the negative plate is pure iron. On discharge, the pure iron of the negative plate is oxidized and the nickel dioxide of the positive plate is changed to nickel oxide. On charge, the reverse action takes place. The chemical equation for the reaction of the Edison cell is—

$$8KOH + 6NiO_2 + 3Fe \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftarrows}} Fe_3O_4 + 2Ni_3O_4 + 8KOH$$

$8KOH+$	$6NiO_2+$	$3Fe$		Fe_3O_4+	$2Ni_3O_4+$	$8KOH$
Potassium hydroxide (electrolyte)	Nickel dioxide (positive plate)	Pure iron (negative plate)		Iron oxide (negative plate)	Nickel oxide (positive plate)	Potassium hydroxide (electrolyte)

d. As in the case of the lead-acid cell, charging requires direct current. If by accident, should the Edison cell be charged in the wrong direction, no permanent damage will result, as long as the temperature does not rise above 115° F. The state of charge cannot be determined with a hydrometer, since the density of the electrolyte does not change between the charge and discharge cycles. Testing the condition of the battery is done by taking the full-load voltage. The voltage per cell of a fully charged Edison battery is 1.37 volts. The voltage of a discharged cell is 1 volt.

e. Even though Edison batteries are lighter in weight, require less attention, and have a longer life (from 15 to 20 years) than lead-acid batteries, their use is limited because of their higher initial cost and their reduced efficiency at low temperatures. Their primary use is for portable lighting units and some marine installations. They are not used by the Signal Corps.

108. Summary

a. When a storage battery furnishes current, it is said to be discharging; when it receives current, its plates and electrolyte are restored electrochemically and it is said to be charging.

b. The open-circuit voltage of the lead-acid cell is about 2.1 volts. The closed-circuit voltage is about 2 volts, depending on the load and state of charge.

c. The term *active material* refers to the spongy lead and lead peroxide which are pressed into the grids of the plates.

d. Specific gravity is the ratio of the weight of a given volume of liquid to the weight of the same volume of water.

e. When a lead-acid storage battery discharges, the active material of the positive and negative plates is converted to lead sulphate and the specific gravity of the electrolyte decreases.

f. When a lead-acid storage battery charges, the lead sulphate on both the plates is converted back into active material, the specific gravity of the electrolyte increases, and the water content of the electrolyte is decreased.

g. The capacity rating for storage batteries is usually based on a steady 20-hour discharge. A 100 ampere-hour lead-acid battery will furnish 5 amperes for 20 hours.

h. The normal rate of discharge is the ampere-hour rating divided by 20.

i. Hydrometers are used to measure the specific gravity of the electrolyte of a lead-acid battery which is an excellent indication of the state of charge of the battery.

j. The specific gravity of the electrolyte in a fully charged cell portable battery should usually be between 1.270 and 1.300 at 80° F. A discharged battery has a specific gravity of 1.150 to 1.210. Batteries having a specific gravity of

1.175 or less should not be further discharged and should be immediately recharged.

k. For use in low temperatures, the specific gravity of a fully charged battery should be 1.350, and the specific gravity of a discharged battery should not be permitted to fall below 1.250, as corrected to 80° F.

l. Batteries are kept in the best condition by constant use.

m. Use only distilled water to replace the water which has evaporated from a cell.

n. Never pour water into acid; instead, acid should be poured slowly into the water.

o. The mixture of hydrogen and oxygen gas given off by a storage battery on charge, is highly inflammable.

p. Sulphation, which reduces the ampere-hour capacity of a lead-acid battery, results from allowing the cell to remain discharged, from habitual undercharging, from the use of too dense an electrolyte, or from low level of electrolyte.

q. Use a solution of ammonia water or bicarbonate of soda to neutralize any spilled sulphuric acid electrolyte.

r. In an Edison battery, the active material of the positive plate is an oxide of nickel and the active material of the negative plate is iron.

s. The electrolyte of an Edison battery is a 21-percent solution of potassium hydroxide to which has been added a small amount of lithium hydrate. The specific gravity does not change during charge and discharge.

t. A hydrometer cannot be used to test the charge of an Edison battery. Instead, a voltage reading must be taken.

109. Review Questions

a. What is the essential difference between a dry cell and a lead-acid battery?

b. What name is given to the liquid in a lead-acid storage battery? What does it consist of?

c. Of what material are the positive plates of a lead-acid battery composed? The negative plates?

d. What is meant by *active* material?

e. Does the storage battery store electricity?

f. What chemical compound is formed on the plates of a lead-acid cell during discharge?

g. What gases are liberated during charge of a lead-acid battery?

h. What are the grids of a battery and why are they used?

i. What does a hydrometer measure?

j. Why is the specific gravity of the electrolyte an indication of the state of charge of a lead-acid cell?

k. Adding sulphuric acid to a discharged storage cell increases the specific gravity of its electrolyte. Does this action recharge the cell? Explain.

l. How are hydrometer readings corrected for temperature?

m. Using the chart in figure 90, what is the corrected reading for a specific gravity of 1.290 taken at 10° below zero? At 110° above zero? Is the correction large for only small changes in temperature?

n. What is the approximate specific gravity of a fully charged lead-acid storage battery of the portable type? Of a fully discharged battery?

o. Is the specific gravity of all types of fully charged lead-acid batteries the same?

p. What effect does raising the discharge rate have upon the capacity of a lead-acid battery?

q. Why does an excessive rate of charge or discharge injure the cell of a lead-acid battery?

r. What objection is there to the gases given off by the storage battery when charging?

s. What is added to a lead-acid battery to replace the part of the electrolyte that evaporates?

t. What is the proper level for the electrolyte of a lead-acid battery.

u. What is meant by sulphation?

v. What are some of the causes of sulphation?

w. What results from the freezing of a lead-acid battery?

x. What is the relation of the specific gravity of the electrolyte used in lead-acid batteries to its freezing point?

y. Why is it that a hydrometer cannot be used to test Edison batteries?

z. What is the electrolyte of an Edison cell composed of?

aa. What is the active material of the negative plate of an Edison battery?

CHAPTER 9

CIRCUITS

110. General

An electric circuit is a path, or group of interconnected paths, capable of carrying electric currents. In this chapter, three basic types of circuits are considered; the series circuit, the parallel circuit, and the series-parallel circuit.

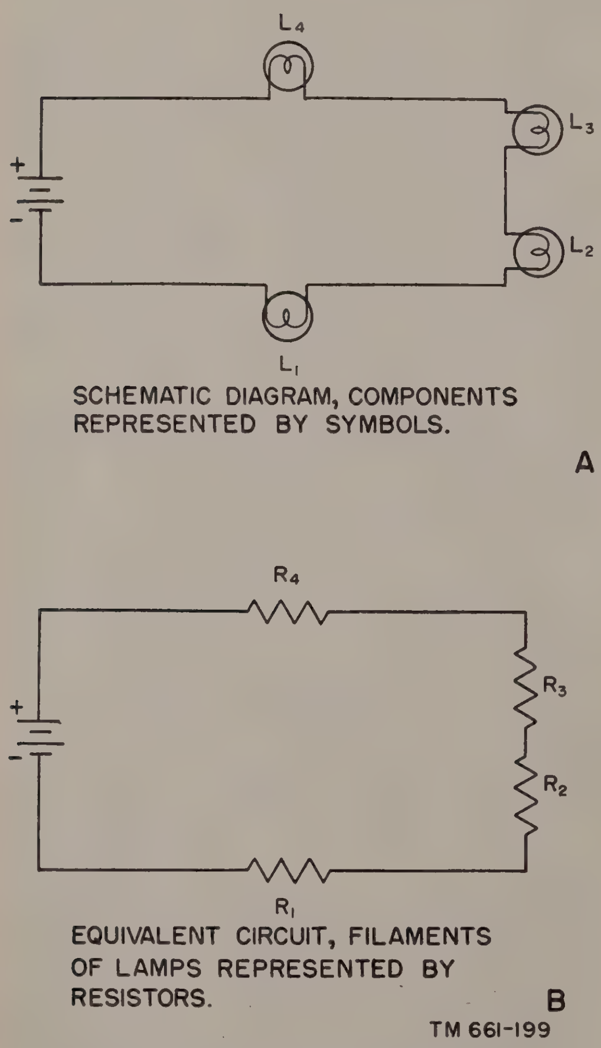


Figure 91. Schematic and equivalent diagrams.

111. The Series Circuit

A series circuit is a circuit supplying energy to a number of devices in series; that is, *the same current passes through each device in completing its path to the source of supply.*

a. SIMPLE SERIES CIRCUIT. Almost everyone is familiar with the string of Christmas tree lamps in which *all* of the lamps go out when any *one* of the lamps burns out. These lamps are connected in series as shown in A of figure 91. In completing its path to the source of supply, the current through lamp L_1 must also pass through the filaments of lamps L_2, L_3, and L_4. It is apparent that if the filament of any one lamp burns out (opens), the current path is no longer complete and the other lamps must also go out.

b. SCHEMATIC AND EQUIVALENT DIAGRAMS. As drawn in A of figure 91, the lamps and supply (battery) are shown by symbols; that is, the circuit *parts* are represented by symbols. To study the electrical action, it is convenient to draw an *equivalent* circuit which represents the *electrical properties* of the circuit parts. Thus, in B of figure 91, A of figure 91 is redrawn and the *resistance* of each lamp L_1, L_2, L_3, and L_4 is represented by the symbol for resistance (R_1, R_2, R_3, and R_4, respectively).

c. CONCEPT OF VOLTAGE DROP. Consider the circuit of A of figure 92. With the switch *SW* open, voltmeter V_1 indicates 150 volts. This is the *emf* of the battery. From Ohm's law, with an emf of 150 volts applied across R_1 and R_2 in series, we would expect that the current I would be—

$$I = emf/R = \frac{150}{50+50} = 1.5 \text{ amperes.}$$

However, on closing switch *SW*, we find that ammeter I indicates a current of only 1 ampere, and the voltmeter reading drops from 150 to 100 volts, a drop of 50 volts. Apparently everything is not included in the diagram of A of figure 92.

We have, in fact, forgotten that the circuit current must flow *through* the battery, and the battery, or any supply source, offers resistance to the flow of current. Because it is the opposition to current from one terminal to the other *within the source,* the resistance of the source is called the *internal resistance.* In B of figure 92, this internal resistance *Rg* is shown in *series* with the source, because it has the same effect as an equivalent series resistor.

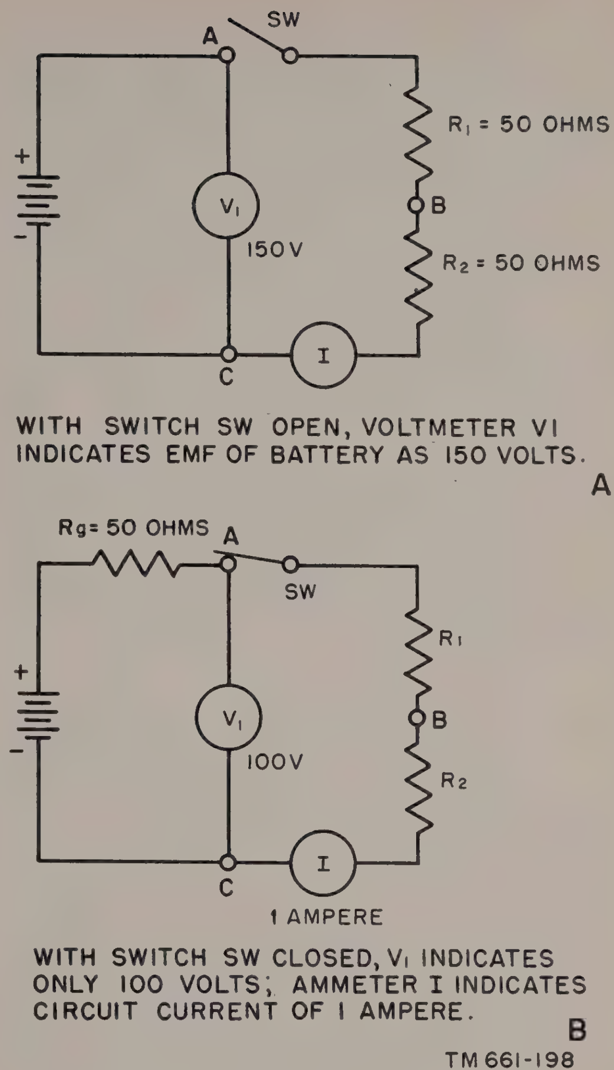


Figure 92. Voltage drop across internal resistance.

(1) *Internal voltage drop.* Internal resistance is difficult to measure directly, but it can be calculated readily. On closing the switch *SW*, it is found that the voltage at the battery terminals (*A–C*) drops from 150 to 100 volts. This means that the internal resistance of the battery consumes sufficient energy to account for 50 volts of the emf. Ammeter *I* tells us that the current through the entire circuit, including the internal resistance, is one ampere. The difference in the voltmeter readings tells us that a potential difference, or voltage, of 50 volts is required to produce this 1 ampere through the internal resistance *Rg* of the battery. From Ohm's law, the internal resistance *Rg* must be—

$$Rg = E/I = 50/1 = 50 \text{ ohms.}$$

(2) *External voltage drops.* Now that 50 volts of the emf has been accounted for, let us account for the remaining 100 volts. In the external circuit, R_1 equals R_2. The voltage across R_1 will be, from Ohm's law,

$$E = IR = 1 \times 50 = 50 \text{ volts.}$$

And, since R_1 equals R_2, the voltage across R_2 will also be 50 volts. On adding the calculated voltages we get—

$$50 + 50 + 50 = 150 \text{ volts.}$$

However, the emf, or battery voltage with switch *SW* open, is 150 volts; therefore, we have now accounted for the entire emf. From this, we can define a *voltage drop* as the potential difference required to produce the current through the portion of a circuit under consideration. In the example given, a potential difference of 50 volts is required to produce 1 ampere through the internal resistance *Rg* of the generator, a potential difference of 50 volts is required to produce 1 ampere through R_1, and a potential difference of 50 volts is required to produce 1 ampere through R_2. We can also state that *the sum of the voltage drops is equal to the emf.*

(3) *Practical considerations. The 50-ohm internal resistance in the above example indicates that the battery is either very weak or is not designed to supply as high a current as 1 ampere.* In general, a suitable source must be chosen for the work to be done by the circuit. In power circuits that furnish electric energy to homes and

factories, the internal resistance of the source is usually a very small fraction of an ohm. In certain radio and telephone circuits, the internal resistance of the source may be relatively high. In circuit *design* work the internal resistance *must* be considered. In most practical work, however, our interest lies primarily in circuit conditions with the circuit in operation. For example, referring again to figure 92B, it is not necessary to know the internal resistance in practical work. Suppose that only an ammeter is available, and we wish to know the voltage across the battery terminals. With switch *SW* closed, the current is 1 ampere and the external resistance across terminals *A–C* is the *sum* of R_1 and R_2, or 100 ohms. From Ohm's law, the voltage required to produce 1 ampere through 100 ohms is—

$$E=IR=1\times 100=100 \text{ volts.}$$

Similarly, if only a voltmeter is available, the current through the circuit can be determined from Ohm's law.

$$I=E/R=100/100=1 \text{ ampere.}$$

It is well to note here that the voltage of a source may be referred to in two ways:

(*a*) *Emf.* This is the *open-circuit* or no-load voltage.

(*b*) *Closed-circuit or load voltage.* This is the source voltage with the load connected.

In general, the closed-circuit voltage is meant unless otherwise specified. Throughout the remainder of this text, therefore, the unqualified terms source voltage, supply voltage, battery voltage, applied voltage, and generator voltage will mean the voltage with load connected to the source.

112. Rise or Fall of Potential, Voltage Drops

It is important to understand what is meant by a *rise of voltage*, a *fall of voltage*, and a *voltage drop*. In A of figure 93 the battery voltage, or potential difference between the battery terminals, is 200 volts, as indicated by voltmeter V_1. By Ohm's law, the current through resistor *A–B* is—

$$I=E/R=200/100=2 \text{ amperes.}$$

Also by Ohm's law, the voltage across the resistor is—

$$E=IR=2\times 100=200 \text{ volts.}$$

This voltage is, of course, the same as the battery voltage, for it is assumed that the resistance of the connecting leads is negligible.

a. Rise of Potential. The negative terminal of the battery (fig. 93) is being used as the reference or zero point for measuring the potentials of all other points in the circuit. Thus, point *A* is the point of highest potential (200 volts, positive) with respect to the point of lowest potential (zero) at *B*. In B of figure 93, the 100-ohm resistor of A of figure 93 is replaced by four 25-ohm resistors R_1, R_2, R_3 and R_4 which are connected between points, *A–B*, *B–C*, *C–D*, and *D–E*, respectively. Measurements with voltmeters (V_1, V_2, V_3, and V_4) show that the potentials of *E, D, C, B,* and *A* (taken in this order) are 0, 50, 100, 150, and 200 volts positive. This shows that there is an increase or rise of voltage from *E* towards *A*. If the resistance is distributed uniformly along the length *E–A*, the voltage rise will also be uniform. This is indicated by the fact that the rise across each section in B of figure 93 is 50 volts.

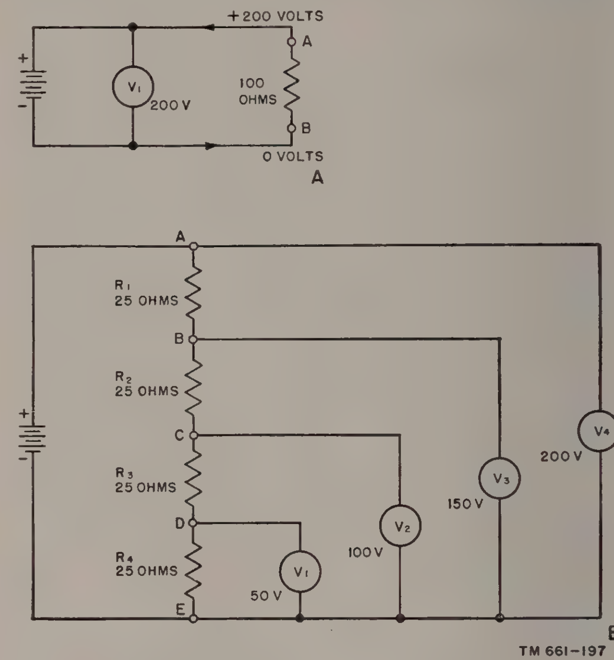


Figure 93. Rise or fall of potential.

b. Fall or Drop of Potential. Suppose that in B of figure 93, the potential at *A* is first measured and that the potential at *B* is then measured.

Obviously, the potential at B is lower than the potential at A, and we say that there is a fall or drop of potential from A to B. Apparently, whether there is a potential rise or fall between two points depends entirely on our point of view. For example, in B of figure 93, it is equally correct to say that there is a voltage rise from B to A, or a voltage fall from A to B.

c. PRACTICAL APPLICATION. In practical work, as previously mentioned, our interest usually is centered on voltages and currents in operating circuits. A voltmeter always indicates the potential difference between the two points to which it is connected. Thus, if a voltmeter is successively connected across A–B, B–C, C–D, and D–E, it will, in each instance, show a potential difference of 50 volts. The voltage across a portion of a circuit is called the voltage drop across that portion of the circuit. Consider points D–E. The resistance between these points is 25 ohms and the current is 2 amperes. From Ohm's law, $E=IR$ and, in this instance,

$$E_{\mathrm{C\text{-}D}}=2\times25=50 \text{ volts.}$$

Similarly considering points C–E, the resistance is 25 ohms (C–D) plus 25 ohms (D–E), or 50 ohms, and the current is two amperes.
Then,

$$E_{\mathrm{C\text{-}E}}=2\times50=100 \text{ volts.}$$

(1) Voltage drops are also called IR drops. The meeting of IR drop is apparent, when it is remembered that the voltage drop E across a resistance is calculated from the Ohm's law formula,

$$E=IR.$$

(2) In practical work the term *voltage drop* is usually used to indicate the potential difference required to produce the current through the two points being considered. As explained in paragraph 111, the sum of the voltage drops around the entire circuit is equal to the e. m. f. When the entire circuit is not considered, the sum of the voltage drops in the external circuit will equal the potential difference between the terminals of the source *with the external circuit connected.* Thus, in B of figure 93,

$$E_{R_1}+E_{R_2}+E_{R_3}+E_{R_4}=E_{battery}$$

and, $50+50+50+50=200$ volts.

113. Precautions in Handling Circuits

Every electric circuit is dangerous. Failure to observe this fundamental principle can be fatal.

a. POINTS OF NEGATIVE POTENTIALS ARE AS DANGEROUS AS POINTS OF POSITIVE POTENTIAL. Consider figure 94. In A the negative terminal is

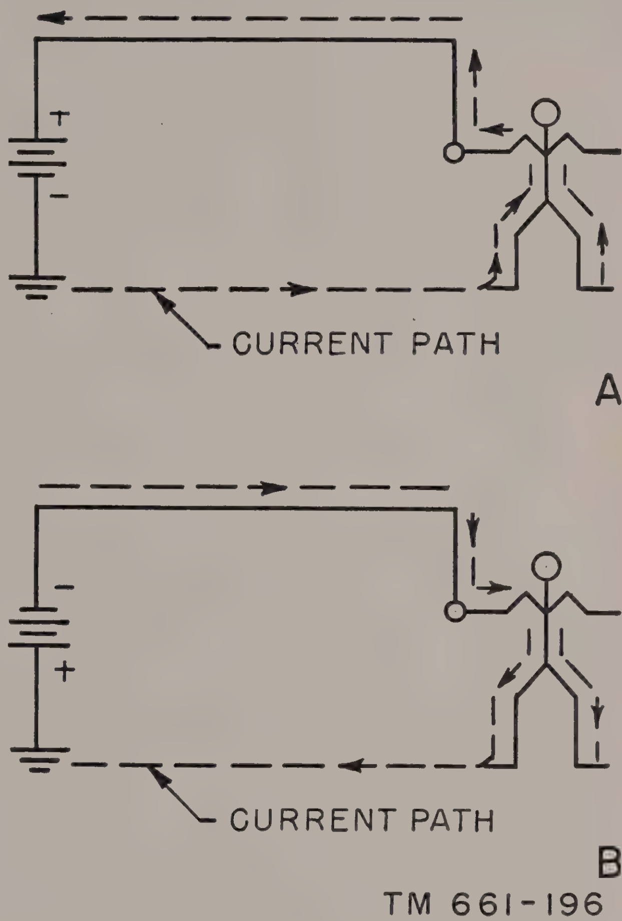


Figure 94. Points of negative and positive potential are equally dangerous.

connected to earth or, as it is more commonly called, *grounded.* Examination of the illustration shows clearly that the *figure* standing on the ground and with one hand on the positive terminal of the battery is connected directly across the battery. In B of figure 94, the positive terminal of the battery is connected to ground. This does *not* mean that it is safe to handle the negative terminal. Comparison of A and B of figure 94 shows that in both illustrations, the figure is connected directly across the battery. The negative terminal in B of figure 94 is just as *hot* as the positive terminal in A of figure 94.

The only difference is the direction of current through the figure, and current from head to toes can be just as fatal as current from toes to head. *Current through the body is limited only by the resistance of the body.* Under certain conditions the body resistance may be very low and a very low voltage may produce a fatal current.

b. Ground May Be At Any Point In The Circuit. In the simple circuit of figure 95, the

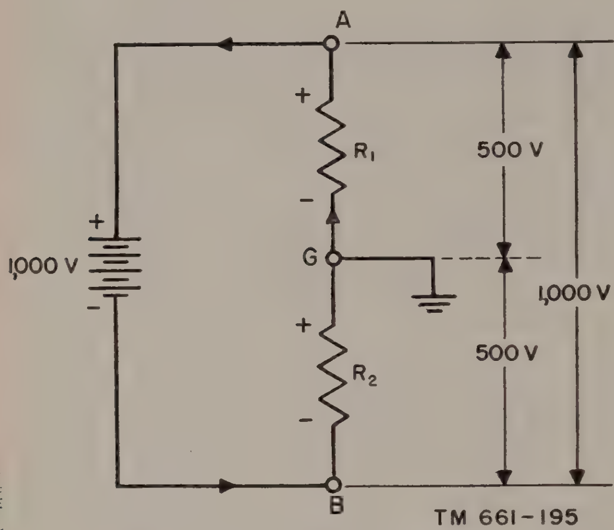


Figure 95. Ground may be at any point.

junction of R_1 and R_2 is grounded. Arrowheads on the lead wires indicate the direction of current Assuming that R_1 equals R_2, point A is 500 volts positive with respect to ground G; point B is 500 volts negative with respect to point G. Although it is negative, point B is as dangerous as point A. Also note that, if a person standing on the ground places one hand on point A and the other hand is on point B, there is a potential difference of 1,000 volts between his hands. Do not *assume* that a certain point is at ground potential. Keep one hand in your pocket to avoid the possibility of connecting yourself between two points between which there is a potential difference.

c. Precautions In Making Voltage Measurements. Detailed instructions for connecting meters into various circuits are given in manuals covering test equipments. Most high voltage and/or high power circuits include a *master switch,* or other circuit breaking device (fig. 96) for disconnecting the circuit from the source of supply. Always open this switch or device *before* attempting to handle any circuit components. Never attempt to work on a high-voltage generator while it is running. Also remember that a voltmeter is always connected between two points which are *not* at the same potential. In general, potential differences in excess of 300

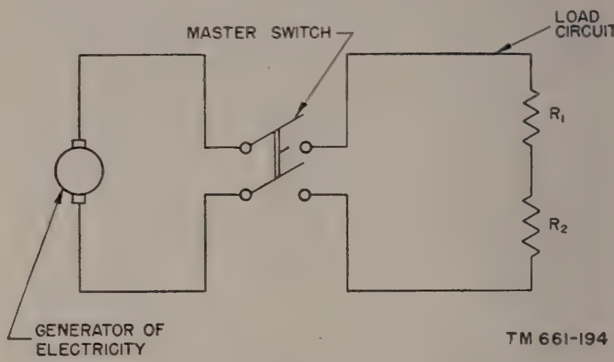


Figure 96. Open master switch to disconnect load circuit from source.

volts are considered to be extremely dangerous. Always obey the following rule when inserting or removing meters from such a circuit: *When the approximate voltages are not known, always shut down the generator and/or open the master switch before attempting to connect or remove meters or any other components from the circuit.* Be sure that the circuits are *dead before* you attempt to touch components.

Note. Students should always obtain instructions before attempting to make any circuit changes.

114. Laws of Series Circuits

a. Resistance. As shown in figure 93, a 100-ohm resistor can be replaced by four 25-ohm resistors in series. Similarly, a 10-ohm resistor and two 20-ohm resistors in series can be replaced by a single 50-ohm resistor. This illustrates a law for resistances in series: *The total resistance is the sum of the individual resistances.* In B of figure 93 there are 4 resistances R_1, R_2, R_3, and R_4 in series. If R_T represents the total resistance from A to E, then

$$R_T=R_1+R_2+R_3+R_4$$

If R_T is the total resistance and E is the applied (in this instance) battery voltage, the current I can be determined from Ohm's law:

$$I=\frac{E}{R_T}$$

b. CURRENT. In figure 97, current flow is from the negative terminal of the battery through ammeter A, resistance R_1, ammeter A_1, resistance R_2, ammeter A_2, resistance R_3, ammeter A_3, and through the battery from the positive to the negative terminal. Obviously, there is but *one* path for the current and the *same* current must be indicated by ammeters A, A_1, A_2, and A_3. This illustrates a law for series circuits: *In a series circuit the current is everywhere the same; or, the same current flows in each part of the circuit.* Thus, in figure 97,

$$I=I_1=I_2=I_3.$$

c. VOLTAGE. *Energy* is expended in the resistance of an electric circuit; that is, in a resistance electric energy is converted to heat energy. In figure 98, a 24-volt battery is connected to

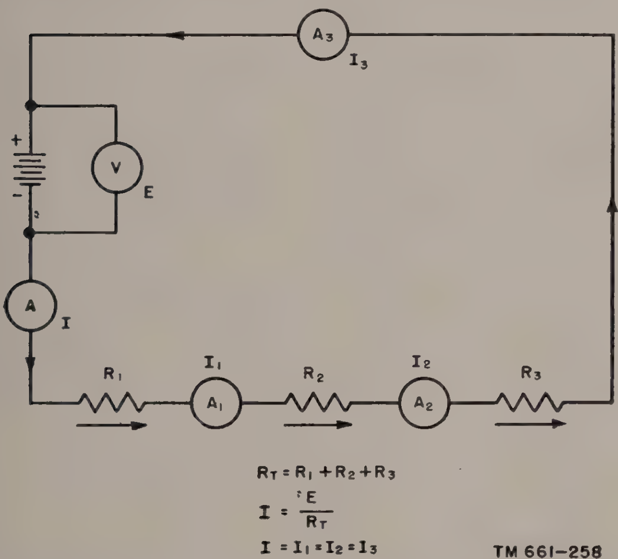


Figure 97. The same current flows through every part of a series circuit.

three resistances in series; R_1 of 3 ohms, R_2 of 6 ohms, and R_3 of 3 ohms. The voltage required to produce the current in each of these resistances is called the voltage drop (par. 111) across each resistor. These voltage drops may be measured by connecting a voltmeter across each resistor. The voltage drops may also be calculated from Ohm's law. The total resistance R_T is the sum of the individual resistances,

$$R_T=R_1+R_2+R_3=12 \text{ ohms}.$$

E is given as 24 volts, therefore,

$$I=E/R_T=24/12=2 \text{ amperes}.$$

By now applying Ohm's law to each part of the circuit,

$$E_1=R_1\times I=3\times 2=6 \text{ volts (across } R_1\text{)}$$
$$E_2=R_2\times I=6\times 2=12 \text{ volts (across } R_2\text{)}$$
$$E_3=R_3\times I=3\times 2=6 \text{ volts (across } R_3\text{)}$$

On adding the individual drops, we find that their sum is equal to the battery voltage,

$$E=E_1+E_2+E_3$$
$$E=6+12+6=24 \text{ volts}.$$

This illustrates a law for series circuits: *In a series circuit the sum of the individual voltage drops is equal to the applied voltage.*

d. CONSUMPTION OF ENERGY. Students sometimes get the impression that a voltage drop indicates that the voltage is produced by the current. Actually, the current is due to the potential difference across the resistance. Students also ask the following question: "I understand that the voltage produces the current through a resistor, but isn't the voltage used up?" The question will not arise, if it is understood that voltage or potential difference is a *measure* of the amount of work required to move unit charge from one of the points to the other (ch. 3). Electric *energy* is consumed in a resistance; that is, electric energy is converted to heat energy as current flows through a resistance. However, as long as the source produces electric energy as rapidly as it is consumed, the potential difference across the resistance will remain constant. Thus, in the battery of figure 98, chemical energy is converted to

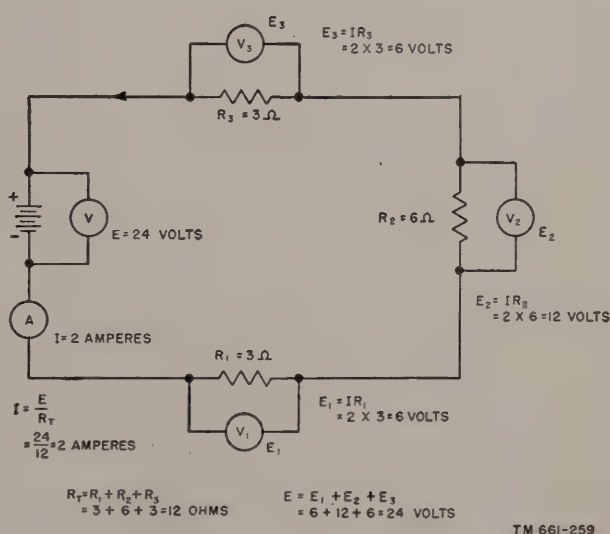


Figure 98. In a series circuit, R_T *equals the sum of the individual resistances.*

electric energy as rapidly as the electric energy is converted to heat energy in the resistances. Consequently, the potential difference or voltage drop across each resistor will remain constant until the battery runs down.

e. LAWS OF SERIES CIRCUITS. The three laws of series circuits are summarized below and should be memorized:

(1) *In a series circuit, the total resistance is equal to the sum of the individual resistances.*

(2) *In a series circuit, the same current flows in all parts of the circuit.*

(3) *In a series circuit, the sum of the voltage drops across the individual resistances is equal to the applied voltage.*

115. Simple Parallel Circuit

a. In A of figure 99, there is but *one* path for current I_1; therefore, R_1 and the battery are *in series*. In B of figure 99, however, there are two *parallel* paths *C–E–F–D* and *C–G–H–D*. Current I_1 goes through path *C–E–F–D* and an additional current I_2 follows path *C–G–H–D*. The total current from the battery is I_1+I_2. This current divides at *C*, I_1 going through R_1 and I_2 going through R_2. The two currents recombine at *D*, so that I_1+I_2 flows from *D* to *A and* through the battery from *A* to *B*. In practical circuits, any number of parallel paths or *branches* may be present and such paths form a parallel circuit.

b. The mere fact that one circuit component is connected *across* another does not mean that the components are *in parallel*. For example, in A of figure 99 resistor R_1 is connected *across* the battery. However, R_1 and the battery are in *series* because the *same* current I_1 passes through each of them. In B of figure 99, R_2 is connected across R_1 and the *combination* is connected across the battery.

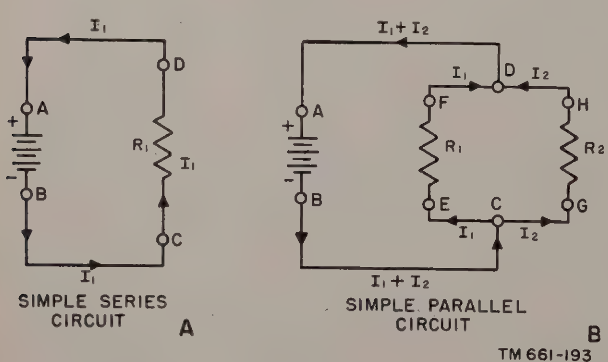


Figure 99. Comparison of simple series and parallel circuits.

R_1 and R_2 are *not* in series with *one another* because *different* currents, I_1 and I_2 pass through them. This *parallel combination* is in *series* with the battery, because the *same* current, I_1+I_2, passes through the *combination* and the battery. In other words, to determine whether a circuit or part of a circuit is a *parallel* circuit or a *series* circuit, apply the definition (par. 111) of a series circuit. If the circuit meets the specification of this definition, it is a series circuit; if the circuit does not meet the required specifications, it is a parallel circuit.

116. Series-Parallel Circuit

Although many simple circuits can be classified as being either a series or a parallel circuit, more complicated circuits are usually *combinations* of series and parallel circuits. Thus, in figure 100,

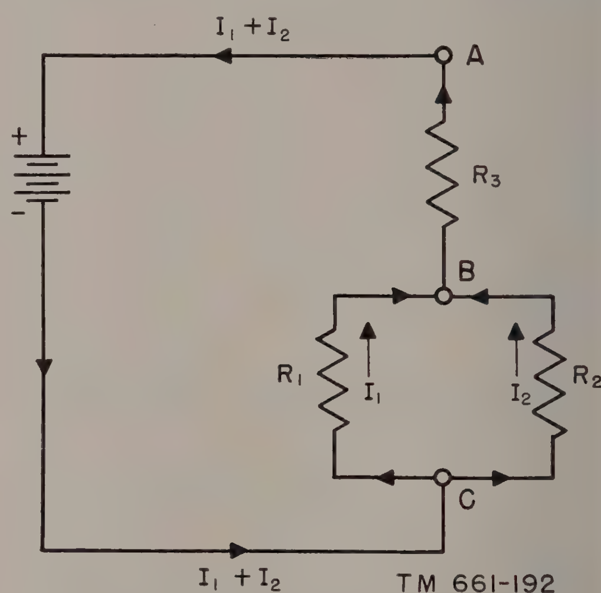


Figure 100. Simple series-parallel circuit.

resistors R_1 and R_2 are in parallel with *one another* and this parallel combination is in series with R_3 and the battery. Probably the simplest way to learn to recognize series, parallel, and series-parallel circuits is to study the voltage and current distributions in various circuits. This is essentially a study of laws applicable to these circuits.

117. Laws of Parallel Circuits

The laws governing the distribution of *current* and *voltage* in parallel circuits will now be developed.

a. VOLTAGE. A of figure 101 is the schematic diagram of a circuit in which the negative terminal

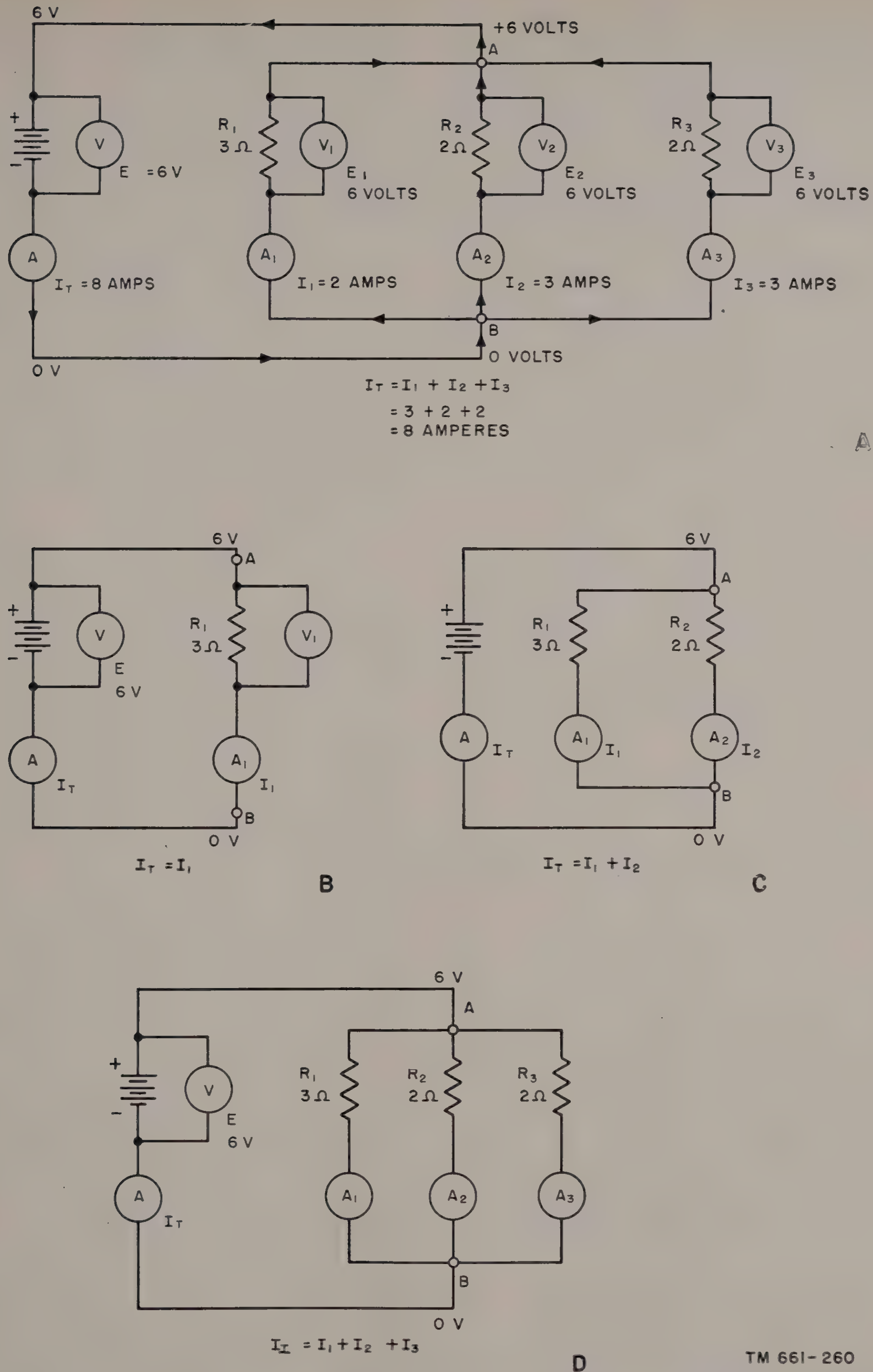


Figure 101. Voltage and current distribution in simple parallel circuit.

of a 6-volt battery is connected to point B, the positive terminal is connected to point A, and resistors R_1, R_2, and R_3 are connected *in parallel* between points A and B. Since A and B are directly connected to the battery terminals, the potential difference between A and B is 6 volts. Each resistor is also connected between points A and B and the *same* voltage is applied to each resistor. Thus, voltmeters V_1, V_2, and V_3 indicates the *same* voltage. This illustrates a law for voltages in a parallel circuit: *In a parallel circuit, the same voltage is applied to each branch.* Thus E, E_1, E_2, and E_3 are the *same* voltage.

b. CURRENT. If A of figure 101 is rearranged so that only R_1 is in the circuit (B of fig. 101), we have a simple series circuit consisting of the 6-volt battery and R_1. From Ohm's law the current I_1 through R_1 is —

$$I_1 = E/R,$$

$$\text{and, } I_1 = 6/3 = 2 \text{ amperes.}$$

This current is indicated by ammeters A and A_1. When the 2-ohm resistor R_2 is placed in parallel with R_1 (C of fig. 101), the *same* voltage is applied to both resistors. (This assumes that the battery voltage remains constant at 6 volts.) The current I_2 through R_2 is—

$$I_2 = E/R = 6/2 = 3 \text{ amperes.}$$

This 3-ampere current is indicated on ammeter A_2. The current I_T from the battery is the *sum* of the two currents, $I_1 + I_2$, or—

$$I_T = 2 + 3 = 5 \text{ amperes.}$$

When the 2-ohm resistor R_3 is connected in parallel with R_1 and R_2, as shown in D of figure 101, the same 6 volts is applied to R_3. The current I_3 through R_3 is—

$$I_3 = E/R = 6/2 = 3 \text{ amperes.}$$

This 3-ampere current is indicated by ammeter A_3. The total current from the battery is thus increased by 3 amperes and now is—

$$I_T = I_1 + I_2 + I_3$$

$$\text{and, } I_T = 2 + 3 + 3 = 8 \text{ amperes.}$$

This 8-ampere current is indicated on ammeter A. From these results, a rule for current in parallel circuits may be stated: *The total current in a parallel circuit is equal to the sum of the currents in the individual branches.*

Note that the current from the negative terminal of the battery divides to follow three paths from point B (A of fig. 101), recombines at point A, and returns to the positive terminal of the battery. The current that returns to the battery flows through the battery and it is exactly equal to the current that leaves the battery. The current through any *branch* may be computed from Ohm's law, $I = E/R$, and depends on the amount of resistance in the branch.

c. RESISTANCE. The total or *effective* resistance of the circuit (A of fig. 101) may be computed from the Ohm's law formula $R = \frac{E}{I}$. Since the applied voltage E is 6 volts and the total current I_T is 8 amperes,

$$R_T = E/I_T$$

$$\text{and, } R_T = 6/8 = .75 \text{ ohm,}$$

where R_T is the effective resistance of the three resistors R_1, R_2, and R_3 in *parallel;* I_T is the total current from the battery. Note that the effective resistance of R_1, R_2, and R_3 is only .75 ohm, considerably *less* than the resistance of any *one* of the resistances. The rule for resistances in parallel may be stated as follows: *The effective resistance of a parallel circuit may be determined by dividing the applied voltage by the total current; it is always less than the resistance of the lowest resistance in the circuit.*

d. LAWS FOR PARALLEL CIRCUITS. These laws are summarized below and should be memorized:

(1) *In a parallel circuit, the same voltage is applied to each branch.*

(2) *In a parallel circuit, the total current is equal to the sum of the currents in the individual branches.*

(3) *In a parallel circuit, the effective resistance is equal to the applied voltage divided by the total current; it is always less than the lowest resistance in the circuit.*

118. Combining Parallel Resistances

As mentioned in paragraph 117, the Ohm's law formula $R = E/I$ is satisfactory for finding the effective resistance when the voltage E and the total current I_T are known. However, it is often necessary to determine R when neither the voltage nor the current is known.

a. The simplest parallel arrangement consists of equal resistances in parallel. To obtain the effective resistance of such a combination, it is only necessary to divide the resistance of *one* resistor by the *number* of resistances. Thus, in A of figure 102 two 10-ohm resistors are connected in parallel, and the effective resistance is 10 divided by 2, or 5 ohms. With three 12-ohm resistors in parallel (B of fig. 102), the effective

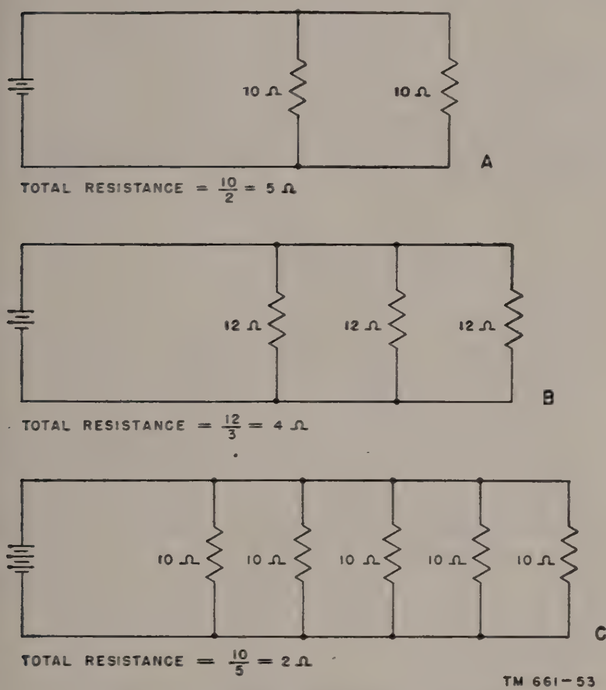


Figure 102. Equal resistances in parallel.

resistance is 12/3, or 4 ohms. With five 10-ohm resistors in parallel (C of fig. 102), the effective resistance is 10/5, or 2 ohms. This method for determining the effective resistance is called the *like* method because all the resistances *must* be equal.

b. In figure 103, two unequal resistances, *A* of 12 ohms and *B* of 4 ohms, are connected in parallel across a 24-volt battery. Obviously, the rule for

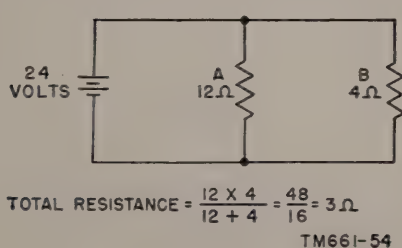


Figure 103. Product/sum method.

like resistances cannot be applied. Since the *same* voltages is applied to *each* branch, the current through any branch is a function of the resistance in that branch. In this example, the current through resistor *A* is—

$$I_A=E/R_A=24/12=2 \text{ amperes.}$$

The current through resistor *B* is—

$$I_B=E/R_B=24/4=6 \text{ amperes.}$$

The total current I_T is—

$$I_T=I_A+I_B=2+6=8 \text{ amperes.}$$

The effective resistance of the circuit can be determined from the Ohm's law formula $R=\frac{E}{I}$.

$$R_T=E/I_T=24/8=3 \text{ ohms.}$$

When the applied voltage is *not* known, *assume* any convenient voltage. For example: *Assume* that a voltage of 6 volts is applied to the parallel resistors *A* and *B*. The current through R_A will be—

$$I_A=E/R_A=6/12=.5 \text{ ampere.}$$

The current through R_B will be—

$$I_B=E/R_B=6/4=1.5 \text{ amperes.}$$

The total current I_T will be—

$$I_T=I_A=.5+1.5=2 \text{ amperes.}$$

The effective resistance can now be calculated—

$$R=E/I_T=6/2=3 \text{ ohms.}$$

c. When only *two* parallel resistances are involved, another method, called *the product over the sum (product/sum) method,* may be the simplest to use. The rule for this method may be stated as follows: *The effective resistance of two resistances in parallel is equal to their product divided by their sum.* Applying this rule to the circuit in figure 103

$$R=\frac{Product}{Sum}=\frac{12\times4}{12+4}=\frac{48}{16}=3 \text{ ohms.}$$

Notice that this result agrees with that obtained when an applied or an assumed voltage is divided by the total current (*b* above). However, only

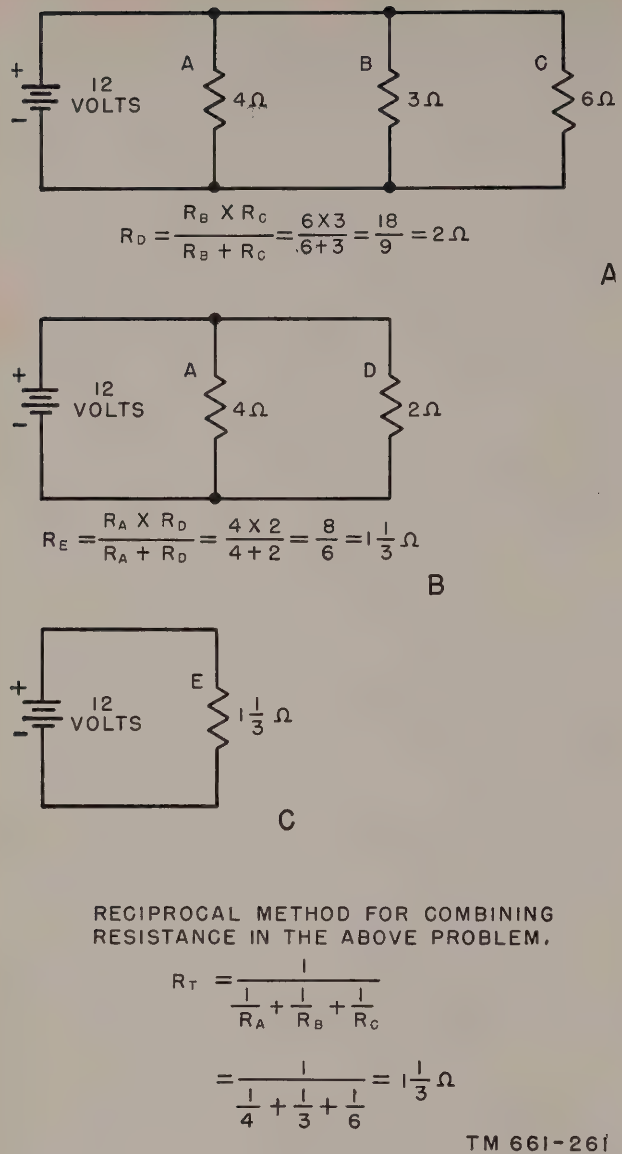


Figure 104. Product/sum and reciprocal methods.

resistance values are used in this computation. (This method can, of course, be applied to *two* resistances of equal value.)

d. The product/sum method may be used to solve problems involving more than two resistances in parallel. To do this, first determine the effective resistance of any *two* of the parallel resistors, then combine the result of this calculation with any *one* of the remaining resistances. Continue the process of combining the calculated resistance with one of the remaining resistances until all of the resistances have been included in the calculations. For example, figure 104 shows a circuit with three unequal resistances connected in parallel. The problem is to find the effective resistance. In other words, what is the resistance of a single resistor which offers the same opposition to current as the parallel combination? Apply the product/sum rule to resistors B and C and call the resultant resistance R_D,

$$R_D = \frac{Product}{Sum} = \frac{B \times C}{B + C}$$

$$R_D = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2 \text{ ohms.}$$

Since R_D is the effective resistance of B and C in parallel, R_D may now be substituted for B and C, as indicated in B of figure 104. Applying the product/sum method to A and D, and calling the result R_E,

$$R_E = \frac{4 \times 2}{4+2} = \frac{8}{6} = 1\tfrac{1}{3} \text{ ohms.}$$

A single resistance R_E of 1⅓ ohms can be substituted for A, B, and C (A of fig. 104) as shown in C of figure 104; that is, the effective resistance of A, B, and C *in parallel* is 1⅓ ohms.

e. The product/sum method can be used only for two resistances at a time. If a circuit contains three or more resistances, the product/sum method of solution becomes long and tedious. The *reciprocal method* may be used to determine the effective resistance of any number of parallel resistances. Stated as a rule: *The effective resistance of parallel resistances is equal to the reciprocal of the sum of the reciprocals of the individual resistances.*

The reciprocal of a number is 1 divided by the number. Thus, the reciprocal of 2 is ½, the reciprocal of ½ is 2; the reciprocal of 5 is ⅕, the reciprocal of ⅕ is 5; and so on. The reciprocal method is stated by the following formula:

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots}$$

where R is the effective resistance and R_1, R_2, R_3, and so on, the parallel resistances. Substituting the values of A, B, and C (A of fig. 104) in the formula,

$$R = \frac{1}{\frac{1}{4} + \frac{1}{3} + \frac{1}{6}},$$

$$\text{and, } R = \frac{1}{\frac{3}{12} + \frac{4}{12} + \frac{2}{12}} = \frac{1}{\frac{9}{12}} \text{ (adding fractions).}$$

Then, $R = 1 \times 12/9 = 1\tfrac{1}{3}$ ohms (dividing 1 by $\frac{9}{12}$).

This result is the same as that obtained by using the product/sum method. Students should be familiar with all of these methods for determining the effective resistance of resistances in parallel. If necessary, one method may be used to prove results obtained with the other. Remember, too, that the effective resistance is *always* lower than the resistance of the lowest individual resistance in the circuit.

119. Series-Parallel Circuits

As the name implies, series-parallel circuits are combinations of series and parallel circuits.

a. A of figure 105 is a schematic of a simple series-parallel circuit. This circuit is solved by applying the rules for simple series and simple parallel circuits. In such circuits we are concerned with:

(1) The applied or source voltage and the voltage drops across each part of the circuit.

(2) The total current from the source and the current in each part of the circuit.

(3) The effective resistance and the resistance of each part of the circuit. In A of figure 105, a 10-volt battery is connected to a 5-ohm resistor, R_A, in series with two parallel resistors, R_B and R_C: A of figure 105 is redrawn in B of figure 105 to show clearly that R_A is in series with the parallel resistances R_B and R_C, and that there are two current paths between point X and point Y. The current (I_T) from the battery goes through R_A to point X. At point X the current divides, part (I_B) going through R_B and the remainder (I_C) going through R_C. At Y these two currents reunite ($I_B+I_C=I_T$) and go to the positive terminal and through the battery to the negative terminal.

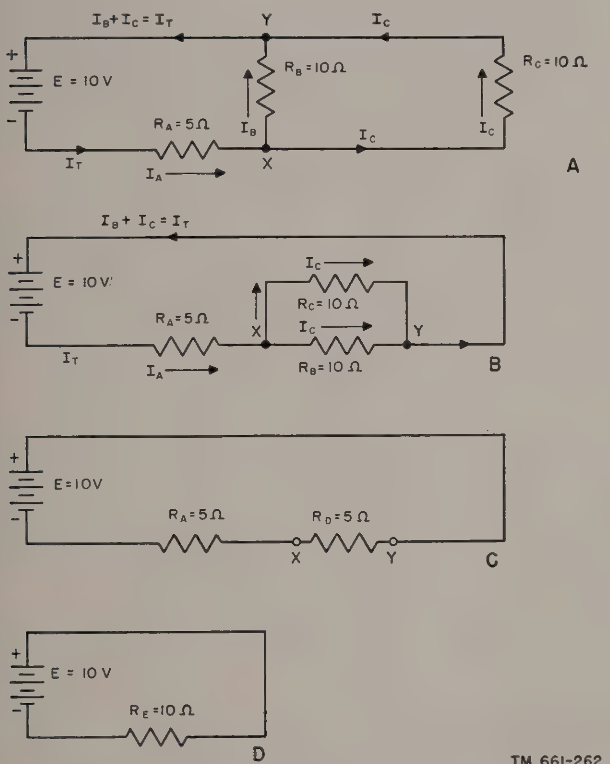


Figure 105. Simple series-parallel circuit, solution.

b. PROBLEM. The battery voltage and the resistance of each resistor is given (A of fig. 105) and it is necessary to determine the currents I_T, I_A, I_B, and I_C, the voltage drops E_A, E_B, and E_C, and the effective resistance R_E.

(1) Since all of the individual resistances are known, the first step is to determine the effective resistance of the circuit. R_B and R_C are like resistances and *their* effective resistance (R_D) is found by the like method (par. 118).

$$R_D=\frac{10}{2}=5 \text{ ohms.}$$

(2) This effective resistance R_D is in *series* with R_A as shown in C of figure 105. The effective resistance R_E of R_A and R_D in series is found by adding the individual resistors:

$$R_E=R_A+R_D=5+5=10 \text{ ohms.}$$

(3) R_E is the effective resistance of the circuit; that is, as far as the battery is concerned, the circuit of figure 105D is the equivalent of that shown in, figure 105A. Therefore, the current from the battery is—

$$I_T=\frac{E}{R_E}=\frac{10}{10}=1 \text{ ampere.}$$

(4) Referring to figure 105A, we see that I_T must go through R_A, that is, $I_A=I_T$. Since we now know the current through R_A, we find the voltage drop E_A from Ohm's law formula $E=IR$:

$$E_A=I_A\times R_A=1\times 5=5 \text{ volts.}$$

(5) The voltage drop across points X and Y can be found in either of two ways.

(*a*) The battery voltage E is equal to the *sum* of the voltage drop E_A across R_A plus the voltage drop E_{XY} across X and Y; that is, $E=E_A+E_{XY}$. Therefore,

$$E_{XY}=E-E_A=10-5=5 \text{ volts.}$$

(*b*) Since R_B and R_C are equal (10 ohms each), the 1-ampere current I_T divides equally between them, so that—

$$I_B=I_C=I_T/2=\tfrac{1}{2} \text{ ampere.}$$

Then,

$$E_{XY}=I_B\times R_B=I_C\times R_C=$$
$$.5\times 10=5 \text{ volts.}$$

This agrees with the rule that the same voltage is across each *branch* of a parallel circuit.

(6) In this circuit we determined the branch currents, I_B and I_C, by dividing I_T by 2. Usually, however, the current in each branch must be determined by dividing the voltage across the branch by the resistance in the branch. Thus,

$$I_B=\frac{E_{XY}}{R_B}=\frac{5}{10}=.5 \text{ ampere}$$

$$I_C=\frac{E_{XY}}{R_C}=\frac{5}{10}=.5 \text{ ampere.}$$

120. Calculations in Series-Parallel Circuit

The series-parallel circuit in figure 106 is more complicated than that in figure 105, but the same rules apply. The solution of this problem illus-

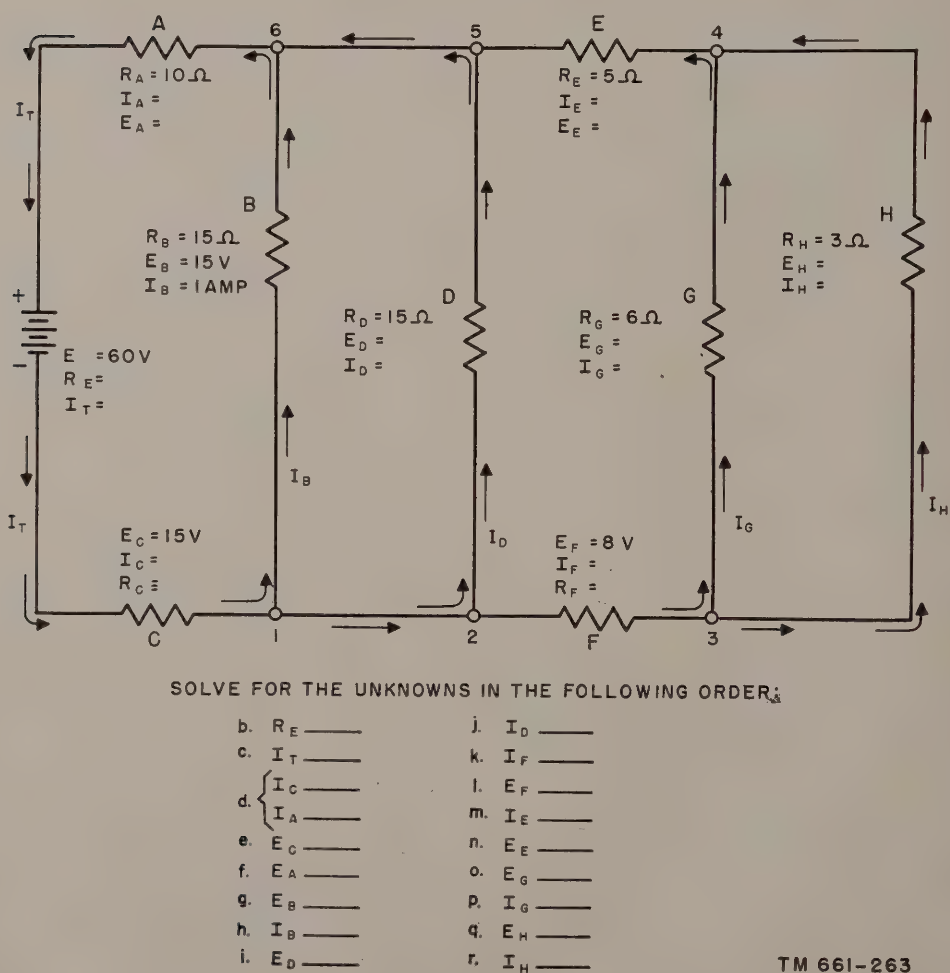


Figure 106. Order of solution.

trates proper methods of solving this type of problem. Always inspect a circuit carefully before starting on the problem. First determine what is known and then determine the best order of procedure. The best order of procedure is given in figure 106 and *keyed* with the text; that is, *b* in figure 106 corresponds to subparagraph *b*, and so on.

a. CURRENT PATHS. Arrows in figure 106 indicate current paths and directions of flow. Clearly marking these paths is always helpful, because *IR* drops cannot be calculated unless the currents through the resistances are known.

b. R_E, EFFECTIVE RESISTANCE. Since all the resistance values are given, the first step is to determine the effective resistance R_E. Figure 106 is redrawn in figure 107 and figure 107 is used to illustrate the method of calculating the effective resistance.

(1) In general, it is best to start combining resistances at the point farthermost from the source of voltage. Thus, in figure 107, we start with resistor *H*, noticing that it is in parallel with resistor *G*. As *G* and *H* are unequal, the product/sum method (par. 118*c*) is used to calculate their equivalent resistance *R*.

$$R=\frac{G\times H}{G+H}=\frac{6\times 3}{6+3}=\frac{18}{9}=2 \text{ ohms.}$$

In B of figure 107, *G* and *H* are replaced by their equivalent resistance *I* of 2 ohms.

(2) In B of figure 107, resistances *E*, *I*, and *F* are in series; therefore, these resistances may be replaced by a single resistance equal to the *sum* of the individual resistances:

$$R=E+I+F=5+2+8=15 \text{ ohms.}$$

In C of figure 107, *E*, *I*, and *F* are replaced by a single equivalent resistance *J* of 15 ohms.

(3) In C of figure 107, three equal resistances, *B*, *D*, and *J*, are in parallel. Using the like method, the equivalent resistance of *B*, *D*, and *J* is—

$$R=\frac{\textit{Resistance of any one}}{\textit{Number of resistances}}=\frac{15}{3}=5 \text{ ohms.}$$

In D of figure 107, resistors *B*, *D*, and *J* are replaced by the 5-ohm equivalent *K*.

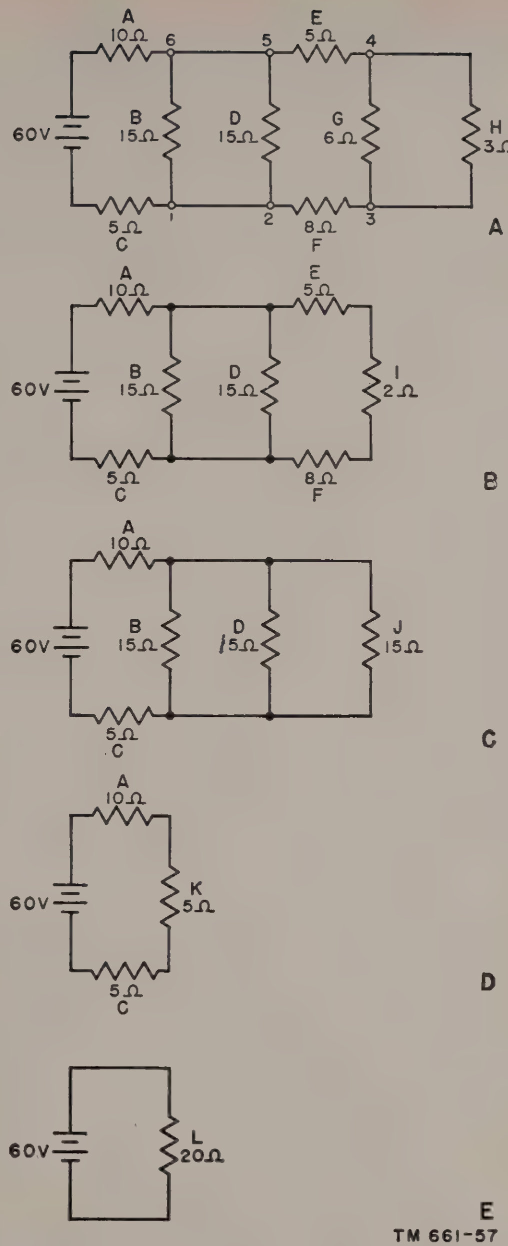


Figure 107. Reduction to equivalent series circuit.

(4) In D of figure 107 resistances *A*, *K*, and *C* are in series; therefore, a single equivalent resistance is equal to the sum of their individual resistances.

$$R=A+K+C=10+5+5=20 \text{ ohms.}$$

In E of figure 107, resistances *A*, *K*, and *C* are replaced by the equivalent 20-ohm resistance *L*.

(5) The circuit of A of figure 107 is now reduced to a simple series circuit (E of

fig. 107). Since every resistance in A of figure 107 has been included in the above calculations, resistance L (E of fig. 107) is the *effective resistance* R_E of the circuit. It might be well to mention here that any electrical circuit can be reduced to an equivalent series circuit.

c. I_T, TOTAL CURRENT. Now that R_E is known, the Ohm's law formula $I=\frac{E}{R}$ can be used to calculate the total current I_T.

$$I_T=\frac{E}{R_E}=\frac{60}{20}=3 \text{ amperes.}$$

d. I_C AND I_A, CURRENT THROUGH RESISTORS A AND C. Examination of the current paths in figure 106 shows that the same current I_T goes through resistors A and C; therefore, $I_A=I_C=I_T=$ 3 amperes.

e. E_C, VOLTAGE ACROSS RESISTOR C. The voltage drop E_C across resistor C can be calculated from Ohm's law formula, $E=IR$:

$$E_C=I_C\times R_C=3\times 5=15 \text{ volts.}$$

f. E_A, VOLTAGE ACROSS RESISTOR A. The voltage drop E_A across R_A is similarly calculated:

$$E_A=I_A\times R_A=3\times 10=30 \text{ volts}$$

g. E_B, VOLTAGE ACROSS RESISTOR B. The voltage drop across resistance B is found by applying the rule for voltage drops: The sum of the voltage drops is equal to the applied voltage. In this circuit $E_A+E_B+E_C$ equals the battery voltage E. Therefore,

$$E_B=E-(E_A+E_C)$$

$$E_B=60-(30+15)=15 \text{ volts.}$$

If there is any doubt about E_B being the only voltage drop in addition to the two drops E_A and E_C, examine C of figure 107. In this illustration, resistances E, G, H, and F of A of figure 107 have been replaced by their equivalent resistance J. Since B, D, and J are in parallel, the *same* voltage E_B is applied to all of them. Also notice in A of figure 106 that the current I_T divides at point 1; part of the current (I_B) goes through resistance B and the remaining current goes to point 2. Thus the voltage E_B, between points 1 and 6, can be considered as a voltage *applied* to points 2 and 5.

h. I_B, CURRENT THROUGH RESISTOR B. Now that the voltage across resistance B is known, the current I_B through B can be calculated from Ohm's law formula, $I=\frac{E}{R}$:

$$I_B=\frac{E_B}{R_B}=\frac{15}{15}=1 \text{ ampere.}$$

i. E_D, VOLTAGE ACROSS RESISTOR D. Since resistance D is in parallel with resistance B, the *same* voltage is applied to both; therefore,

$$E_D=E_A=15 \text{ volts.}$$

j. I_D, CURRENT THROUGH RESISTOR D. Resistance D is equal to resistance B and the same voltage is applied to both resistors. Therefore, the current through D must be equal to the current through B; that is, $I_D=I_B=1$ ampere. If desired, this current also can be calculated from the Ohm's law formula, $I=\frac{E}{R}$.

k. I_F, CURRENT THROUGH RESISTOR F. At point 1 (A of fig. 106), the 3-ampere current I_T divides; 1 ampere (I_B) goes through resistance B, and the remaining 2 amperes goes to point 2. At point 2 there is a second division, 1 ampere (I_D) goes through resistance D and, therefore, the remaining 1 ampere must go through resistance F.

l. E_F, VOLTAGE ACROSS RESISTOR F. Now that the current through F is known, the voltage drop across F is found by using Ohm's law for voltage, $E=IR$. $E_F=I_F\times R_F=1\times 8=8$ volts.

m. I_E, CURRENT THROUGH RESISTOR E. At point 3 (A of fig. 106) the current through F divides, one part (I_G) goes through resistance G and the remainder (I_H) goes through H. These two currents recombine at point 4; therefore, the current through E must be the same as that through F; that is, 1 ampere.

n. E_E, VOLTAGE ACROSS RESISTOR E. The voltage drop E_E across E, is, of course,

$$E_E=I_E\times R_E=1\times 5=5 \text{ volts.}$$

o. E_G, VOLTAGE ACROSS RESISTOR G. The voltage E_D is equal to the sum of the voltage drops E_E, E_G, and E_F. Since E_D, E_E, and E_F are known,

$$E_G=E_D-(E_E+E_F)$$

$$E_G=15-(5+8)=2 \text{ volts.}$$

p. I_G, CURRENT THROUGH RESISTOR *G*. Now that the voltage drop across *G* is known, the current I_G can be calculated from Ohm's law formula, $I=E/R$.

$$I_G=\frac{E_G}{R_G}=\frac{2}{6}=\frac{1}{3} \text{ ampere.}$$

q. E_H, VOLTAGE ACROSS RESISTOR *H*. Since *G* and *H* are in parallel, the *same* voltage is applied to both; therefore,

$$E_H=E_G=2 \text{ volts.}$$

r. I_H, CURRENT THROUGH RESISTOR *H*. The current I_H through *H* is found from Ohm's law formula, $I=E/R$:

$$I_H=\frac{E_H}{R_H}=\frac{2}{3} \text{ ampere.}$$

Or, since it is known that I_F is 1 ampere and I_G is $\frac{1}{3}$ ampere,

$$I_H=I_F-I_G=1-\tfrac{1}{3}=\tfrac{2}{3} \text{ ampere.}$$

121. Series-Parallel Circuit Problem

This problem (fig. 108) differs from the two previous examples in that all of the resistance values are not given; instead, certain resistances, voltage drops, and currents are given.

a. A small arrow indicates the direction of current through each resistance.

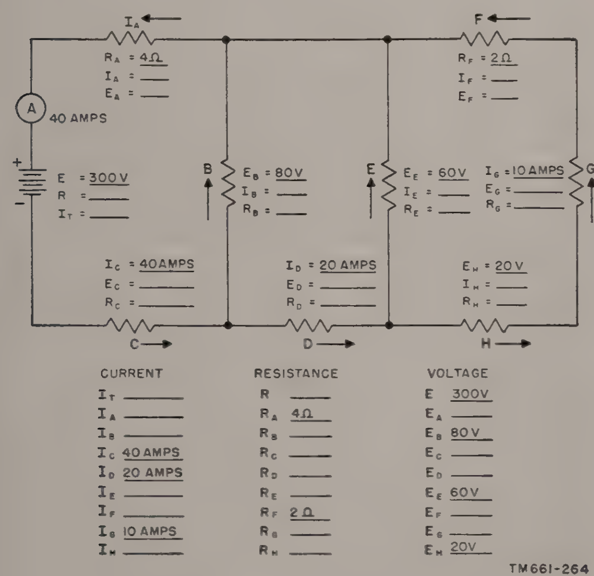


Figure 108. Series-parallel problem.

b. The current I_C through resistance *C* is given as 40 amperes. Since resistance *C* is in series with the battery, I_C and the current I_T from the battery must be the *same* current; therefore,

$$I_T=I_C=40 \text{ amperes.}$$

c. This same 40-ampere current must return to the battery through resistance *A*; therefore,

$$I_A=I_T=40 \text{ amperes.}$$

d. Now that the voltage *E* and the total current I_T are known, the effective resistance R_E of the circuit is calculated by substituting these known values into the Ohm's law formula $R=E/I$.

$$R_T=\frac{E}{I_T}=\frac{300}{40}=7.5 \text{ ohms.}$$

e. The current I_B through resistance *B* can now be calculated. The 40-ampere current I_T divides at the junction of resistors *B* and *D*. The current I_D through *D* is given as 20 amperes; therefore, the current I_B through *B* must be the difference between the total current and the current through *D*.

$$I_B=I_T-I_D=40-20=20 \text{ amperes.}$$

f. Since the voltage drop across *B* is given as 80 volts and the current is 20 amperes.

$$R_B=\frac{E_B}{I_B}=\frac{80}{20}=4 \text{ ohms.}$$

g. Since R_A is given as 4 ohms and I_A is 40 amperes, the voltage drop E_A across resistance *A* is calculated from the Ohm's law formula $E=IR$.

$$E_A=I_A\times R_A=40\times 4=160 \text{ volts.}$$

h. The voltage drop E_C across *C* can now be determined by applying the rule for voltage drops: *The applied voltage is equal to the sum of the voltage drops.* In this instance the applied voltage *E* is given as 300 volts and the drops E_A are and E_B are now known.

$$E=E_A+E_B+E_C$$

and, $E_C=E-(E_A+E_B)$

Solving, $E_C=300-(160+80)=60$ volts.

i. Now that the voltage drop across resistance C is known, the resistance of C is determined by using the Ohm's law formula $R=E/I$

$$R_C=\frac{E_C}{I_C}=\frac{60}{40}=1.5 \text{ ohms.}$$

j. Examination of figure 108 shows that the voltage drop across resistance B is applied to resistances E and D *in series*. Consequently, the voltage drop E_D across D is determined by applying the rule for voltage drops; that is,

$$E_B=E_E+E_D$$

$$\text{and, } E_D=E_B-E_E.$$

$$\text{Therefore, } E_D=80-60=20 \text{ volts.}$$

k. The resistance of D is now easily determined from the Ohm's law formula $R=E/I$:

$$R_D=E_D/I_D=20/20=1 \text{ ohm.}$$

l. Examination of the diagram (fig. 108) shows that resistances F, G, and H are *in series*. Since the current through G is given as 10 amperes, this *same* current must go through the other two resistances; therefore,

$$I_F=I_H=I_G=10 \text{ amperes.}$$

m. The 20-ampere current through resistance D divides at the junction of E and H. Since 10 amperes of this current goes through H, the other 10 amperes must go through E; that is, $I_E=$ ~~20~~ amperes.

10 *n.* The resistance of E is now readily determined from the Ohm's law formula $R=E/I$.

$$R_E=E_E/I_E=60/10=6 \text{ ohms.}$$

o. Similarly, since E_H is given and I_H is 10 amperes,

$$R_H=E_H/I_H=20/10=2 \text{ ohms.}$$

p. Since R_F is given as 2 ohms and the current I_F is known to be 10 amperes, the Ohm's law formular $E=IR$ is used to determine E_F.

$$E_F=I_F\times R_F=10\times 2=20 \text{ volts.}$$

q. Since $E_F+E_G+E_H$ equals the voltage drop E_E,

$$E_G=E_E-(E_F+E_H)$$

$$\text{and, } E_G=60-(20+20)=20 \text{ volts.}$$

r. The voltages drops E_F, E_G, and E_H are equal and the same current (10 amperes) flows through resistances F, G, and H. Therefore, without further calculation, we know that $R_F=R_G=R_H=$ 2 ohms; otherwise, *unequaled* voltage drops would be obtained across these resistances.

s. Check the above solution in terms of the following rules.

(1) In a parallel circuit the total current is equal to the sum of the branch current.

(2) The same voltage is applied to each branch of a parallel circuit.

(3) The current in a series circuit is the same everywhere.

(4) In a series circuit the sum of the voltage drops is equal to the applied voltage.

122. Complex Circuits; Kirchhoff's Laws

Ohm's law is not sufficient for determining currents in complicated circuits. The reason is that Ohm's law is a special case of much more general relations. Methods of treating complicated circuits are based on Kirchhoff's laws. These laws are simple, but the methods of applying them are difficult. Kirchhoff's laws and other methods for solving complex circuits are discussed in appendix III.

123. Circuit Tracing

The circuits in electrical equipment cannot be identified by a casual examination of the equipment. Most of the parts do *not* look like the schematic symbols representing them, many of the components are encased, and lead wires are run in cables. For example, a number of the circuit resistors may be mounted on a terminal or resistor board (A of fig. 109). At first glance it is not apparent that these resistors are connected as shown in B of figure 109. By carefully tracing the connections of each lead and resistor, the schematic diagram of C of figure 109 can be made. In general, manufacturers furnish a schematic diagram with equipment. Even with such a diagram available, a technician could not trace the actual circuit, unless he had considerable experience with equipment. In other words, circuit tracing cannot be learned solely from a book; the book must be supplemented by experience.

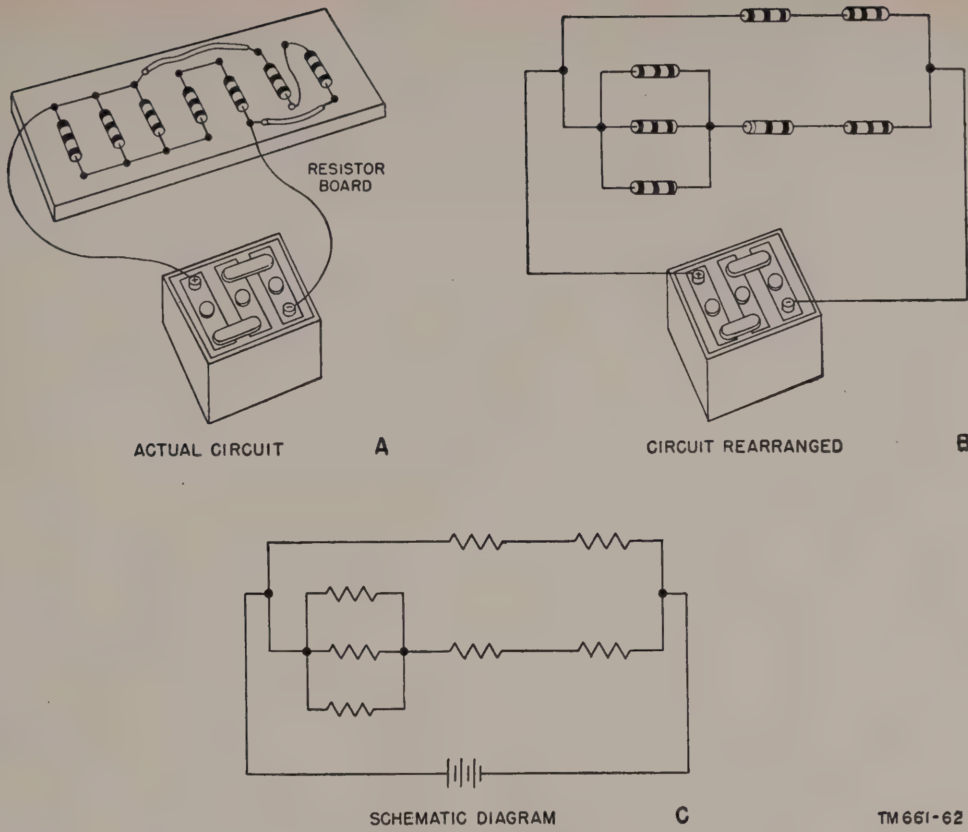


Figure 109. Tracing a simple circuit.

124. Summary

a. There are three basic types of circuits:

(1) Series circuits.

(2) Parallel circuits.

(3) Series-parallel circuits.

b. In a series circuit the same current passes through each device in completing its path to the source of supply.

c. In a parallel circuit the same current does not flow through each device in completing its path to the source of supply; the current divides to follow two or more parallel paths.

d. A series-parallel circuit is a combination of series and parallel circuits.

e. In more complicated circuits it is impossible to identify the series and parallel circuits. Such circuits are solved by applying Kirchhoff's laws (app. III).

f. There is a rise of potential in the direction of (electron) current flow; a fall of potential in the opposite direction.

g. The voltage drop between two points is the potential difference required to produce the current between the two points.

h. Voltage drops across resistances are called *IR* drops, since they are computed from the Ohm's law formula $E=IR$.

i. In a simple series circuit, the sum of the voltage drops in the external circuit is equal to the applied voltage.

j. When the internal resistance of the source is included in the calculations, the sum of the voltage drops around the entire circuit is equal to the emf of the source.

k. In a series circuit the effective or total resistance is equal to the sum of the individual resistances.

l. In a parallel circuit the same voltage is applied to each parallel branch.

m. In a parallel circuit the total current is equal to the sum of the currents in the individual branches.

n. In a parallel circuit the effective resistance of the parallel branches is equal to the voltage applied to the branches divided by the total current through the branches.

o. In a series circuit, the effective resistance is equal to the voltage divided by the current.

p. Parallel resistance may be combined to obtain the effective resistance by—

(1) The like method.
(2) The product/sum method.
(3) The reciprocal method.

q. Any circuit may be reduced to an equivalent series circuit.

r. The sum of the currents arriving at any point in a circuit is equal to the sum of the currents leaving that point (app. III).

125. Review Questions

a. Refer to figure 110 and determine the following:

(1) Voltage drop across *A* 12 V
(2) Resistance of *A* 3 Ω
(3) Resistance of *E* [illegible]
(4) Voltage drop across *C* 2.5 V.
(5) Current through *D* 1.25 A.
(6) Resistance of *C* 10 Ω
(7) Current through *B* 2 A.

b. Three tubes, their filaments, and a battery are shown in A of figure 111. Connect the filaments in *series* with the battery. If each filament has a resistance of 20 ohms and is designed to draw .3 ampere of current, what is the correct battery voltage? 12 V

c. Three tubes, their filaments, and a battery are shown in B of figure 111. Connect the filaments in *parallel* with the battery. If each filament has a resistance of 20 ohms and is designed to draw .3 ampere of current, what is the correct battery voltage? 6 V

d. Find the unknowns in figures 112 through 117.

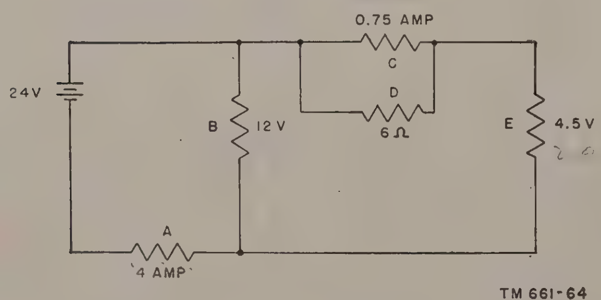


Figure 110. Problem 1.

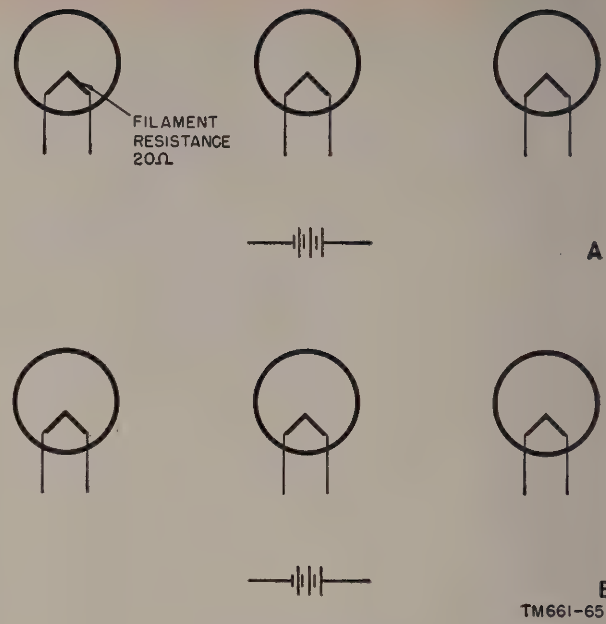


Figure 111. Problem 2.

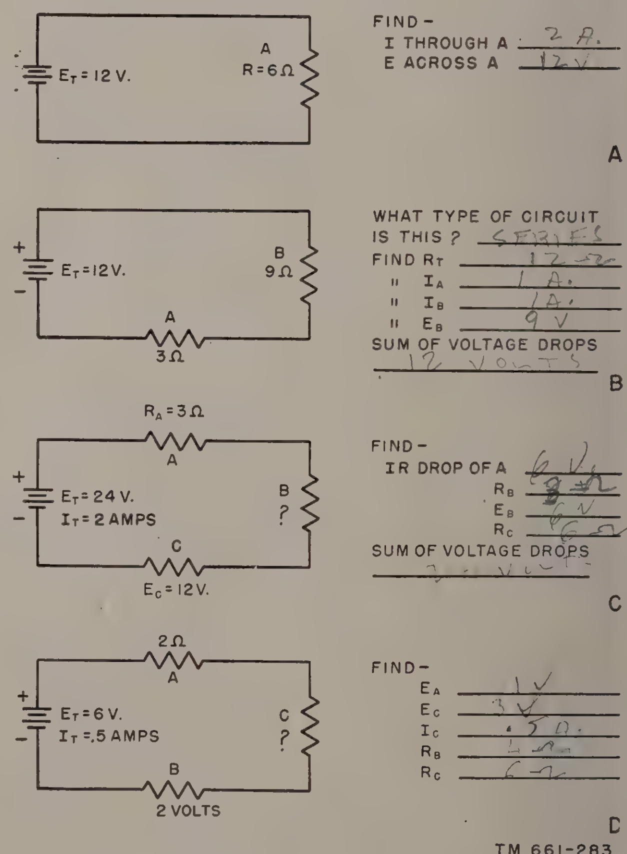


Figure 112. Problem 3.

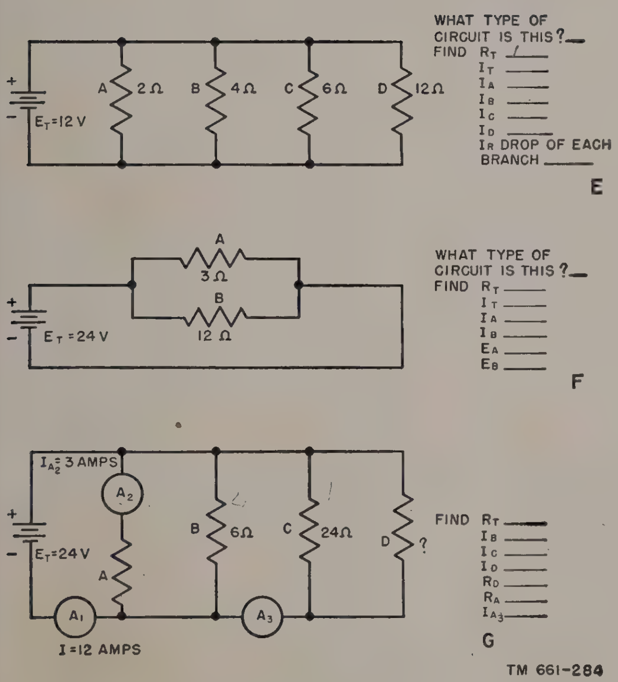


Figure 113. Problem 4.

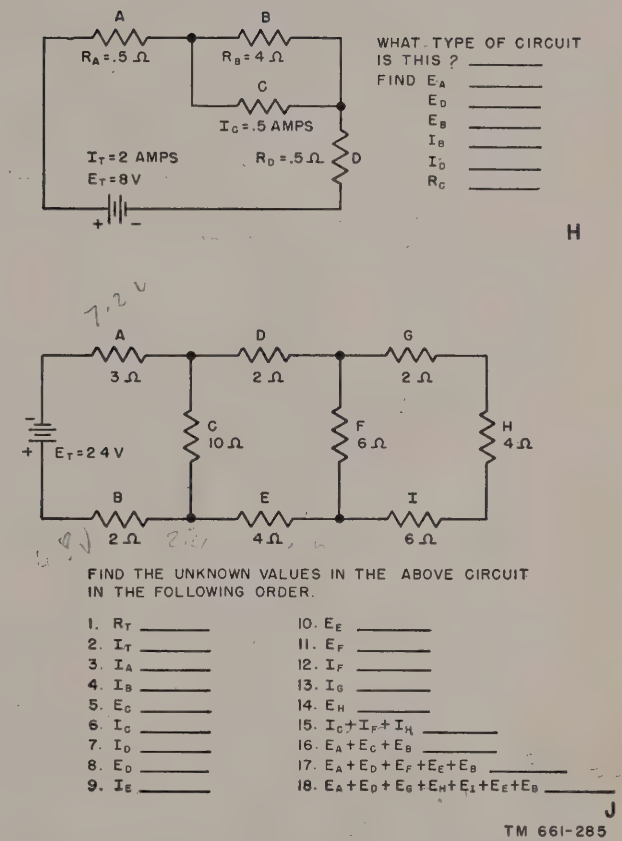
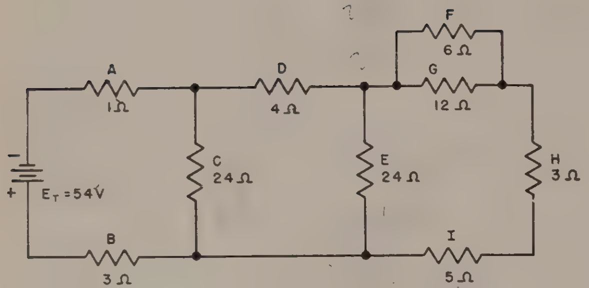


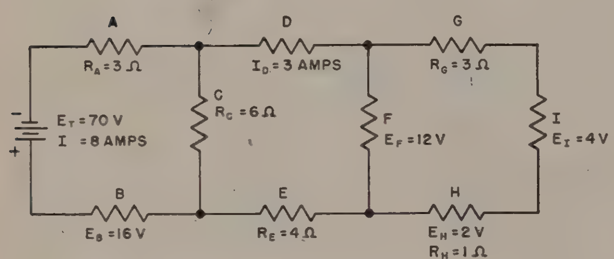
Figure 114. Problem 5.



FIND THE UNKNOWN VALUES IN THE FOLLOWING ORDER

1. R_T ______
2. I_T ______
3. E_A ______
4. I_B ______
5. E_C ______
6. I_D ______
7. E_E ______
8. I_E ______
9. I_F ______
10. I_G ______
11. I_H ______
12. E_F ______
13. E_G ______
14. $E_A + E_C + E_B$ ______
15. $E_A + E_D + E_E + E_B$ ______
16. $E_A + E_D + E_F + E_H + E_I + E_B$ ______

K



FIND THE UNKNOWN VALUES IN THE FOLLOWING ORDER

1. E_A ______
2. E_C ______
3. R_B ______
4. I_C ______
5. I_E ______
6. E_E ______
7. E_D ______
8. R_D ______
9. I_H ______
10. I_F ______
11. R_F ______
12. R_I ______
13. E_G ______

L

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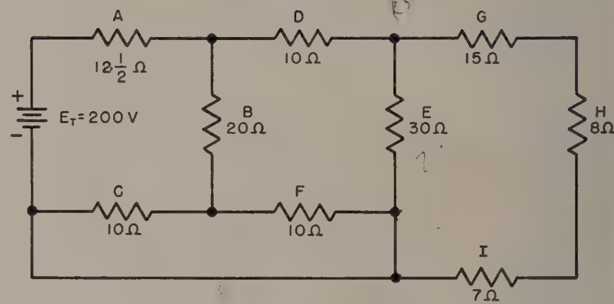
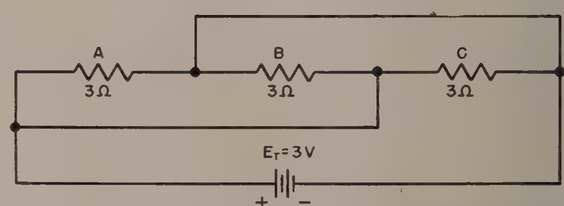
Figure 115. Problem 6.



FIND THE UNKNOWN VALUES IN THE FOLLOWING ORDER.

1. R_T ______
2. I_T ______
3. I_A ______
4. E_A ______
5. E_B ______
6. E_C ______
7. E_F ______
8. I_B ______
9. I_C ______
10. I_F ______
11. I_D ______
12. E_D ______
13. E_E ______
14. I_E ______
15. I_G ______
16. E_G ______
17. E_H ______
18. E_I ______

M



WHAT TYPE OF CIRCUIT IS THIS ?

FIND THE UNKNOWN VALUES IN THE FOLLOWING ORDER.

1. R_T ______
2. I_T ______
3. E_A ______
4. I_A ______
5. E_B ______
6. I_B ______
7. E_C ______
8. I_C ______

N

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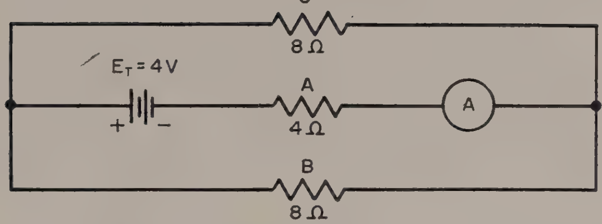
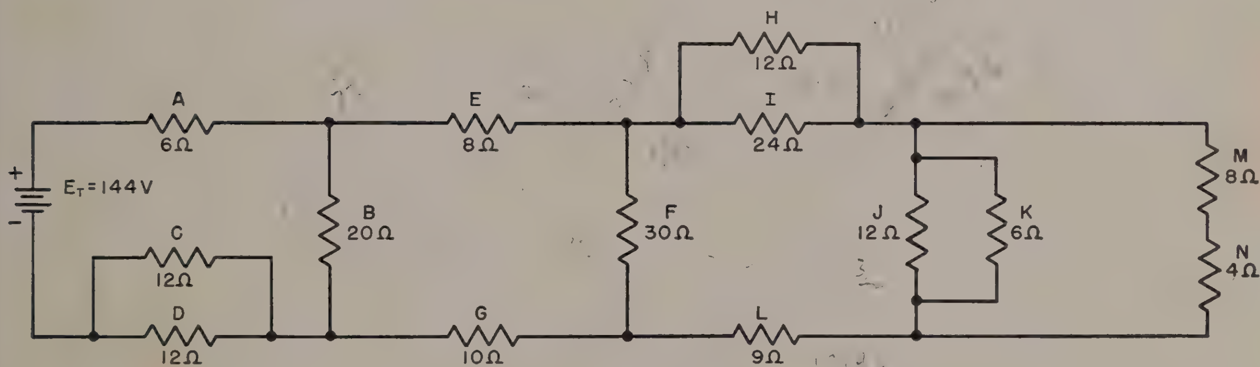
Figure 116. Problem 7.



WHAT TYPE OF CIRCUIT IS THIS ? ______
SOLVE FOR THE FOLLOWING VALUES.

1. R_T ______
2. I_A .5
3. E_A 2 V
4. I_T ______
5. I_B ______
6. E_B ______
7. I_C ______
8. E_C ______

P



FIND THE UNKNOWN VALUES IN THE FOLLOWING ORDER

1. R_T 24 Ω
2. I_T 6 A.
3. I_A 6 A
4. E_A 36 V
5. E_C 36 V
6. E_D 36
7. I_C 3 A
8. I_D 3 A.
9. E_B 72 V
10. I_B 3.6
11. I_E 2.4
12. E_E 19.2
13. I_G 2.4
14. E_G 24 V
15. E_F ______
16. I_F .96
17. E_H ______
18. I_H .96
19. I_I ______
20. I_L 1.44
21. E_L 12.96
22. E_K 4.32
23. I_K .72
24. I_J ______
25. I_M ______
26. E_M ______
27. E_N ______

Q

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Figure 117. Problem 8.

CHAPTER 10

ELECTROMAGNETISM

126. Oersted's Experiment

In chapter 1 of this text, electricity and magnetism were mentioned as the two fundamental forces which were utilized in the development of modern communication equipment and electronics. Since ancient times, certain facts had been known about both magnetism and electricity; but it was not until 1819 that Hans Christian Oersted, a native of Denmark and a professor of physics at the University of Copenhagen, actually discovered that there is a direct connection between these two basic forces. Up to this time (1819), Oersted believed and thought he had proved that there was no connection between electricity and magnetism. His usual custom was to connect a wire conductor to a voltaic cell and then place it at right angles to, and directly over a compass needle. In this position there is no movement of the compass needle. However, one day while discussing the experiment with his students, he accidentally placed the current carrying conductor parallel to the compass needle, and to his amazement he saw the compass needle turn at right angles to the conductor and come to rest. This excited his interest, since up to this time he was convinced that there was no connection between these two forces. Continuing his experiments, he found that if the current was reversed in the conductor, the needle would swing at right angles in the other direction; and if the needle was placed over the conductor, the entire action was reversed. Figure 118 shows a conductor and a compass in three different positions. Note that the compass needle comes to rest at right angles to the current-carrying conductor, regardless of the position of the conductor. By this experiment, Oersted proved that a wire carrying a current of electricity has around it a field of force which acts on a compass needle in a manner similar to the field of force around a permanent magnet. Thus, *electricity produces magnetism.* The operation of motors, generators, door bells, and countless other types of electrical equipment in common use today depends on the application of this discovery; that is, the relationship between magnetism and electricity.

127. A Magnetic Field Is Produced by Electric Current

a. The magnetic field produced by a current of electricity always lies at right angles to the current that produces it. The truth of this statement can be illustrated by the following experiment: Pass a current-carrying conductor through a hole in a piece of cardboard or other nonmagnetic material, so that the cardboard can be held horizontal (*A* of fig. 119). When iron filings are sprinkled on the cardboard, the filings become arranged in the form of concentric circles about the wire, thus proving that the magnetic field around the wires is circular. The direction of this field can be found by placing a compass at the various positions shown.

b. When magnetism is caused by an electric current flowing through a conductor, it is called *electromagnetism.*

c. Since a magnetic field has both intensity and direction, it can be represented by lines of force. *B* of figure 119 shows the lines of force closely concentrated near the conductor and gradually becoming less concentrated as the distance from the conductor increases. The lines of force are only a representation of the field since the magnetic field itself cannot be seen.

d. For a very short straight conductor carrying current, the magnetic field strength at any distance d from the wire is given by the formula—

$$H=\frac{2I}{d^2}.$$

H=magnetic field strength,
I=current flowing through conductor, and
d=perpendicular distance from the conductor of the point at which the magnetic field is to be evaluated.

Note. For a long straight conductor carrying current under the conditions set up in *d* above, the formula becomes $H=\frac{2I}{d}$.

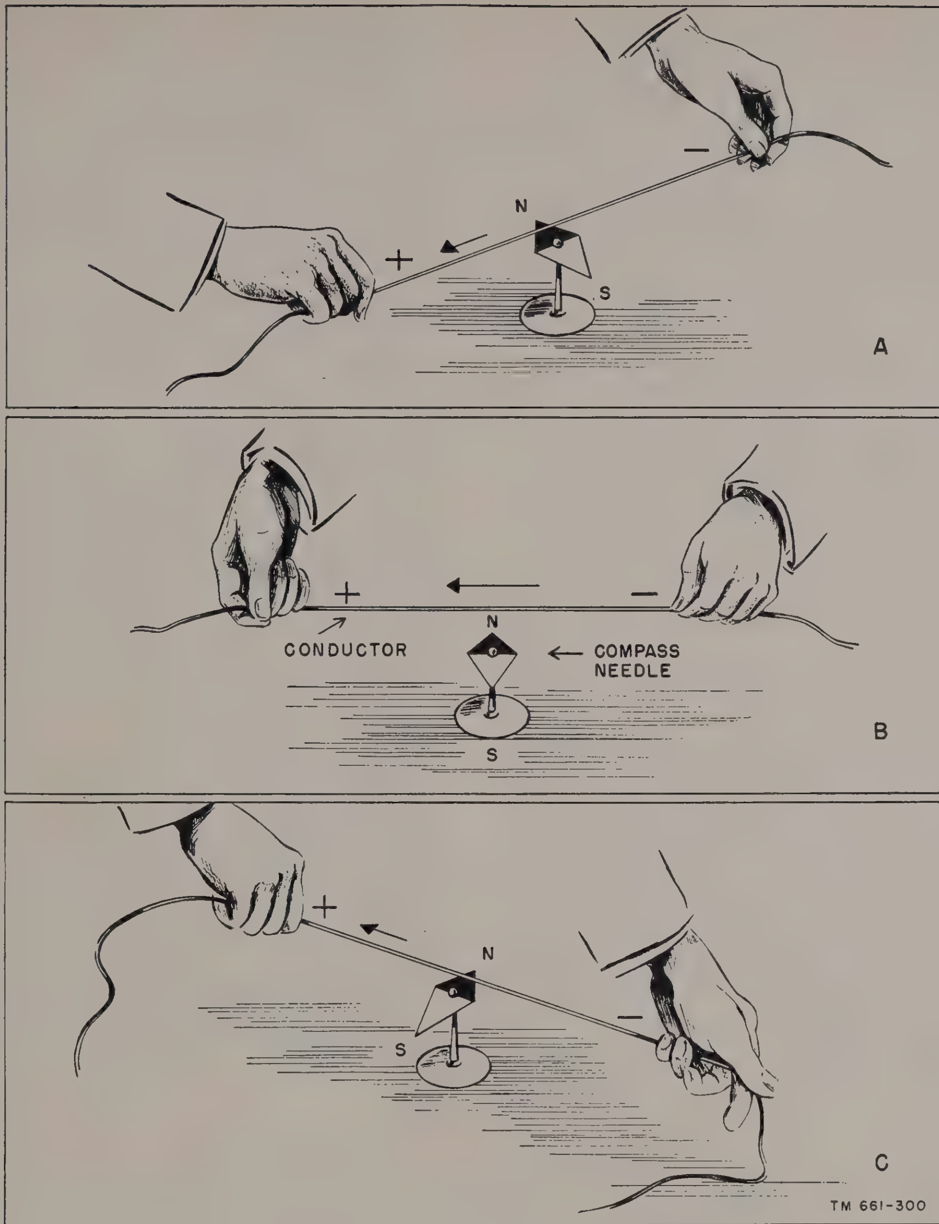


Figure 118. Oersted's experiment.

128. Relationship Between Current and Its Magnetic Field

a. Theoretically, the magnetic field of every magnet extends throughout space. Practically, if one of the compasses in A of figure 119 is moved farther away from the conductor a point would soon be reached where the effect of the field would no longer be felt and the north pole of the compass would point to the north pole of the earth. If the current in the conductor is now increased, the compass needle will again be affected and it will point the direction of the magnetic field. From this it is evident that *the strength or intensity of the magnetic field around a current-carrying conductor increases when the current increases and decreases when the current decreases.*

b. If the direction of the flow of current in the conductor is reversed, it will be found that the compasses in A of figure 119 will also reverse and point in the opposite direction. This indicates that the direction of the magnetic field around a current-carrying conductor is dependent on the direction of the current through the conductor.

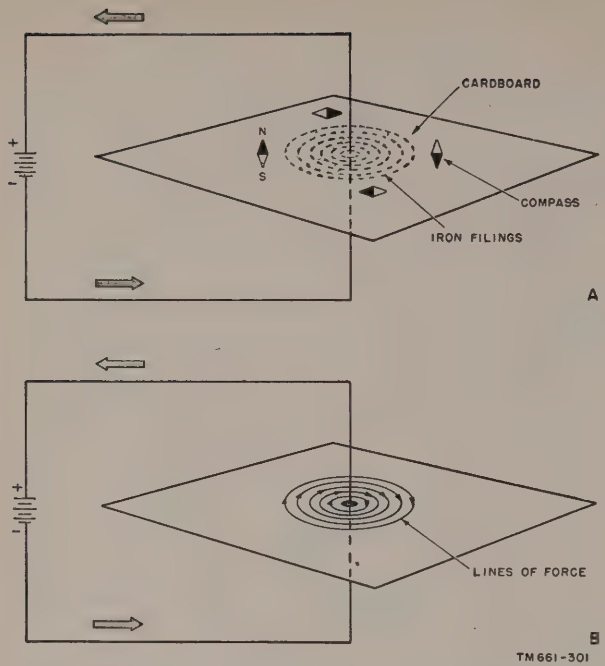


Figure 119. Magnetic spectrum showing intensity and direction of field.

Reversing the direction of current in the conductor reverses the direction of the magnetic field.

c. Actually the magnetic field around a conductor extends the complete length of the conductor and not at only one point as illustrated in A of figure 119. B of figure 120 shows a wire conductor with the lines of force indicating the direction of the field. Remember that while lines of force show direction, they are *only an indication* that a magnetic field is present.

d. A of figure 120 is the end view of the conductor and magnetic field showing the current flowing away from the observer. Notice the symbol ⊕, a plus sign, inside a circle, representing the tail of an arrow to indicate the direction of the current (into the paper). The arrows on the lines of force show the direction of the magnetic field to be counterclockwise. The end of the conductor, showing the current flowing toward the observer is shown in C of figure 120. In this case the symbol ⊙, a circle with a dot, representing the point of an arrow to indicate the direction of the current (out of the paper). The magnetic field viewed from this end is in a clockwise direction.

129. Left-Hand Rule

After the discovery of the relationship between the direction of the current through a wire and the direction of the magnetic field caused by the current, a simple rule was set up to find the direction of the magnetic field when the direction of current was known. This rule is known as the *left-hand rule* for determining the direction of the magnetic field about a current carrying conductor, and may be stated as follows: *Grasp the conductor in the left hand with the thumb pointing in the direction of the current (electron current) flow (fig. 121). The fingers will point in the direction of the magnetic field.* Using this rule, if either the direction of the current or the direction of the field is known, the other may be obtained.

Note. If conventional current flow (positive to negative) is being considered, use the right hand and apply the rule in the same manner. The rule is then known as the *right-hand rule.* Some of the older texts use this convention.

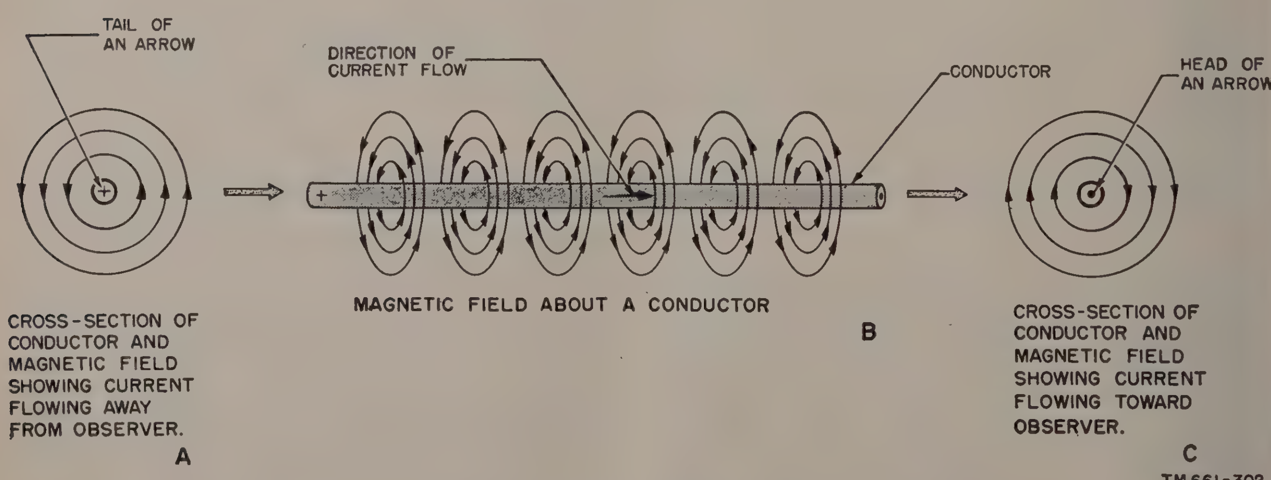


Figure 120. Magnetic field about a conductor.

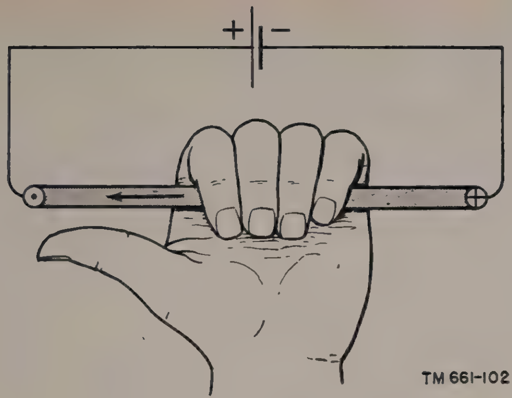


Figure 121. Use of the left-hand rule.

130. Magnetic Field Between Conductors Carrying Currents

a. The magnetic fields produced by electric currents in conductors cause attraction or repulsion, depending on the direction of the current flow. There are three important laws governing this action which will be given below.

(1) *Parallel currents flowing in the same direction cause attraction.* A of figure 122 shows two conductors with the current flowing in the same direction. When applying the *left-hand rule*, notice the direction of the lines of force (magnetic field) in the area between the two conductors. Remember that the field of force is at right angles to the plane of the paper, therefore the lines of force are directed either away from or toward the observer, as shown by the pointer or the tail of the arrow in the same manner as the direction of current. In the area between the conductors, the lines of force oppose each other, thus causing the field to be weakened, while in the area outside the conductors the field is strengthened.

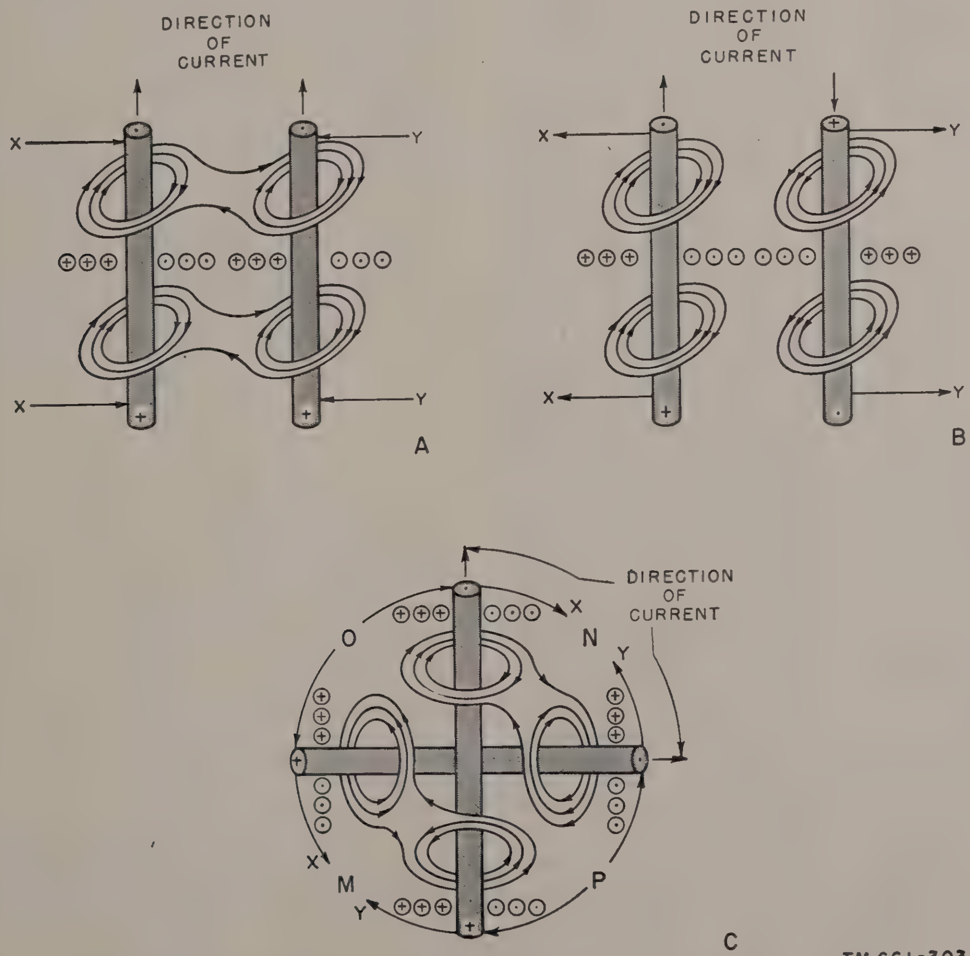


Figure 122. Attraction and repulsion between conductors carrying currents.

Due to the weakened field the lines of force tend to encircle both the conductors like rubber bands, and the conductors are attracted to each other in the direction of the arrows X and Y.

(2) *Parallel currents flowing in opposite directions cause repulsion.* B of figure 122 shows two conductors with current flowing in opposite directions. By applications of the left-hand rule, the lines of force indicate that the two magnetic fields aid each other in the region between the conductors. Since this field is strengthened, compared with the field outside of the conductors, it tends to push the conductors apart as illustrated by the direction of the arrows X and Y.

(3) *Currents through conductors making an angle with each other produce fields which tend to force the conductors into parallel paths with their currents flowing in the same direction.* C of figure 122 shows two conductors placed at right angles to each other and carrying current in the direction indicated by the arrows. If the left-hand rule is applied to each of these two conductors, the magnetic fields as indicated by the lines of force, in the quadrant marked M and N, will be similar to the magnetic field between the conductors in A of figure 122. This means that the field in quadrants M and N will be weakened. For the same reason, if the field in the quadrants O and P is compared to the field between the conductors in B of figure 122, the lines of force indicate that the magnetic field in these quadrants is strengthened. Therefore the two conductors will tend to move toward a parallel position, the ends of the conductors moving in the direction of the arrows X and Y. The current flows in the same direction in both conductors.

b. The three laws developed in this paragraph may be incorporated into one statement as follows: *Any two circuits carrying current tend to so place themselves that they will include the largest possible number of lines of force.* The student should keep in mind that a conductor carrying current will always tend to move from a strong magnetic field to a weak magnetic field and the direction of the movement is the resultant of the forces caused by all the magnetic fields acting on a conductor.

131. Magnetic Field of a Single Coil

a. If a conductor, similar to the one represented in figure 120 and carrying current, is bent in the form of a loop (A of fig. 123), the same

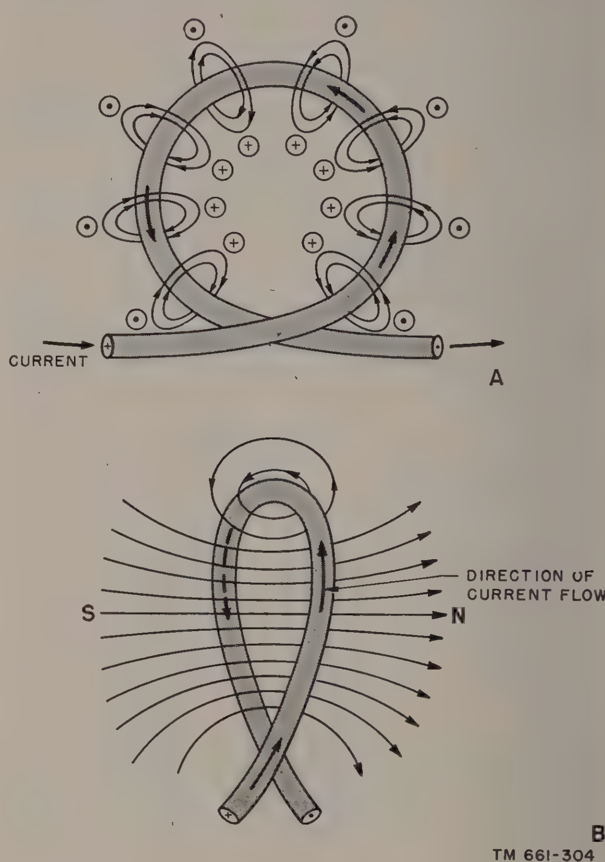


Figure 123. Single loop showing lines of force.

circular lines of force or magnetic field surround the conductor as when it was straight. Also, when the conductor is bent into a loop, all the lines of force will enter on one side (face) of the loop and leave on the other side (face) of the loop (B of fig. 123). This is in accordance with the left-hand rule. Thus, a north pole is created on one face of the loop and a south pole on the other face of the loop.

b. If several loops or turns of wire are so wound as to form a coil, it may be called a *helix* or *solenoid*. Actually, a helix is any coil of wire that is carrying current, while a solenoid is a coil that has considerable length as compared with its diameter. A of figure 124 shows a solenoid wound

loosely. Since the current is flowing through each turn in the same direction, the effect of the magnetic field produced is similar to that of parallel conductors (A of fig. 122). Thus the field is weakened due to the opposition of the flux between the turns, causing some of the lines of force to encircle several turns of the entire coil. Those lines of force produce a magnetic field similar to the magnetic field of a bar magnet (B of fig. 124), making a north pole at one end of the coil, and a south pole at the other end of the coil. Now if the coils are pushed as close together as possible (C of fig. 124), a great many

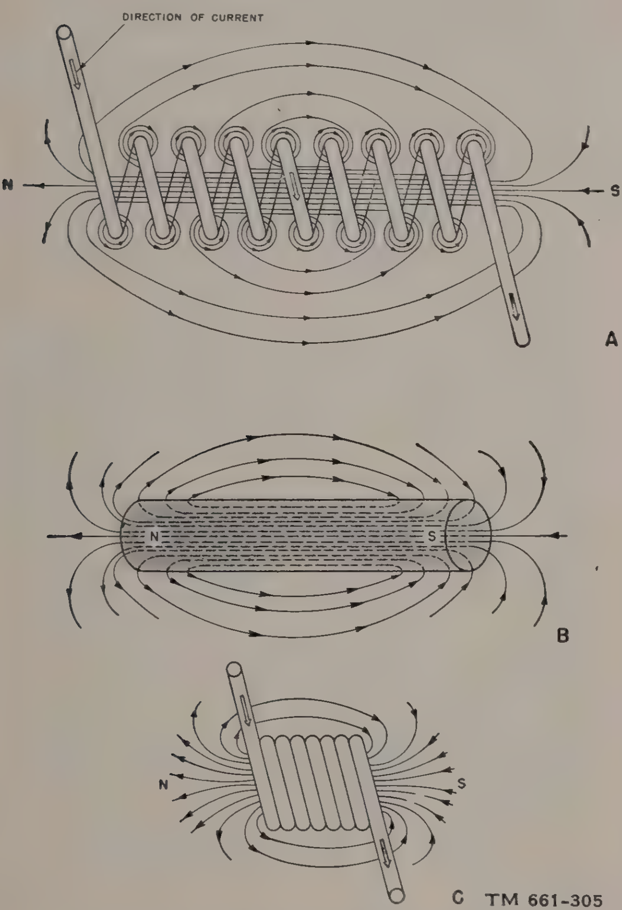


Figure 124. Magnetic field about a solenoid.

more lines will encircle the entire coil. Obviously the field within the coil and at the poles is much stronger than the field illustrated in A of figure 124, since practically all the lines of force which previously encircled the individual loops will now encircle the entire solenoid. Therefore, while current is flowing, the solenoid has the properties of a permanent magnet.

c. The polarity of a coil or solenoid may be determined by a left-hand rule as follows: *Grasp the coil with the left hand so that the fingers will point in the direction of the flow of current around the coil and extend the thumb at right angles to the fingers. The thumb will then point in the direction of the north pole of the coil (fig. 125).* If the current

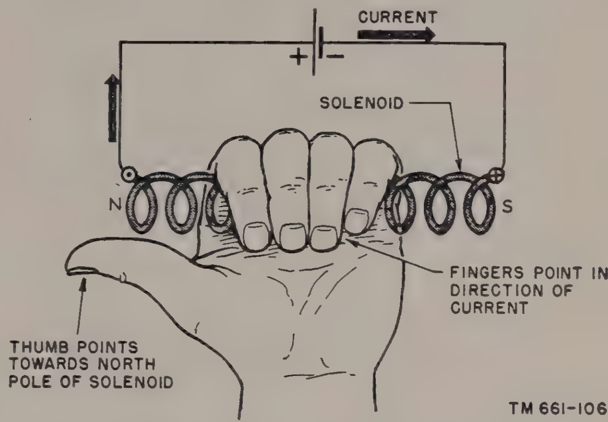


Figure 125. Left-hand rule for a coil or solenoid.

in the coil is reversed, the polarity of the coil will reverse also.

132. Electromagnets

a. If a piece of magnetic material, usually soft iron, is placed within a solenoid through which current is flowing the magnetic properties of the solenoid are tremendously increased. The reason for this increase in magnetic strength is because soft iron is more permeable than air (ch. 1), and therefore provides a better path for the lines of force (flux) than does air. The inside of any coil is considered as the core whether it be air or of some magnetic material. If a coil is wound on a core of magnetic material, it is called an *electromagnet* (A of fig. 126). The coil may be wound with one or more layers of wire from one end to the other and back, providing, of course, that the current flows around the core continuously in the same direction. The rule for determining the polarity of an electromagnet is the same as for a solenoid, that is, by use of the left-hand rule.

b. Any magnetic material may be used as the core for an electromagnet. However, soft iron or soft steel generally is used because the retentivity of those materials is so low that they have the characteristic of retaining very little residual magnetism when the current stops flowing. This is a very important quality of electrical equipment such as

relays. On the other hand, a piece of hard steel when inserted into a coil in which current is flowing becomes a *permanent* magnet. That is, due to its *retentivity,* it will retain a large amount of *residual magnetism* when the current ceases to flow. Electromagnets are widely used in electrical equipment, including relays, motors, generators, radios, and transformers. The U-type electromagnet illustrated in B of figure 126 is used in the receiver of a telephone.

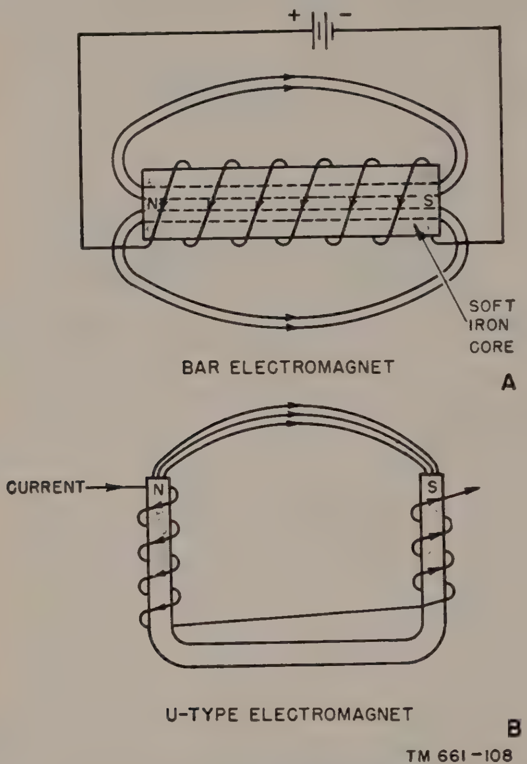


Figure 126. Types of electromagnets.

c. The circuit of a simple relay is shown in figure 127. When the switch is closed current flows through the coil of the electromagnet, and is said to *energize* it. That is, it becomes a magnet and the magnetic field attracts the soft iron armature and overcomes the pull of the spring so that the armature is drawn up against the contact. When the contact is closed, it completes the circuit through the load and the 12-volt battery, and current flows. The current will flow from the negative terminal of this battery through the armature and the load back to the positive terminal of the battery. When the switch is opened, the current flow through the electromagnet stops and it is said to become *de-energized.* Since the magnetic field disappears, the armature is pulled away from the contact by the spring, breaking the circuit and removing the power from the load.

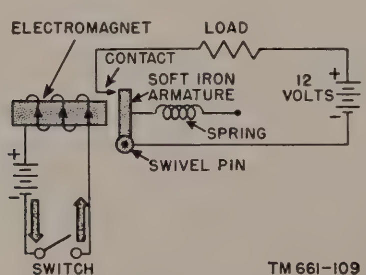


Figure 127. Simple relay.

133. The Magnetic Circuit

a. The laws which apply to the magnetic circuit are similar to (but not the same as) those of the electric circuit. It has previously been shown (ch. 1) that magnetic flux forms closed loops. The path that the flux takes, whether through air or through metal, is called the magnetic circuit. The magnetic circuit will now be compared with the electric circuit.

b. In order to produce an electric current, a voltage, known as the electromotive force, is required. Similarly, in order to produce a magnetic flux, a force known as the magnetomotive force is required. In an electric circuit, for a given electromotive force, the current depends on the amount of resistance. Likewise in a magnetic circuit, for a given magnetomotive force, the flux depends on the amount of opposition (reluctance). It is then evident that in the magnetic circuit, as in the electric circuit, the relationship expressed in the following statement is true: *The result produced is directly proportional to the force that produces it and inversely proportional to the opposition encountered.* For the electric circuit, it has been shown in chapter 6 that —

$$\textit{current } (I) = \frac{\textit{electromotive force } (E)}{\textit{resistance } (R)}$$

For the magnetic circuit a similar law may be stated as follows:

$$\textit{flux} \text{ (lines of force)} = \frac{\textit{magnetomotive force } (m.m.f.).}{\textit{reluctance}}$$

Using symbols instead of words, the following formula may be written:

$$\phi = \frac{F}{R}$$

where ϕ=flux,

F=m.m.f. in gilberts, and

R=reluctance in oersteds.

If F is expressed in ampere-turns (par. 134), R is usually in rels. Ampere-turns may be changed to gilberts by multiplying by 1.26 which gives R in oersteds.

c. There are important differences in the relationship of the electric circuit to the magnetic circuit.

(1) In the electric circuit, the resistance is a constant and can be determined by the ratio of voltage to current (discounting the heating effect). In a magnetic circuit, however, the reluctance is not a constant but depends on the flux (strength of the field).

(2) In electric circuits, current actually flows (electron flow) from one point to another. In the magnetic circuit, there is no actual flow of flux, but merely an indication of the intensity and direction of the magnetic field.

134. Magnetomotive Force (M. M. F.)

a. The magnetizing force set up due to current flowing in a coil or solenoid is known as the magnetomotive force. Since the strength of the magnetic field about a conductor increases when the current through the conductor increases (par. 128), the magnetic field about a coil or solenoid will also increase when the current through the coil is increased. In fact, for any coil, if the current is doubled the strength of the field will also double. Also, since the total magnetic field above a coil is a summation of the field of the individual loops or coils, if the number of loops is increased, the strength of the magnetic field will increase. Therefore, the amount of flux (lines of force) about a helix, whether it have an air core or an iron core is proportional to two factors:

(1) The current in amperes flowing in the coil.

(2) The number of loops or TURNS in the coil.

b. The word "TURN" means just one wrap of a conductor around a core which may be either air for a solenoid or in the case of an electromagnet, a piece of soft iron (A of fig. 128).

c. The magnetomotive force is proportional to the current (in amperes) in the circuit and to the number of turns of the coil.

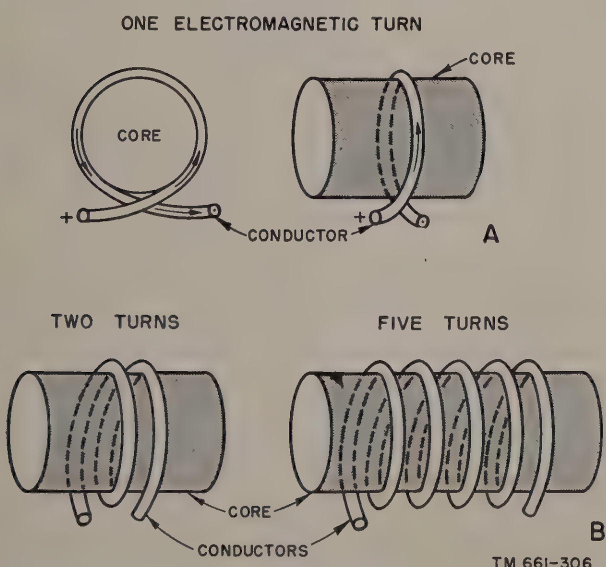


Figure 128. Electromagnetic turns.

135. Magnetization Curves

a. In order to simplify magnetic circuit computations, *magnetization curves* (see note below) of certain materials have been prepared (fig. 129).

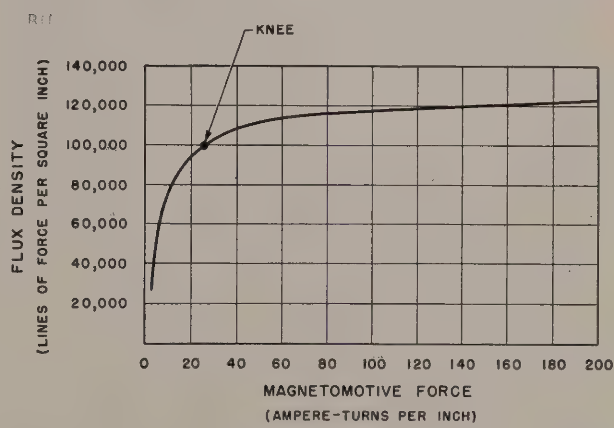


Figure 129. Magnetization curve for annealed sheet steel.

The magnetomotive force, expressed in ampere-turns per inch, is plotted against the flux density (flux per square inch). Examination of the curve shows that when the magnetomotive force is 20 ampere-turns, the flux density is about 95,000 lines of force per square inch.

Note. Magnetization curves are also known as *B–H* curves.

b. When the magnetomotive force is increased from 20 to 40 ampere-turns per inch, the flux

density increases from 95,000 to about 108,000 lines of force per square inch.

c. As the ampere-turns per inch continue to increase, the flux density also increases but at a lesser rate. Whereas 40 ampere-turns per inch produce 108,000 lines of force per square inch, note that increasing the ampere-turns to 160, or four times as much, only produces 120,000 lines of force per square inch.

d. It is evident then that a magnetization curve provides a quick and simple method of calculating flux density if the type of core and magnetomotive force in ampere-turns per inch is known. Of course, other magnetic substances do not follow the same magnetization curve as that of annealed sheet steel. In fact, a new magnetization curve must be made for every type of *magnetic* substance, for no two curves will be exactly alike.

e. The point where the curve begins to flatten out is called the *knee* of the magnetization curve (fig. 129). This point is also called the *saturation point* since beyond this point it becomes increasingly difficult to increase the flux density. For example, there is a large increase in flux density when the ampere-turns per inch are increased from 0 to 20, but there is only an extremely small relative increase in flux density when the ampere-turns per inch are increased from 160 to 180. In radio circuits, it is usually desirable that the flux density remain below the knee so that a change in the magnetomotive force will produce a more or less proportional change in the flux density.

136. Hysteresis Loss

a. Hysteresis is a lagging of the magnetizing effect behind the magnetizing cause. This phenomena will be explained with the aid of figure 130. If a constantly increasing magnetizing force (ampere-turns per inch) is applied to a core that

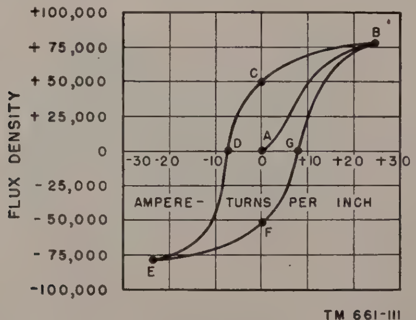


Figure 130. Typical hysteresis loop or magnetic cycle.

has not been magnetized previously, the magnetization curve *A–B* shows how the flux density increases. If, at point *B*, the magnetizing current is reduced, the core will start to lose its magnetism as shown by curve *B–C*. The flux density becomes smaller as the magnetomotive force is reduced until point *C* is reached when the magnetomotive force is 0. Note that the flux density at point *C* is 50,000 even though there is no applied magnetomotive force. This lagging or hysteresis is due to the residual magnetism remaining in the core.

b. If a negative magnetomotive force is then applied, by reversing the direction of the current, the core will lose its magnetism as shown by curve *C–D*. It is evident that 8 ampere-turns of negative magnetomotive force are required to completely demagnetize the iron at point *D*. These 8 ampere-turns of magnetomotive force, which are necessary to overcome the residual magnetism, represent a distinct loss of power and are referred to as *hysteresis loss.*

c. If the negative magnetomotive force is then increased beyond *D* to a numerical value equal to the positive value at *B*, the core will become magnetized in the opposite direction, as shown by curve *D–E*. The flux density at *E* will then be equal to the flux density at *B*.

d. If the negative magnetomotive force is then decreased to 0, the flux density will vary according to the curve *E–F*. At point *F* the magnetizing force is 0. The flux density, due to residual magnetism, is −50,000. The minus sign merely indicates that the direction of the field at point *F* is opposite to the direction of the field at point *C*.

e. If a positive magnetomotive force is then applied and gradually increased, the core will lose its magnetism along curve *F–G*, become completely demagnetized at point *G*, and become magnetized again in the opposite direction along curve *G–B*. This completes the hysteresis loop.

f. It can be shown that the area within the hysteresis loop is proportional to the amount of *work* done against the residual magnetism. The energy or power thus lost (hysteresis loss) is considerable and is transformed into heat in the core. The hysteresis loss is therefore an important factor in the design of iron core transformers in which the direction of the flux is continually reversed. A hysteresis loop of small area means a relatively small hysteresis loss while a loop of large area indicates a large loss. This point will be treated more fully in the manual on alternating current.

137. Relationship of Electric and Magnetic Fields

a. Up to now, electromagnetism has been attributed to the flow of current in a conductor. However, current flow also means *charges in motion*. It follows, therefore, that electromagnetism is the result of charges in motion and that a charge moving with respect to the observer is always accompanied by a magnetic field. This fact can be verified by the following experiment.

(1) Suppose that several pieces of tinfoil are fastened to the outer edge of a round glass disk and the tinfoil is then charged with negative electricity, that is, each piece of tinfoil is touched by a previously charged negative body (A of fig. 131). The negative charges on the tinfoil are represented by the electrons. If the glass disk is then rotated, these electrons will also be rotated as shown in B of figure 131.

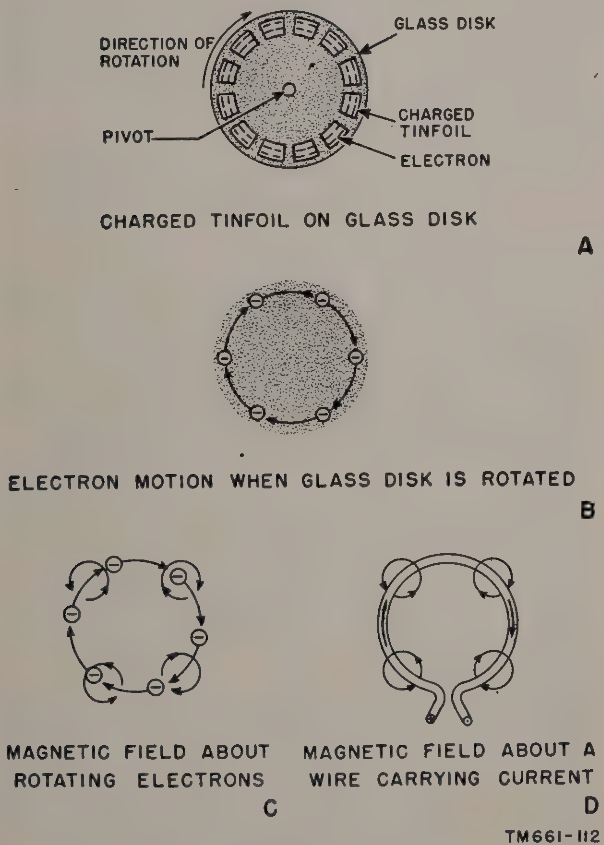


Figure 131. Electrons in motion produce an electric field.

(2) If a compass is then held above the rotating electrons, it will be deflected, thus showing the presence of a magnetic field. By placing the compass in various positions, the magnetic field about the rotating electrons will be found to be that shown in C of figure 131. Note that this field is the same as that about a wire carrying current (D of fig. 131).

b. The above experiment shows that *a charge in motion produces a magnetic field*. This is one of the most important of the fundamental laws of electricity, and is the basis upon which rests the operation of generators and the production of radio waves.

c. The fact that a charge in motion produces a magnetic field leads to some very important conclusions. It will be recalled that an electric field exists in the space about a positive or negative charge (fig. 31). Therefore, when a charge is moved its electric field moves with it, since the two are inseparable. It follows that instead of a charge in motion, it can be said that an electric field in motion produces a magnetic field. The previous conclusion can then be restated as follows: *A moving electric field creates a magnetic field*. This action is reversible, that is, *a moving magnetic field creates an electric field*.

d. The force of the magnetic field acts at right angles to the electric field and also at right angles to the direction in which the charge is moving. This fact is illustrated in figure 132. It will be recalled that the electric field about a negative charge such as an electron is represented as indicated in A of figure 132. Also, the magnetic field about an electron flowing away from the observer is shown in B of figure 132. When both figures are combined as in C of figure 132, the relationship between the electric and magnetic fields about a moving charge can be seen easily. Note that whenever an electric line of force crosses a magnetic line of force, the two are always perpendicular (at right angles) to each other. Also note that the electric and magnetic lines of force are at right angles to the direction in which the charge is moving. The theory of wave propagation in the study of radio shows that this relationship between the electric and magnetic fields is similar to the relationships existing in an electromagnetic or radio wave if the direction of propagation is substituted for the moving charge.

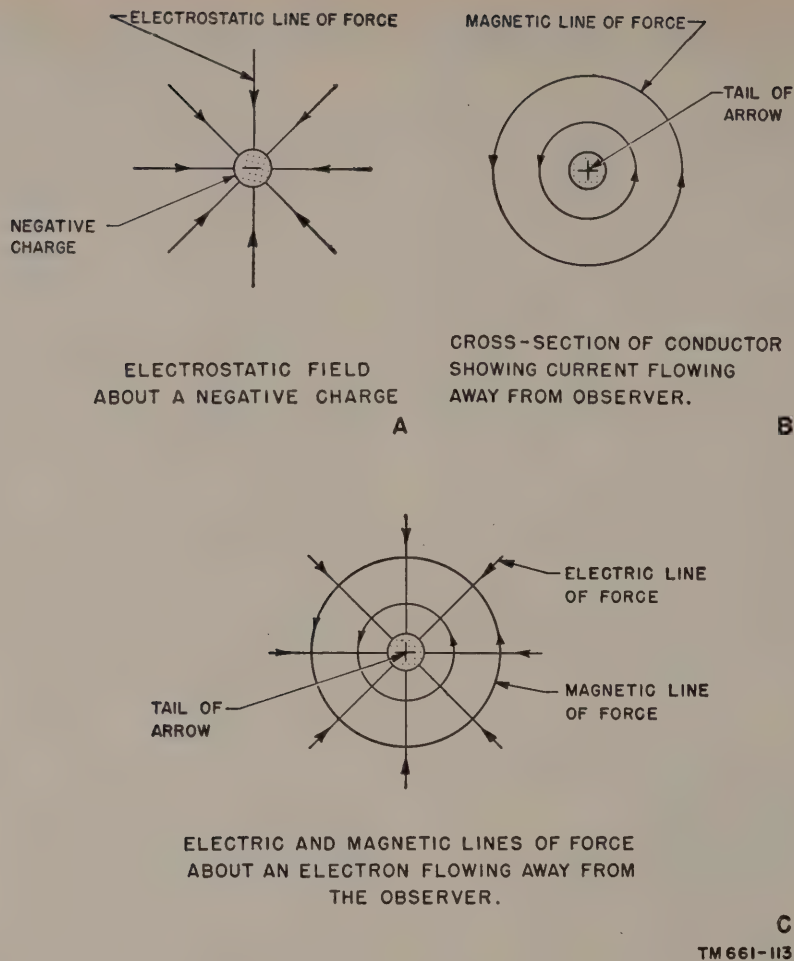


Figure 132. Electric and magnetic lines of force.

138. Energy in Magnetic and Electric Fields

a. When a magnet is moved over iron filings, the magnetic field of the magnet causes the iron filings to move although the iron filings and magnet are not in contact. Thus, the magnetic field does work on the iron filings. Since the magnetic field has the ability to do work, and since energy is defined as the ability to do work, *the magnetic field therefore possesses energy.* This energy is appropriately called *magnetic energy.*

b. Since magnetic fields are created by the *motion* of charges, the magnetic field is said to represent *kinetic* energy. This is equivalent to saying that the kinetic energy of an electric current is stored as magnetic energy in the magnetic field the electric current creates.

c. In practice, all circuits have some amount of resistance, such as the resistance of the wire used for connections, the resistance of the wire of which a coil is made, or the resistance of the resistors in the circuit. Any current flowing through this resistance causes an I^2R power loss which appears in the form of heat. When power is supplied to a simple circuit that contains resistance, some of the power appears in the form of heat. The remainder of the power becomes the kinetic energy of the current and is stored as magnetic energy in the magnetic field. When the power is removed from the circuit, interrupting the current flow, the magnetic field collapses and returns its energy to the circuit by inducing an emf in it, as will be explained in chapter 11. In other words, it requires energy to establish a magnetic field, and this energy is returned to the circuit when the current stops flowing and magnetic field collapses.

d. Just as a magnetic field of force represents energy, so also does an electric field of force represent energy. It will be recalled that if an isolated, or free, electron is placed in an electric field, the electron will be driven from a lower to a higher

potential point by the electric field of force. *The electric field thus does work on the electron and therefore possesses energy.*

e. Thus, both the magnetic and the electric fields possess energy. The total energy of a moving charge is therefore the sum of the energy in its electric and magnetic fields.

139. Summary

a. An electric current in a conductor is surrounded by a magnetic field.

b. The magnetic lines of force around a current-carrying conductor take the form of concentric circles around the conductor.

c. The strength of the magnetic field about a current-carrying conductor increases when the current increases and decreases when the current decreases.

d. Reversing the direction of the current through a conductor reverses the direction of the magnetic field about that conductor.

e. The left-hand rule for determining the direction of the magnetic field about a current-carrying conductor is: Grasp the conductor in the left hand with the thumb pointing in the direction of the current (electron current) flow. The fingers will point in the direction of the magnetic field.

f. The magnetic field about an energized coil or solenoid is similar to that about a bar magnet.

g. The left-hand rule for solenoids and electromagnets is: Grasp the coil with the left hand so that the fingers point in the direction of the flow of current. The thumb will then point in the direction of the north pole of the solenoid.

h. Reversing the direction of the current through a solenoid reverses the direction of the magnetic field about that solenoid.

i. Either end of an energized solenoid attracts iron filings.

j. An electromagnet consists of a solenoid and an iron or steel core placed within it.

k. The magnetizing force set up by current flowing through a solenoid is known as the magnetomotive force.

l. Magnetomotive force equals the product of the current and the number of loops or turns and is expressed by a unit called the ampere-turn.

m. In an *electric* circuit, *current*= $\dfrac{\textit{electromotive force}}{\textit{resistance}}$	In a *magnetic* circuit, *flux*= $\dfrac{\textit{magnetomotive force}}{\textit{reluctance}}$

n. A magnetization curve shows how the flux density varies as the magnetomotive force increases.

o. Hysteresis is the lagging of the magnetizing effect behind the magnetizing cause.

p. Hysteresis loss refers to the power required to overcome the residual magnetism in the core of an electromagnet.

q. A charge in motion creates a magnetic field about it.

r. A moving electric field creates a magnetic field. A moving magnetic field creates an electric field.

s. The electric and magnetic lines of force about a moving charge are at right angles to each other and at right angles to the direction in which the charge is moving.

t. A magnetic field possesses *magnetic energy.* An electric field possesses *electric energy.*

u. When a magnetic field collapses its energy is returned to the circuit.

v. The total energy of a moving charge is the sum of the energy in its electric and magnetic fields.

140. Review Questions

a. Is an electric current always surrounded by a magnetic field?

b. What forms do the magnetic lines of force take around a conductor carrying an electric current?

c. Distinguish between magnetism and electromagnetism.

d. If the current through the conductor is increased, will the strength of the magnetic field around the conductor increase?

e. Reversing the direction of the current through a conductor has what effect on the direction of the magnetic field?

f. What is the conventional symbol for representing current flowing into a conductor away from the observer? Out of a conductor toward the observer?

g. State the left-hand rule for determining the direction of the magnetic field of force around a conductor carrying a current.

h. Using the left-hand rule, find the resultant field about two parallel conductors in which current flows in opposite directions. In the same direction.

i. Describe the magnetic field about a loop of wire through which current flows.

j. Indicate the magnetic field of a solenoid and show that this field is the sum of the fields produced by the individual loops.

k. Has a solenoid, when carrying an electric current, a definite north and south pole?

l. State the left-hand rule for determining the polarity of a solenoid.

m. What happens to the poles of a solenoid if its current is reversed?

n. Why does an energized solenoid attract magnetic material placed at either end of the solenoid?

o. What is an electromagnet?

p. Is residual magnetism desirable in an electromagnet?

q. When is an electromagnet energized? De-energized?

r. Is the rule for determining the polarity of an electromagnet the same as that for a solenoid? Explain.

s. What is magnetomotive force?

t. Magnetomotive force is proportional to what factors?

u. In what units is magnetomotive force measured? Give an example.

v. Magnetomotive force establishes flux in a magnetic circuit, the amount of flux depending on the reluctance in the circuit. How does this statement compare with electromotive force, current, and resistance in an electric circuit?

w. Does flux actually flow, the same as current?

x. What substance is generally used for the cores of electromagnets?

y. How can the strength of an electromagnet be increased?

z. What is a magnetization curve and what is it used for?

aa. What is the knee of a magnetization curve? Saturation point?

ab. What is hysteresis?

ac. Describe the formation of a hysteresis loop.

ad. What is a hysteresis loss?

ae. How does hysteresis loss effect the core of an electromagnet?

af. Describe an experiment that proves that a charge in motion produces a magnetic field. From this fact show that a moving electric field creates a magnetic field.

ag. Does a moving magnetic field create an electric field?

ah. What is the relationship of the electric and magnetic fields about a moving charge?

ai. Show that a magnetic field possesses energy. Likewise for an electric field.

aj. What is the total energy of a moving charge?

ak. What happens to a magnetic field about a conductor or coil when the circuit is broken? What becomes of the energy in the magnetic field?

CHAPTER 11
INDUCED ELECTROMOTIVE FORCE

141. Electromagnetic Induction Due to Cutting of Magnetic Field by a Conductor

a. As a source of commercial electrical energy on a large scale basis, batteries are relatively expensive and unwieldy. Therefore, other means are used to obtain electrical energy in large quantities. Probably the most common source of electric energy is the generator, which depends on the principle of *electromagnetic* induction for its operation.

b. This principle was discovered by Michael Faraday in 1831 and can be demonstrated easily as follows:

(1) Connect a highly sensitive current-indicating device, such as a zero-center galvanometer (fig. 133), to the ends of a conductor, such as a copper wire, and move the conductor downward through a magnetic field, so that the conductor cuts across the magnetic flux.

(2) When this is done, a momentary voltage will be *induced* in the conductor with polarity as indicated. This *induced voltage,* in turn, will cause current (electrons) to flow through the galvanometer and move its indicating needle. This momentary deflection of the galvanometer needle is an indication that a voltage has been induced in the conductor. The current caused by the induced voltage is called the *induced current.*

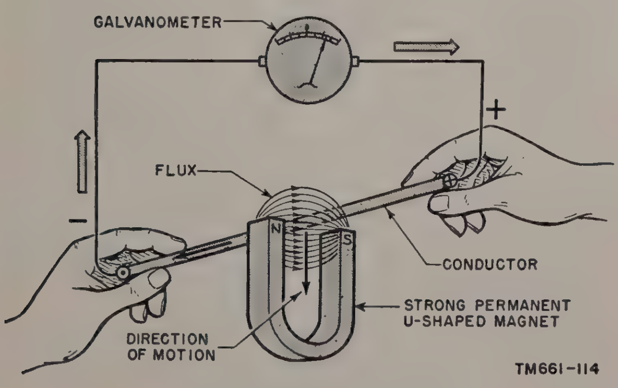


Figure 133. Induced current.

(3) If the conductor is held stationary in the magnetic field, the galvanometer needle will indicate zero, thus showing that no voltage is being induced in the conductor.

(4) If the conductor is moved upward through the magnetic field, the galvanometer needle will again be deflected, but in the *opposite* direction to that of when the conductor was moved downward. This indicates that the *polarity* of the induced voltage has been reversed, thus reversing the direction of the induced current through the galvanometer.

(5) If the conductor is moved in the field in a sidewise direction, from the north pole to the south pole of the magnet or vice versa, so that no lines of force are cut, the galvanometer needle will remain at zero, indicating that no voltage is being induced in the conductor. As the angle between the flux lines and the path of the conductor is increased, the deflection of the galvanometer needle increases until the maximum deflection is obtained when the conductor is swept through the magnetic field at right angles (90°) to it.

c. If the polarity of the magnetic field is changed, that is, if the north and south poles of the magnet are interchanged, the polarity of the induced voltage will be reversed also.

d. The above experimental facts lead to the conclusion that the conductor must cut through the magnetic field in order to have a voltage induced in it and, also, that the polarity of the induced voltage depends on the direction of motion of the conductor and the direction of the magnetic field.

142. Left-Hand Generator Rule

The relationship between the motion of the conductor, the magnetic flux it cuts, and the direction of the induced current is summed up by

the left-hand generator rule (fig. 134). With the thumb, forefinger, and middle finger of the left hand set perpendicularly to each other as shown, *point the forefinger in the direction of the flux, and point the thumb in the direction of motion of the conductor; the middle finger will then point in the direction in which the generated or induced voltage tends to send the current through the circuit.*

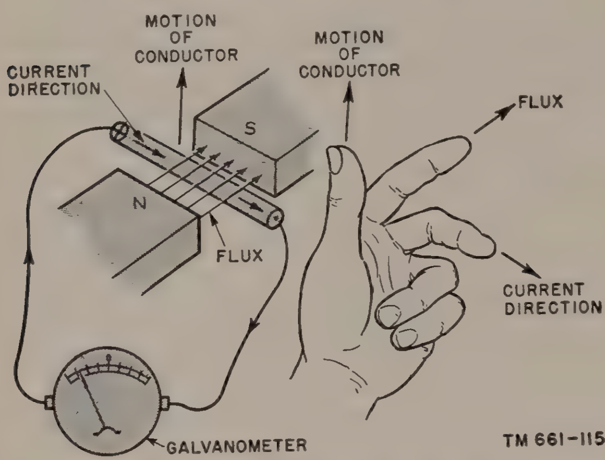


Figure 134. Left-hand generator rule.

143. Electromagnetic Induction Due to Cutting of Conductor by Magnetic Field

a. It was shown previously that a voltage is induced in a conductor when it is moved through a magnetic field. The reverse of this is also true. That is, if the conductor in figure 133 is held stationary, and the magnet is moved so that its field cuts the conductor, a voltage will be induced in the conductor. In other words, it makes no difference if the magnetic field is kept stationary and the conductor cuts it in a downward direction, or if the conductor is held stationary and the magnetic field cuts it in an upward direction; the induced voltage and current will be exactly the same and have the same polarity. This is to be expected since the *relative* motion of the magnetic field with respect to the conductor is the same in either case. To apply the left-hand generator rule, however, it is always necessary to assume that the magnetic field is stationary and that the conductor is cutting it.

b. Another example of inducing a voltage by cutting a conductor with a moving magnetic field is illustrated in figure 135. In this case, the conductor is a loop of wire, the ends of which are connected to a galvanometer. When a permanent bar magnet is thrust through the stationary loop, the field of the magnet cuts the loop as shown, and induces a voltage in it. The resulting current through the galvanometer causes a deflection of its needle. If the magnet is held stationary in the stationary loop, no voltage is induced. If the magnet is withdrawn from the loop, a voltage of opposite polarity will be induced in the loop.

c. The principle of electromagnetic induction can then be stated as follows: *Whenever there is relative motion between a conductor and a magnetic field, a voltage will be induced in the conductor; the motion may be produced either by moving the conductor through a stationary magnetic field or by making the magnetic field move through and cut a stationary conductor.* The induced voltage is also referred to as the *induced emf.*

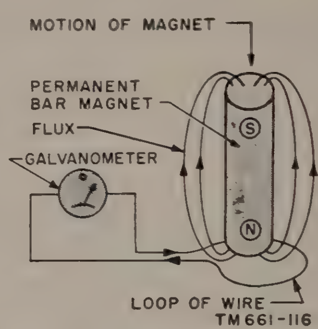


Figure 135. Inducing an emf. in a loop.

144. Factors Determining Magnitude of Induced Electromotive Force

a. There are three factors that determine the magnitude (strength) of the induced emf. The first factor is the number of turns in the conductor. In figure 135, only one loop of wire is used. If two loops of wire are used it will be found that the motion of the magnet through the loops of wire will cause twice the deflection on the galvanometer that one turn will cause, everything else remaining equal. If more loops or turns of wire are added (fig. 136), the motion of the magnet will further increase the deflection of the galvanometer needle. It is apparent, then, that *increasing the number of turns of wire increases the magnitude of the induced emf.* The reason for this is that the turns of wire are actually connected in series and the induced voltages in each turn of wire are additive. The total induced voltage is the sum of the voltages induced in the individual turns.

b. The second factor in increasing the induced emf is to increase the strength of the magnetic field. This can be done by utilizing a winding

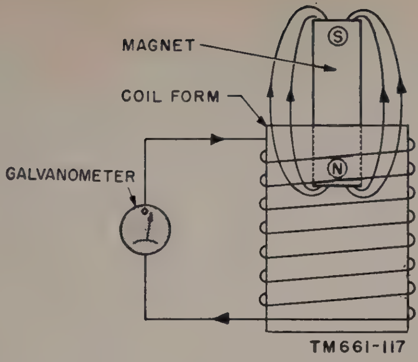


Figure 136. Inducing an emf. in a coil.

around the magnet and making it an electromagnet with a greatly increased field strength (fig. 137). The cutting of this field by a conductor will produce a greatly increased emf. *Increasing the field strength increases the induced emf.*

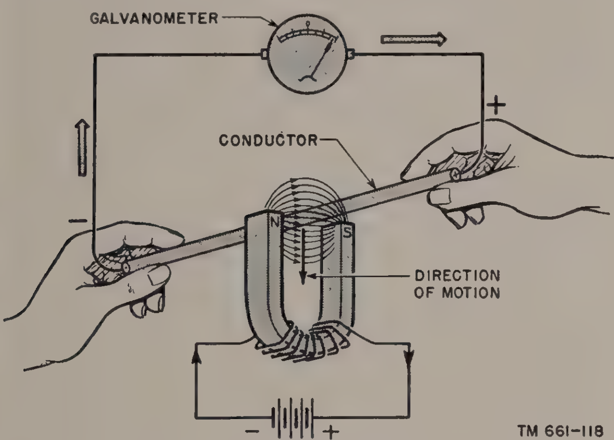


Figure 137. Increasing the field strength increases the induced emf.

c. The third factor that determines the magnitude of the induced emf is the *speed* of the relative motion of the conductor and the magnetic field. If the conductor is passed through the magnetic field, first slowly and then rapidly, it will be found that a much greater emf is induced when the conductor is moved rapidly. *Increasing the speed of the conductor (or magnetic field) increases the induced emf.*

d. Summing up, whenever there is a relative motion between a conductor and a magnetic field, the magnitude of the emf induced in the conductor is directly proportional to—

(1) *The number of turns in the conductor.*
(2) *The strength of the magnetic field.*
(3) *The speed of the relative motion between the conductor and the magnetic field.*

145. Explanation of Electromagnetic Induction

a. In the preceding paragraphs, the principle of electromagnetic induction and the factors that determine the magnitude of the induced voltage have been set forth, but no attempt has been made to show actually why there should be such a thing or phenomenon as an induced voltage. The cause of the induced emf is not very mysterious, but logically is derived from previously explained electrical principles.

b. For example, consider the now familiar conductor cutting the stationary magnetic field as shown in A of figure 138. The direction of motion of the conductor is downward and the polarity of the induced voltage or emf is indicated.

c. If an observer were stationed on the north pole of the magnet and could see lines of force, he would see the lines of force flowing from the north to the south pole as indicated in B of figure 138.

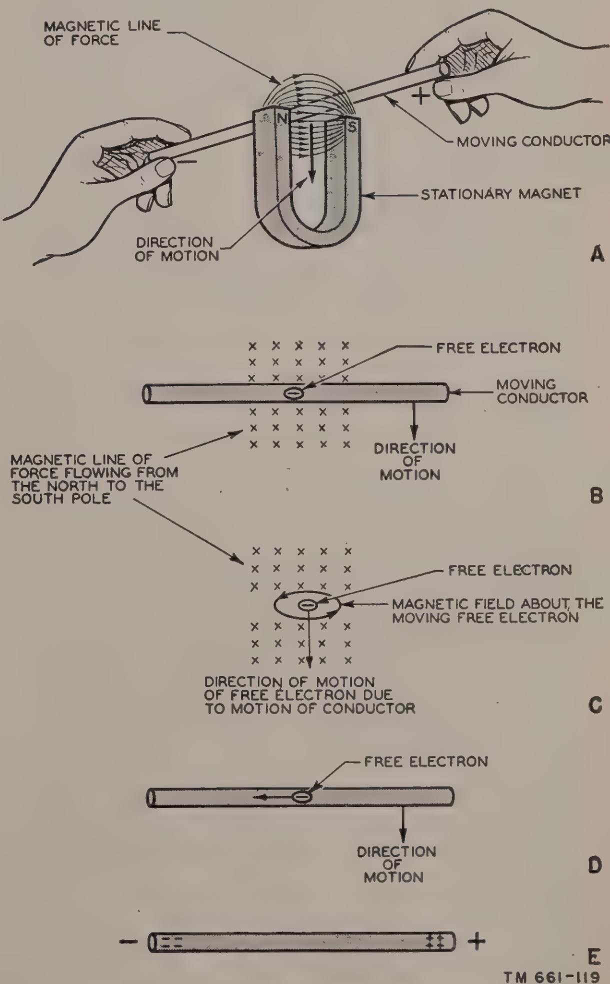


Figure 138. Electron theory of induced emf.

(The crosses represent lines of force flowing from the north to the south pole.) The conductor is indicated as moving downward and cutting the lines of force or flux. B of figure 138, therefore, is only another view of the cutting of flux shown in A of figure 138.

d. The conductor in B of figure 138 contains free electrons, one of which is indicated. When the conductor is moved downward through the magnetic field, the free electrons in the conductor must necessarily move with the conductor. The direction of motion of the free electron indicated in B of figure 138 is then downward as shown in C of figure 138. *This electron is then a charge in motion.*

e. It was shown previously that a charge in motion produces a magnetic field about it (fig. 131). Therefore, there is a magnetic field about the moving free electron in C of figure 138, as indicated. The direction of this field is determined by the use of the left-hand rule.

f. Note that at the right of the moving free electron its field is in the *same* direction as the field from the north to the south pole of the magnet. The resultant magnetic field on the right side of the moving free electron is therefore strengthened.

g. On the left side of the moving free electron, however, its field *opposes* the field from the north to the south pole of the magnet. The resultant magnetic field on the left side of the moving free electron is therefore weakened.

h. Under these conditions, the moving free electron is urged from the strengthened part toward the weakened part of the field, or from right to left as shown in D of figure 138. Other free electrons in the conductor are displaced in the same manner, thus causing the left end of the conductor to have an excess of electrons and be negatively charged (E of fig. 138). The right end of the conductor has a deficiency of electrons and is positively charged. Note that the polarity of the charged ends of the conductor agrees with that indicated in A of figure 138 and also with the experimentally determined polarity indicated in figure 133.

i. In the conductor shown in E of figure 138, work was done in moving the electrons from right to left and a potential difference exists between the ends of the conductor. Actually, each electron is moved only an infinitely small distance and there is a potential difference across this small distance. However, this occurs all along the conductor and the individual potential differences add, since they are in series, to produce an over-all potential difference between the two ends of the conductor.

j. This potential difference will be maintained as long as the conductor continues to move downward and keeps the negative and positive charges separated. The negative end of the conductor is at a lower potential with respect to the positive end of the conductor. For this condition, the conductor can be compared with a battery if an external circuit such as the galvanometer is connected to the ends of the conductor (fig. 137), current (electrons) will flow from the negative end of the conductor, through the galvanometer, to the positive end of the conductor.

k. The force that causes electrons to flow through an electric circuit is called electromotive force (emf). Therefore, it is said that whenever there is a relative motion between a conductor and a magnetic field, an emf is induced in the conductor, rather than a potential across it. Actually, the maximum potential difference across the conductor is called the emf, and this maximum potential difference appears only when no current is drawn by an external circuit. (When current is drawn, the potential difference across the conductor is always less than the emf induced in it.) Also, since emf is measured in volts, the induced emf is also referred to as the *induced voltage*.

l. From the foregoing explanation of how an emf is induced in a conductor, it is evident that a complete circuit is not required. When there is no complete circuit, an emf is induced in the conductor but no current flows since there is no complete circuit through which current can flow. Thus, it is possible to have an induced emf without a resulting induced current.

146. Faraday's Law

a. In the previous paragraphs, it was shown that a voltage was induced in a conductor moving with respect to a magnetic field. This result is actually a consequence of a more general experimental law which was first discovered by Faraday.

b. *If the total flux linking a circuit changes with time, there will be an emf induced in this circuit.* This is the law that will be used in developing the principles of mutual induction. Faraday also discovered that if the rate of flux change is increased, the magnitude of the induced emf is increased also. Thus, the induced emf depends on the following two factors:

(1) Amount of flux linking the circuit.
(2) Rate of change of flux linking the circuit.

147. Inducing an Emf in a Neighboring Conductor (Mutual Induction)

a. Consider the two coils shown in A of figure 139. In coil (*A*), electrons are moving in the direction shown by the arrowheads and, therefore, constitute a current. From the work in electromagnetism, it is known that this current will produce a flux of magnetic field as shown in the diagram (A of fig. 139). If the current in coil (*A*) has a constant value, the number of flux lines produced is fixed; but if the current in coil (*A*) is changed (for example, by opening the shorting switch), the number of flux lines created in coil (*A*) is decreased and consequently the flux linking coil (*B*) will decrease also. This changing flux in (*B*) will, as explained previously, induce an emf in coil (*B*), which is evidenced by the movement of the needle of the current indicating device (B of fig. 139).

b. Thus, it is seen that energy can be transferred from one circuit to another by the principle of electromagnetic induction. The particular circuit shown is called a transformer. Obviously, if conditions are not changing in circuit (*A*), then no emf will be induced in (*B*). Now, when current begins to flow in coil (*B*), it sets up its own flux, a part of which will link coil (*A*) (B of fig. 139). Notice that the direction of flux produced by the induced current in coil (*B*) is the same as that produced by coil (*A*). This is a general principle known as Lenz's law and can be explained physically. Suppose that I_A were increased in the direction shown. Then the changing flux linking (*B*) would induce a voltage in (*B*) and this voltage could produce a current I_B.

c. It is very important that the directions of the induced emf in coupled circuits be determined correctly (fig. 140):

(1) Notice that the sense in which coil (*A*) is wound with respect to (*B*) is shown very clearly. The arrow through battery E_A, means that it is a variable source of voltage.

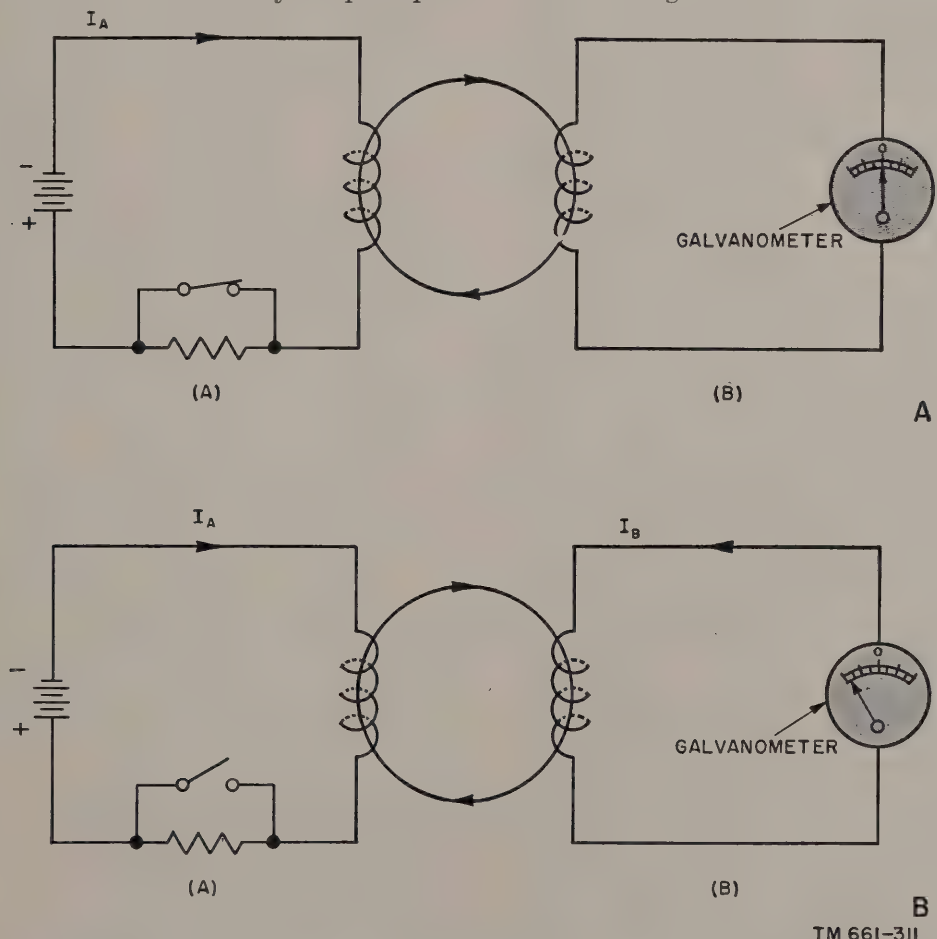


Figure 139. Electromagnetic induction.

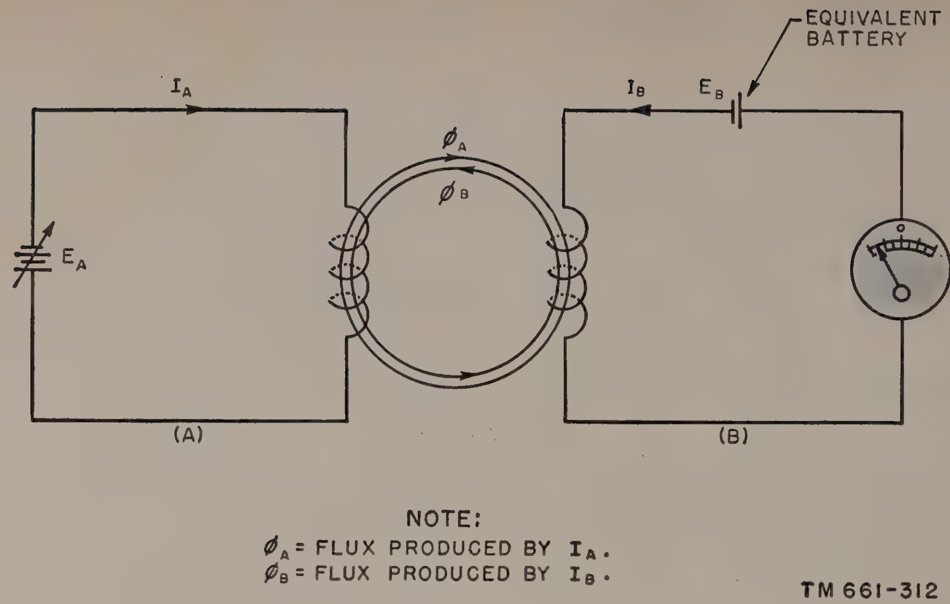


Figure 140. Diagram illustrating Lenz's law.

(2) Suppose E_A is increased; I_A will increase in the direction shown and, therefore, Φ_A will increase. As explained before, this change causes an emf to appear in coil (B). From the physical considerations discussed previously, it is known that the emf induced in (B) is of such polarity as to produce a current creating a flux in a direction which opposes the increase in Φ_A. This means that Φ_B must be in the direction shown; applying the left-hand rule, it is seen easily that the flow of electrons in (B) must be as indicated by the arrow, I_B. It is known that a battery of polarity E_B would give the correct direction of current I_B. Thus, the effects of the emf of mutual induction can be replaced by an equivalent battery, the magnitude of which is that of the induced emf, and the polarity of which is chosen in accordance with Lenz's law.

d. In actual practice, of course, all the flux produced by the primary current does not link all the turns of the secondary; only a fraction of this flux is coupled into winding (B) from (A). This statement is also true as regards the flux linking (A) from (B). Some of the flux produced by I_A links (A) alone and some of the flux produced by (B) links (B) alone. These fluxes are called *leakage fluxes*. The component transformer fluxes are the leakage flux of (A), the leakage flux of (B), and a mutual flux linking (A) and (B). This separation of the fluxes is shown in figure 141.

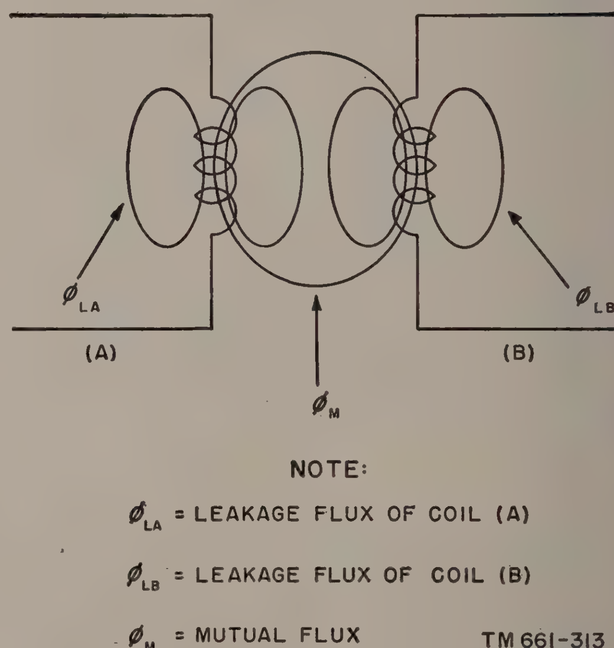


Figure 141. Component fluxes of a transformer.

If ϕ_A=total flux produced by I_A,
ϕ_B=total flux produced by I_B,
ϕ_{LA}=leakage flux of (A),
ϕ_{LB}=leakage flux of (B),
ϕ_{AB}=flux which links (B) due to I_A,
ϕ_{BA}=flux which links (A) due to I_B, and
ϕ_M=mutual flux,

then the relations between these quantities are—

$$\phi_A = \phi_{AB} + \phi_{LA},$$
$$\phi_B = \phi_{BA} + \phi_{LB}, \text{ and}$$
$$\phi_M = \phi_{AB} + \phi_{BA}.$$

The last equation is merely a statement of the fact that *the mutual flux is the sum of the flux linking* B *rom* A *plus the flux linking* A *from* B.

e. Since the emf induced in (B) by mutual induction is the sum of the emf's induced in the separate turns of (B), the greater the number of turns of coil (B), the greater the induced emf will be. Also, the amount of the flux linking (B) and the rate at which this flux is changing, will influence the magnitude of the emf of mutual induction. These factors are listed below:

(1) Number of turns on secondary winding (B).
(2) Amount of flux linking (B).
(3) Rate of change of flux linking (B).

148. Summary

a. The principle of electromagnetic induction may be stated as follows: *An emf is induced in any circuit in which the amount of flux linking it is changing with time.*

b. If a conductor is moving with respect to a magnetic field, there is an emf induced in the conductor which is directly proportional to the velocity of the conductor with respect to the field.

c. For a given velocity of conductor and direction of magnetic field, the polarity of the induced emf is given by the left-hand rule.

d. Lenz's law, which is merely a restatement of the principle of conservation of energy, states that the direction of the induced emf in a circuit is such that it tends to produce a current, the direction of which is such as to create a flux which is in opposition to the change in flux which was responsible for the induced emf.

e. The voltage induced in a coil by induction is proportional to the number of turns of the coil, the magnitude of the inducing flux and the rate of change of this flux.

149. Review Questions

a. State the principle of electromagnetic induction.

b. When a conductor cuts a magnetic field, does the polarity of the induced voltage depend on the direction of motion of the conductor and the direction of the magnetic field? Explain.

c. State the left-hand generator rule. Give an example.

d. State three factors upon which the emf induced in a conductor depends.

e. Can there be an induced current without an induced emf? Can there be an induced emf without an induced current? Explain.

f. Define mutual induction and give an example.

g. When a current varies in strength, what happens to the magnetic field around it?

h. What happens when a conductor is placed in a moving magnetic field? Near a conductor through which the current is changing?

i. Is there an emf induced in a conductor whenever there is a change in the magnetic flux linked with it? Why?

j. Can current be induced by current? Explain.

k. If the direction of the induced current in a conductor is known, how can the polarity of the induced voltage across that conductor be determined?

l. If the current in the primary is constant, does the magnetic flux that links the secondary also remain constant?

m. What is the only way that the magnetic flux that links the secondary can induce a voltage in it?

n. Give three ways of reversing the polarity of the induced voltage in the secondary.

o. State three factors upon which the emf induced in the secondary depends. Compare with the answer to question 4.

p. What happens to the magnetic field about a conductor or a coil when the current through the conductor or coil stops flowing?

CHAPTER 12
INDUCTANCE

150. Self-Induction

It has been shown in chapter 11 that if the magnetic flux linking a circuit changes, an emf is induced in the circuit. Now suppose we have a coil wound on a core as shown in figure 142.

a. The current flowing through the coil produces a flux in the direction shown by the arrows. If the voltage E is increased, the current I will also increase, and this, in turn, will cause an increase in flux linking the coil. The *change* in flux linking the coil induces an emf in the coil which tends to oppose the increase in flux. From the principles developed in chapter 11, we know that the effect of this induced emf can be replaced by an equivalent battery having the polarity shown in figure 142. Notice that the polarity of this equivalent battery voltage is opposite to that of the driving voltage E. *This emf of induction is spoken of as the emf of self-induction, or cemf (counter emf).* In other words, a *change* of current flowing through the coil produces an induced emf in the coil. By Lenz's law, we know that the polarity of this emf of self-induction is such that, *at every instant of time*, it opposes the change in flux. Since the flux is proportional to the current (in air core and iron core coils without saturation), *the induced emf tends to oppose any change in current.* Obviously, the greater the number of turns in the coil, and the more rapidly the current changes (thus the more rapidly the flux changes), the greater will be the cemf, or emf of self-induction. The factors on which the cemf depend are—

(1) The number of turns on the coil.
(2) The rate of change of current.
(3) The geometry of the circuit and the material from which the core is made.

> *Note.* The geometry of the circuit means the cross-sectional area of the core and the length of the core (in the case of air core, the length of the flux lines).

b. Since a given magnetomotive force acting on an iron core produces a much greater flux than when it acts on an air core, the kind of material used for the core is important. A greater flux will mean a larger induced emf; therefore, the cemf depends on the permeability of the medium. It has been found that the cemf can be expressed by the following equation:

(*Cemf*) equals $-L \times$ (*rate of change of current*), where L is the coefficient of self-induction.

Note. The negative sign indicates that when the current is decreasing (rate of change of current is negative), the cemf is positive; when the current is increasing (rate of change of current is positive), the cemf is negative. This

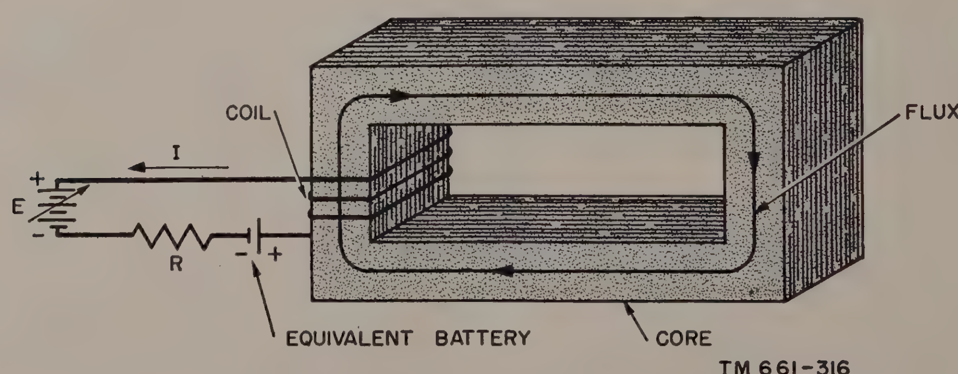


Figure 142. Equivalent battery used to represent the effects of the induced emf.

is in accordance with Lenz's law. Thus, the factors upon which the coefficient of self-induction depend are—

(1) The number of turns.
(2) The cross-sectional area of the core.
(3) The length of the core.
(4) The permeability of the core.

c. For a coil wound on the core shown in figure 143, the inductance L is given by the following formula:

$$L=\frac{0.4\pi\mu AN^2\times 10^{-8}}{l} \text{ henrys,}$$

where L=inductance in henrys,
μ=permeability of core material,
A=cross-sectional area of core in cm² (fig. 143),
N=number of turns, and
l=mean length of core in cm (fig. 143).

Note that the inductance varies directly as the square of the number of turns. Thus, doubling the number of turns quadruples the inductance L.

d. In summarizing, we may say that *all coils have the property of tending to prevent any current changes.* This property is called the *inductance* and depends upon the geometry of the coil.

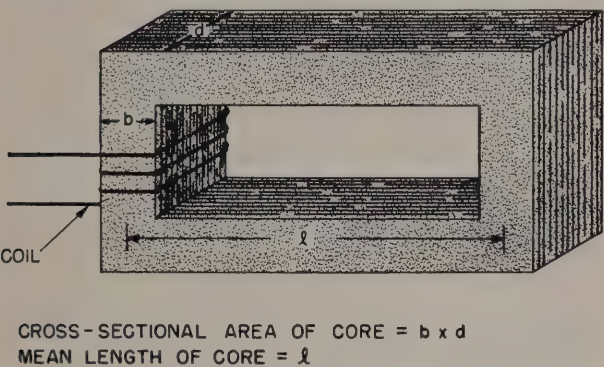


Figure 143. Coil wound on iron core of dimensions b, d, and l.

151. Inductance

a. A device which possesses inductance is called an *inductor.* For example, a coil is an inductor.

b. The greater the cemf produced in a circuit, the greater is the opposition to a change of current in that circuit. Therefore, the cemf produced by a specified change of current is a measure of the inductance of a circuit. This measure is expressed in *henrys.* *A circuit has an inductance of 1 henry when a current change of 1 ampere-per-second causes a cemf of 1 volt to be induced in it (A of fig. 144).* As a formula,

$$\text{Inductance } L \text{ (in henrys)}=\frac{\text{induced voltage } E \text{ (in volts)}}{\text{rate of change of current (in amperes per second).}}$$

For example, if the current in a circuit is changing at the rate of 1 ampere per second, and the induced voltage is 2 volts, the inductance is 2 henrys (B of fig. 144). To take another example, suppose the current in a circuit or coil changes from 4 amperes to 7 amperes in 1 second, which is a rate of change of 3 amperes per second, and the induced voltage is 6 volts; the inductance is then 6 divided by 3, or 2 henrys.

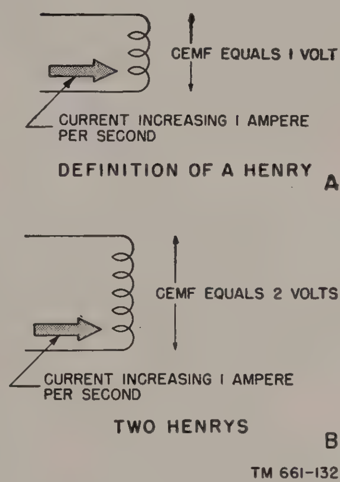


Figure 144. Cemf developed across inductances of 1 henry (A) and 2 henrys (B).

c. In figure 144, the larger inductor is indicated by more turns of wire. This method is not commonly used to indicate relative amounts of inductance. A method in general use is to use the same number of turns for various values of inductance and label each inductor with its correct value. Since radio circuits use inductors as large as 100 henrys and as small as one-millionth of a henry, the millihenry (mh) representing one-thousandth of a henry, and the microhenry (μh) representing one-millionth of a henry, are used frequently.

d. Rearranging the above formula for inductance, induced voltage=inductance $\times$ rate of change of current, or more simply,

$$e=L \times \textit{rate of change of current.}$$

The usefulness and importance of this formula is immediately apparent since it gives the value of

the induced voltage e when L and the rate of change of current are known. For example, if the current through a coil changes from 5 amperes to 7 amperes in 1 second, or a rate of change of 2 amperes per second, and the inductance is 8 henrys, the induced voltage is 8 times 2 equals 16 volts. The formula shows that increasing either L or the rate of change of current *increases* the induced voltage. Likewise a decrease in either L or the rate of change of current *decreases* the induced voltage.

e. The student will find that the inductance and rate of change of current for a particular circuit usually is known and, for this reason, the above formula is used to analyze the behavior of electrical circuits. To further simplify and shorten this formula, the expression *rate of change of current* has been replaced by the symbol $\frac{di}{dt}$. The formula is then—

$$e=L\frac{di}{dt}.$$

f. It will be recalled however, that the voltage induced in the coil is a *cemf* which opposes the change of current. To indicate this, a minus sign is placed in front of the right-hand side of the equation which then reads

$$e=-L\frac{di}{dt},$$

where e equals the induced voltage,

L equals the inductance, and

$\frac{di}{dt}$ equals the rate of change of current $\left(\frac{di}{dt}\right.$ actually represents a small change in current di which takes place within a certain small interval of time $\left.dt\right)$.

The minus sign does not mean that the induced voltage e is negative but merely means that e opposes the change of current.

g. This formula applies to any induced voltage when the induced voltage is in the same circuit or coil in which the current is changing. The formula is simple to visualize since $\frac{di}{dt}$, the rate of change of current, describes the rapidity with which the magnetic field or flux changes and cuts the conductor, in this case, the inductor whose value of inductance is L. The more rapid the cutting, the greater the induced voltage.

152. Growth and Decay of Current in Inductive Circuits

a. Now that the concepts of *cemf* and inductance have been discussed, it is easy to analyze what goes on when current increases in a circuit containing inductance. Consider the circuit of A of figure 145 where a battery (E) is connected in series with a coil (L), a resistor (R), and a switch. To simplify matters, assume that the resistor R includes all the resistance in the circuit, that is, the resistance of any resistors in the external circuit, the resistance of the battery, and the resistance of the windings of the coil.

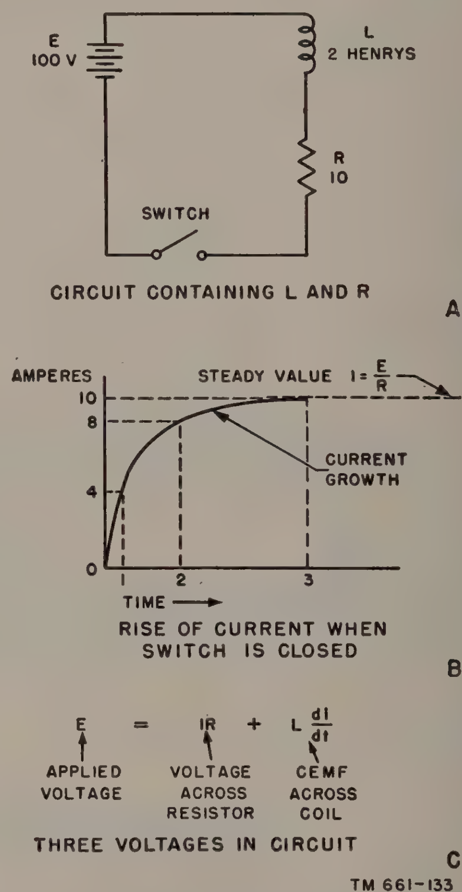


Figure 145. Analysis of circuit containing L and R.

b. When the switch is closed, the current in the circuit does not immediately jump to its final steady value because of the *cemf* in the coil, but rises as shown by the current growth curve in B of figure 145. During the time that the current is rising or increasing in strength, there are three voltages in the circuit:

(1) The applied battery voltage E.

(2) The IR drop across resistor R.

(3) The cemf, $L\frac{di}{dt}$, across the coil.

c. According to Kirchhoff's second law (app. III), the sum of all the voltage drops in any current path must equal the voltage applied to that path. This means that the battery voltage E must equal the sum of the IR drop across the resistor R and the cemf $L\frac{di}{dt}$ across the coil. In equation form,

battery voltage $E=IR$ drop across resistor + cemf across coil or more simply

$$E=IR+L\frac{di}{dt}$$

where E is the applied voltage,
I is the current at any instant,
R is the total resistance in the circuit,
L is the inductance of the coil,
$\frac{di}{dt}$ is the rate of change of current.

This equation, which is a generalization of Ohm's law to include inductance in a circuit, expresses the conditions for *any* instant in the circuit. It shows that part of the battery voltage is used to force current through the coil against the cemf caused by the increasing current, and that the remainder of the battery voltage is used to force current through the *resistance* of the circuit.

d. To illustrate this, suppose that the values of battery voltage, coil inductance, and circuit resistance are 100 volts, 2 henrys, and 10 ohms, respectively (A of fig. 145). At the instant the switch is closed, at time zero (B of fig. 145), the current in the circuit starts to increase from zero. At this time, there is no IR drop across the resistor, and the cemf $L\frac{di}{dt}$ is equal to the applied battery voltage of 100 volts. Since L is 2 henrys, $\frac{di}{dt}$ (the *rate of change of current*), must be 50 amperes per second. That is, in changing from zero current to some value of current, the rate of change of current at the instant the switch is closed is *50 amperes per second.*

e. The current increases with time, as shown. For example, at time 1, the current is 4 amperes. The IR drop is then 4 times 10 equals 40 volts. From the equation in C of figure 145, it is evident that if the applied voltage E is 100 volts and the IR drop is 40 volts, the voltage across the coil, $L\frac{di}{dt}$ equals 60 volts. That means that 60 volts of 100-volt battery voltage is being used to overcome the cemf in the circuit, and that 40 volts is being used to force current through the resistance R. Also, since $L\frac{di}{dt}$ is 60 volts, and since L is 2 henrys, it follows that $\frac{di}{dt}$ is 60/2 equals 30 ampers per second. That is, at time 1, the *current is changing* at the rate of 30 amperes per second.

f. At some later time (time 2 in B of fig. 145), the current increases to 8 amperes. The IR drop across R is then 8 times 10 equals 80 volts. Since E is 100 volts, $L\frac{di}{dt}$ (C of fig. 145) is 100 minus 80 equals 20 volts. This means that, at time 2, only 20 volts of the 100-volt battery is being used to overcome the cemf in the circuit and that 80 volts is being used to make current flow through the resistance R. In addition, since $L\frac{di}{dt}$ equals 20 volts, and L equals 2 henrys, $\frac{di}{dt}$ equals 20/2 or 10. At time 2, then, the rate of change of current is *10 amperes per second.*

g. From the foregoing, it is evident that *as the amount of current in the circuit increases, the rate of current increase becomes smaller*, the voltage across the resistance R increases, and the cemf across the coil decreases. This bears out the previously stated fact that as the rate of increase of current becomes smaller, the cemf induced is smaller. Finally, at time 3, the current reaches its final value, the magnetic field about it ceases to fluctuate, there is no cemf induced in the coil, and the full battery voltage of 100 volts appears across the resistance R. The steady value of current is then, by Ohm's law, the battery voltage divided by the value of resistance or 100/10 equals 10 amperes. This is the maximum current that can flow in the circuit.

h. When L is small, the current in the circuit rises rapidly because the cemf, $L\frac{di}{dt}$, is correspondingly small. When L is large, the cemf is correspondingly larger, and the current therefore rises slowly. Figure 146 shows how the rise of current is affected if the inductance value is changed and the applied voltage and circuit resistance is kept constant. For the particular circuit chosen, with an inductance of 1 henry, the current increased to 10 amperes in 5 seconds. For an inductance of 10

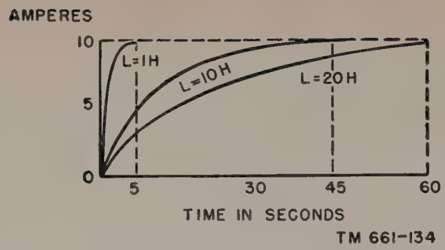


Figure 146. Value of inductance in a circuit containing L and R affects time required for current to reach its maximum value

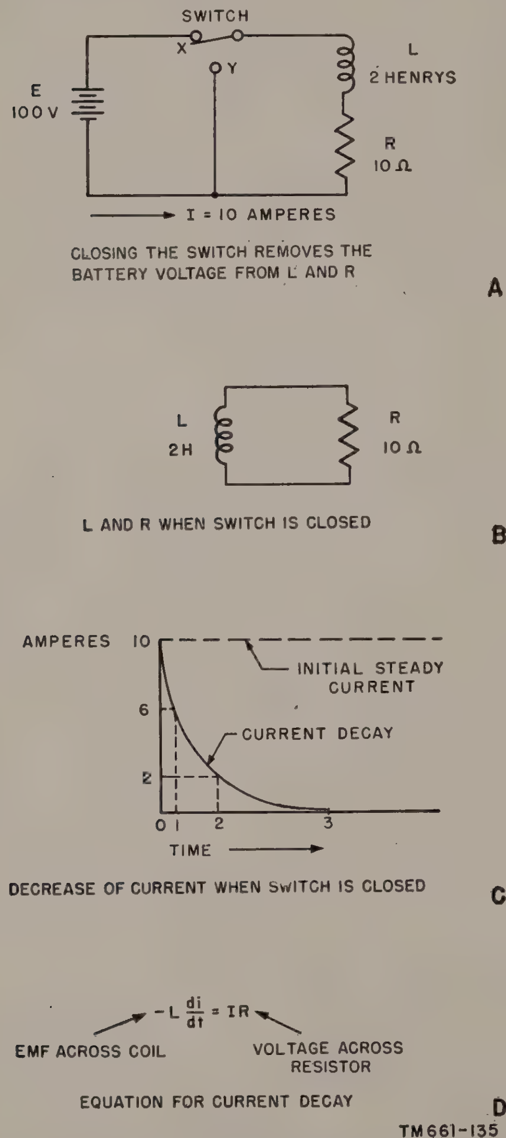


Figure 147. Analysis of circuit containing L and R.

henrys, however, it takes about 45 seconds for the current to increase to this amount. When the inductance is further increased to 20 henrys, it takes 60 seconds for the current to increase to 10 amperes. *Increasing the inductance in a circuit, therefore, decreases the rate at which the current will rise, and increases the time necessary for the current to reach its final steady value.*

i. Starting a current in an inductive circuit causes a cemf to oppose it; stopping a current causes an emf to make the current persist for a small period of time before coming to rest at zero. The shape of the current decay curve is the inverse of the current growth curve. A of Figure 147 shows a battery E furnishing a *steady* 10-ampere current I to the inductance L and resistance R. The 100-volt battery voltage appears across resistor R; no voltage appears across inductance L. If the switch is opened instantaneously at X and closed at Y, battery voltage E will be removed from the circuit and L and R will form a closed circuit (B of fig. 147). Since there is no battery in the circuit, the current will attempt to drop to zero immediately. The collapsing magnetic field about the coil, however, cuts it and induces in it an emf which tends to maintain the flow of current. Therefore, at time zero (C of fig. 147), when the switch is closed, the current remains at 10 amperes, and the IR drop across the resistor remains 100 volts, since 10 amperes times 10 ohms equals 100 volts. But reference to B of figure 147 shows immediately that if 100 volts appear across R, 100 volts also appear across L. In other words, the induced voltage $L\frac{di}{dt}$ is equal to the battery voltage which appears as an IR drop across R. As a formula,

$$-L\frac{di}{dt}=IR.$$

The minus sign merely indicates that $\frac{di}{dt}$ is decreasing.

j. Since at time zero, $-L\frac{di}{dt}$ equals 100 volts, and since L is 2 henrys, $-\frac{di}{dt}$ is 100/2 equals *50 amperes per second*. That is, the current starts to decrease from 10 amperes at the rate of 50 amperes per second.

k. When the current falls to 6 amperes at time 1 (C of fig. 147), IR equal 6 times 10 equals 60 volts. Since R is connected across L (B of fig 147), $-L\frac{di}{dt}$ also equals 60 volts. With L equal to 2 henrys, $-\frac{di}{dt}$, the rate of decrease of current, is 60/2 equals 30 amperes per second.

l. When the current falls to 2 amperes at time 2, the IR drop is 2 times 10 equals 20 volts and this

is also the induced voltage $-L\frac{di}{dt}$. Since L is 2 henrys, $\frac{di}{dt}$ is 20/2 equals *10 amperes per second.*

m. It is evident then that *as the amount of current decreases, the rate of decrease of current decreases,* and the equal and opposite voltages across R and L also decrease. This keeps on until the current decreases to zero at time 3, at which time there is no current or voltage in the circuit.

n. Furthermore, just as increasing the inductance in a circuit decreases the rate at which the current will rise (fig. 146), increasing the inductance also decreases the rate at which the current will decay or fall (fig. 147).

153. Development of High Induced Voltages when an Inductive Circuit is Completely Broken

a. The magnitude of self-induced voltages can be extremely great even though the source of voltage is very low. For example, consider the circuit in figure 148 which consists of a 6-volt battery, a switch, and a 102-henry coil of very many turns wound on a soft iron core. The resistor R represents the resistance of the windings of the coil.

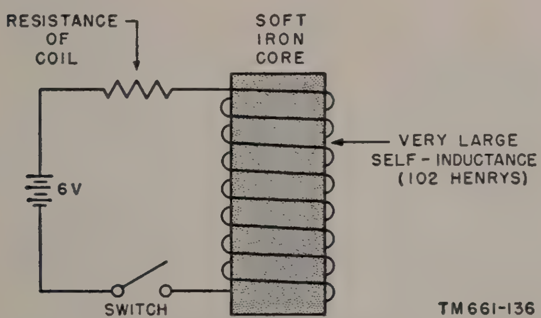


Figure 148. Circuit used for explanation of the development of high induced voltage.

b. At the instant the switch is closed, 6 volts of cemf will appear across the coil. Then, if the switch is opened, the induced emf in the coil can rise to several hundred volts. Since the resistance of the battery is practically negligible, this voltage appears across the switch and causes a strong spark to jump from one terminal of the switch to the other. The energy in the spark is liberated in the form of heat and can cause serious burns. The switch contacts also can be damaged by the excessive heat. Furthermore, the voltage across the coil may be strong enough to puncture its insulation. For these reasons, circuits such as the field windings of generators and motors should never be broken in this way.

c. The reason that the induced emf is greater when the switch is opened than when it is closed is simple. At the instant the switch is closed, the current starts to increase from zero. At this time there is no IR drop across the resistor, and the cemf $L\frac{di}{dt}$ is equal to the applied battery voltage of 6 volts. Since L is 102 henrys, $\frac{di}{dt}$ is 6 divided by 102, or 1/17 ampere per second. That is, the current starts to increase at the rate of 1/17 ampere per second.

d. After a small interval of time, the current rises to its final steady value, a value determined by the resistance of the windings of the coil. The magnetic field about the coil is then fully established and steady. If the switch is now opened, one side of the battery is disconnected and there is no complete path through which the current can flow. As a result, the current stops flowing almost immediately and the magnetic field about the coil collapses, cuts it very rapidly, and induces a voltage in it. Suppose, for example, the *rate of decrease of current* is about 6 amperes per second. With $\frac{di}{dt}$ equal to 6, and with L equal to 102 henrys, the induced voltage $L\frac{di}{dt}$ equals 6 times 102 equals 612 volts. The induced voltage is therefore much greater when the switch is opened than when it is closed.

e. The above explanation also accounts for the fact that a spark is observed when a complete circuit is broken by pulling out a plug from an electrical outlet. This method of developing a high voltage from a 6-volt source is used in the spark coil circuits of automobiles.

154. Types of Inductors

a. An inductor is a device, the primary purpose of which is to introduce inductance into an electric circuit. In radio sets, inductors, are used for many varied purposes such as to couple energy from one circuit to another, match impedances, tune resonant circuits, remove hum, step-up voltages, etc. For this reason, inductors are manufactured in many sizes and shapes, depending on the function for which they are designed.

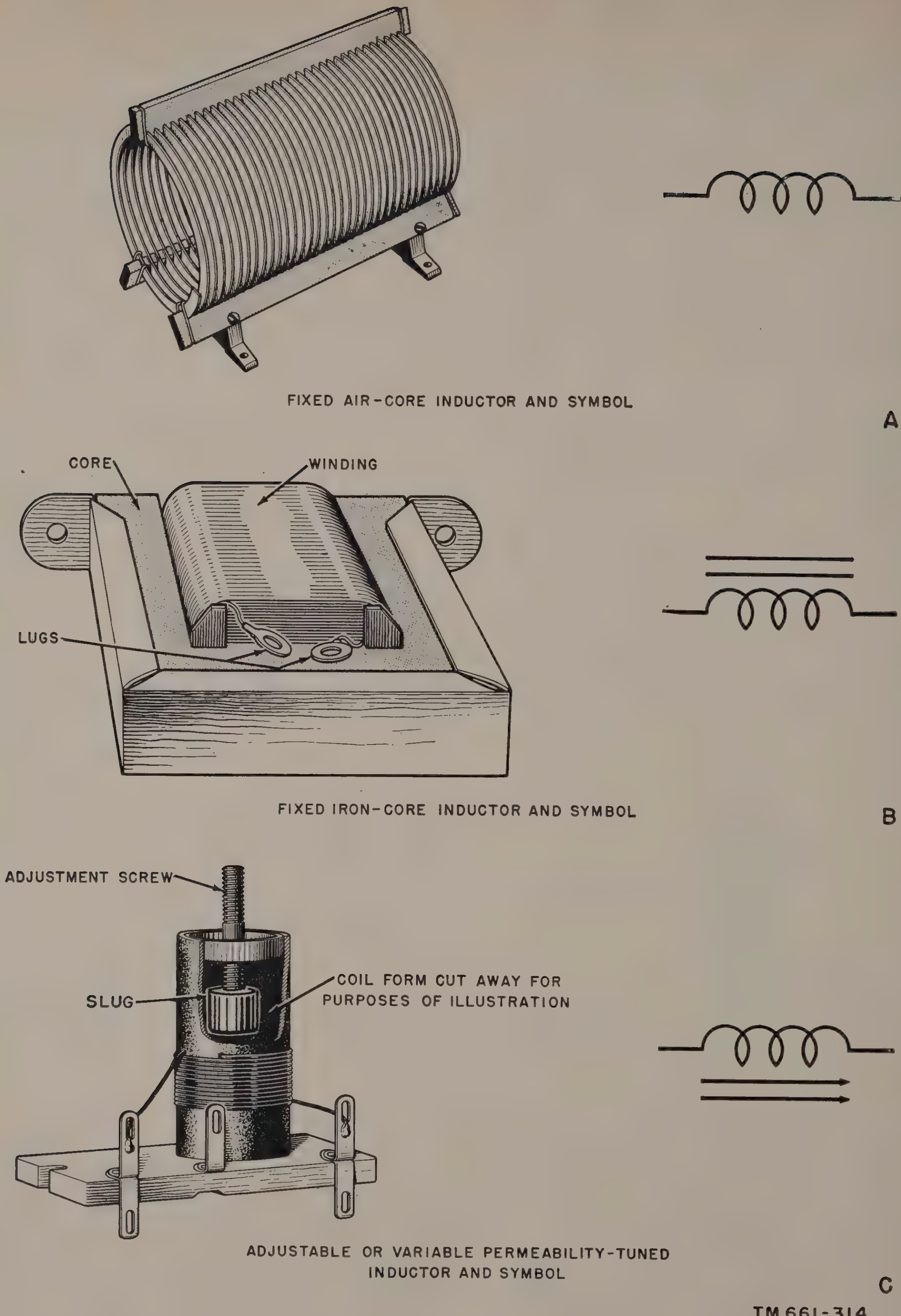


Figure 149. Types of inductors.

b. Inductors are either *fixed* or *adjustable* (*variable*). Most of the coils used in radio work are of the fixed type. A of figure 149 shows a large air core fixed inductor which is used in a radio transmitter. The symbol for an air core fixed inductor is shown also.

c. Fixed *iron-core* inductors are used, generally in power supply circuits. Such inductors are called *chokes* and *reactors*. A typical iron-core inductor and its symbol are shown in B of figure 149.

d. Adjustable or *variable* inductors found in modern radio equipment are of two main types. The first type consists of a coil which is provided with several taps and a switch, or clip, so that the inductance can be adjusted in several steps. This type is found mainly in the antenna circuit of radio transmitters where it is desirable to adjust the inductance of the coil to suit the varying requirements of different antenna lengths. In the second type, the inductor is provided with a magnetic core which can be moved in or out of the core by an adjustment screw. C of figure 149 shows this type of adjustable inductor and its symbol. The coil form is shown cut away in order to show the magnetic core or slug. This type of inductor is usually referred to as *slug tuned* or *permeability tuned*.

155. Mutual Inductance

a. When the circuits involved include a primary and secondary coil, an increase of current in the primary coil induces a voltage in the secondary coil. This induced voltage, in turn, induces a current in the secondary and creates a magnetic field which cuts the primary coil and induces a voltage in the primary. By applying the left-hand generator rule it can be shown that this induced voltage in the primary tends to send current through it in a direction which opposes the *original increase of current* through it.

b. Thus the circuit composed of the primary and secondary coil, acting together, possesses the property of opposing a change in the amount of current flowing in the primary coil. Since inductance is the property of a circuit that opposes any change in the amount of current in it, the primary and secondary coil acting together possess inductance.

c. To distinguish between the inductance of a single coil and the inductance between two coils acting together, the inductance of a single coil is referred to as *self-inductance* whereas the inductance between two coils is termed *mutual inductance*.

d. Just as with self-inductance, mutual inductance is measured in henrys by the emf produced by a specified change of current. For mutual inductance, however, the emf is induced in the secondary by a changing current in the primary. *The mutual inductance between two coils is 1 henry when a current change of 1 ampere per second in the primary induces an emf of 1 volt in the secondary.* As an equation,

$$\underset{\text{(in henrys)}}{\text{Mutual inductance}}\ M=\frac{\text{Induced voltage } e \text{ in secondary (in volts)}}{\text{Rate of change of current in primary (in amperes per second)}}$$

For example, if the current in the primary changes from 4 to 8 amperes in 1 second, which is a rate of change of 4 amperes per second, and the resulting induced emf in the secondary is 12 volts, the mutual inductance is 12/4 equals 3 henrys.

e. Rearranging the above formula for mutual inductance,

$$\text{induced voltage}=\text{mutual inductance}\times\text{rate of change of current.}$$

or more simply

$$e=M\frac{di}{dt}$$

where—e equals the induced voltage in the secondary,

M equals the mutual inductance, and

$\frac{di}{dt}$ equals the rate of change of current in the primary.

This formula is extremely helpful since it clearly and simply describes how much voltage is induced in the secondary of two magnetically coupled circuits, and upon what factors this voltage depends.

f. For example, if the mutual inductance between two coils is 8 henrys, and the primary current is changing from 2 to 4 amperes in 1 second, which is a rate of change of 2 amperes per second, the induced voltage in the secondary is 8 times 2 equals 16 volts.

g. The formula also shows that *an increase in either the mutual inductance or the rate of change of current in the primary will cause an increase in the induced voltage in the secondary.*

156. Factors Affecting Mutual Inductance

a. Since the mutal inductance is so important in determining how much voltage is induced in the secondary of two magnetically coupled coils, the question then arises: "What does the mutual inductance depend on—what determines its magnitude?" The answer is that mutual inductance depends on—

(1) The number of turns in the primary coil.
(2) The number of turns in the secondary coil.
(3) The relative position of the two coils.
(4) The magnetic permeability of the medium between the coils.

b. The first two factors are understandable since the number of turns affect the amount of emf induced for a specified change of current; the mutual inductance is therefore affected.

c. The third factor, the relative positions of the two coils, is also easy to comprehend. Consider the primary and secondary coils in A and B of figure 150. All four coils are of equal inductance, but the coils in B of figure 150 are spread further apart than the coils in A of figure 150. Therefore, if the same magnitude of current is flowing in both primary circuits, for example 5 amperes, the steady magnetic field about the secondary coil in A of figure 150 will be *stronger* than the steady magnetic field about the secondary coil of B of figure 150. This is so because the strength of the magnetic field about a coil decreases as the distance from the coil increases. Then, if the current in both primary coils changes an equal amount, for example, 2 amperes per second, a greater emf will be induced in the secondary coil of A of figure 150 than in the secondary coil of B of figure 150 since the stronger the field that cuts a conductor or coil, the greater is the induced emf. But, since the induced emf in the secondary for a specified change of current in the primary is the measure of mutual inductance, it follows that the mutual inductance is greater between the coils in A of figure 150 than between the coils in B of figure 150. In other words, *two parallel coils have greater mutual inductance when they are closer together than when they are apart.*

d. If the two coils are positioned as illustrated in C of figure 150, no resultant voltage appears across the secondary as previously indicated. The mutual inductance between the two coils is then zero.

e. The fourth factor, the magnetic permeability of the medium between the two coils, is also easy

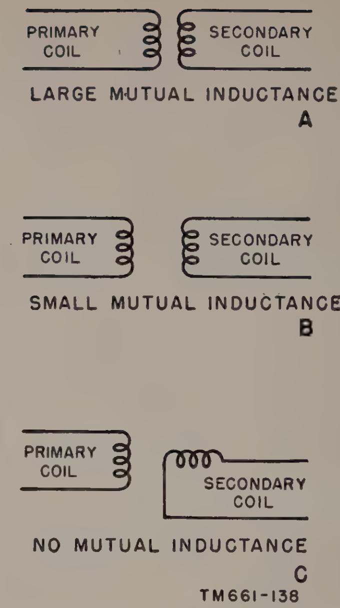


Figure 150. Physical relationship of two coils affects their mutual inductance.

to understand. It will be recalled that the permeability of a substance is the ratio of the number of lines of flux which pass through a given space when it is occupied by that substance, to the number of lines of flux passing through that space when it is occupied by air. In figure 151 two coils are wound on an iron core which has a permeability of 2,000. As a result, practically all the lines of flux due to a steady current in the primary will travel through the iron core rather than through the air, which has a reluctance 2,000 times greater than air. In traveling through the core, these lines of flux thread the secondary and represent a very strong stationary magnetic field.

f. If the iron core is omitted, however, the lines of flux would have to travel through air in order to thread the secondary. Since the permeability of air is 1, and the permeability of the iron core is 2,000, the magnetic field about the secondary will only be about 1/2,000 as strong as it was previously. In other words, the type of medium, such as air or iron, determines the strength of the magnetic field about the secondary due to a steady current in the primary. Then, if the current in the primary changes a like amount for the instance in which the medium is air and for the instance in which the medium is the iron core, it is evident that the stronger magnetic field due to the iron core will induce a greater emf in the secondary than the weaker magnetic field when

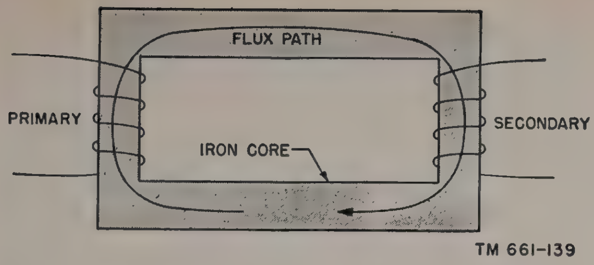


Figure 151. High permeability of iron core produces high mutual inductance for primary and secondary windings.

air, or an air core, is used. Since the induced emf in the secondary for the same change of current in the primary is greater when the medium is iron, the mutual inductance is also greater when the medium is iron. Thus the magnetic permeability of the medium between two coils affects their mutual inductance.

157. Coupling

a. One of the commonly used methods of transferring energy from one circuit to another is by making use of mutual inductance between two coils. The amount of energy that can be transferred depends on how much of the magnetic flux of the two circuits is common to both coils. The student should not make the mistake of assuming that maximum energy transfer will be achieved if all the flux created by the primary links the secondary. Obviously, as soon as current begins to flow in the secondary this current will produce its own flux which will induce an emf in the primary which modifies the primary current. Because of this mutual interaction between primary and secondary the amount of energy transferred will depend very intimately on the degree of coupling from primary to secondary and the degree of coupling from secondary to primary, these two not being necessarily equal. As an illustration of the fact that the two degrees of coupling are not always equal, see figure 152.

b. From the diagram it is seen that all the flux lines produced by coil No. 1 link all the turns of coil No. 2, but that not all the flux lines produced by coil No. 2 link all the turns of coil No. 1. In other words the coupling from 1–2 is not the same as the coupling from 2–1. Maximum coupling is attained when all the flux produced by the primary links all the turns of the secondary and vice-versa. In this case the coupling coefficient is said to be unity or 100 percent. In practice it is impossible to get unity coupling, but by the use of iron core transformers, couplings as high as 99 percent are attained. Naturally one would expect that the mutual inductance of two coils should in some way depend on their coupling. Actually, this has been found to be the case and the formula giving its value is—

$$M = K\sqrt{L_p L_s}$$

$K = \sqrt{k_p k_s}$,

L_p=self-inductance of the primary,

L_s=self-inductance of the secondary,

k_p=the fractional amount of flux created by the primary current which links *all* the turns of the secondary, and

k_s=the fractional amount of flux created by the secondary current which links *all* the turns of the primary.

Thus, using the formula for calculating the emf induced in the secondary, $E = M\frac{di_p}{dt}$ and the expression for $K = \sqrt{k_p k_s}$, it is seen that K can be very small if either k_p or k_s is very small. Therefore the

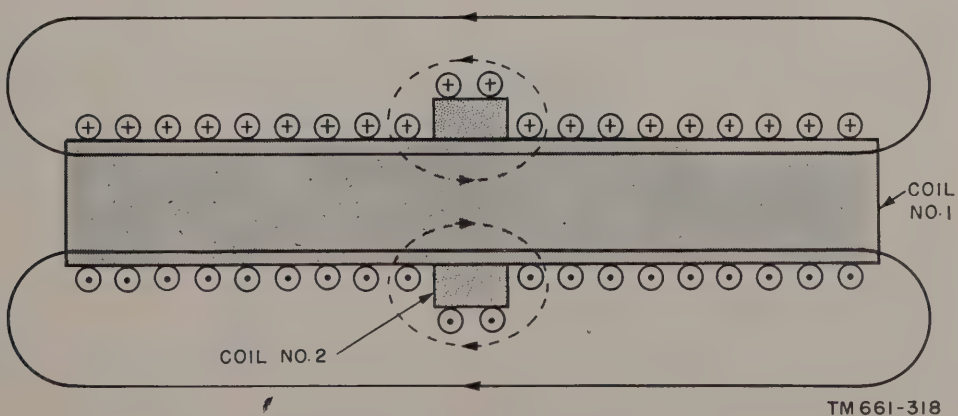


Figure 152. Flux linkage determines coefficient of coupling.

emf induced in the secondary can be very small and consequently the energy delivered to the secondary can be negligible. Of course neither k_p nor k_s can be greater than one, since, at best, all the flux from a coil can link all the turns of the other. For unity coupling,

$$K=1 \text{ and } M=\sqrt{L_p \times L_s}$$

158. Ignition System

Using the principle of mutual inductance, engineers have developed the gasoline engine ignition system, which has made the modern automobile possible (fig. 153). The main shaft of the engine controls a cam, which activates a *breaker* and this in turn opens and closes the *points*. This assembly in an automobile is known as a *distributor*. Its function is to act as a switch. The primary of the coil comprises a few turns of heavy wire, and the secondary is composed of many turns of fine wire, both being wound on the same iron core. The coil is designed to have a high mutual inductance and a high step-up voltage ratio from primary to secondary. During the time that the points are closed, the battery sends current through the primary winding and this creates a magnetic field which links the secondary winding. When the points are opened by the action of the cam, the current flow in the primary winding is suddenly interrupted, which causes a rapid collapse of the magnetic field. From our study of mutual inductance, we know that the voltage induced into the secondary winding will be—

$$E \text{ (voltage induced into the secondary)} = M \text{ (mutual inductance)} \times \frac{di}{dt} \textit{ (rate of current change in primary)}.$$

Thus, a *rapid* change of current in the primary winding will induce a very high voltage in the secondary winding. In fact, several thousand volts may be developed. This high voltage is able to jump the gap in the spark plug, igniting the gasoline mixture in the engine. Although capacitors have not been studied at this point, it is sufficient to say that the capacitor across the points acts to aid and speed the collapse of the magnetic field of the primary winding and to prevent sparking across the points.

159. Inductors in Series and in Parallel

a. Inductance has been defined as the property of any circuit which opposes a change in the amount of current flowing through it. When inductors are connected in *series* in a circuit, each inductor helps to oppose any change of current in that circuit. *Thus the total inductance of several inductors connected in series is equal to the sum of the individual inductances* (A of fig. 154). For example, if the inductances are 4, 2, and 8 henrys the total inductance is—

$$L_{\text{Total}}=L_1+L_2+L_3=4+2+8=14 \text{ henrys.}$$

This formula only applies when there is no magnetic coupling between the inductors.

b. When resistors are connected in parallel, the smallest resistor effectively shorts out the larger resistors and most of the current flows through it since it offers the least opposition. Likewise, when inductors are connected in parallel, and the

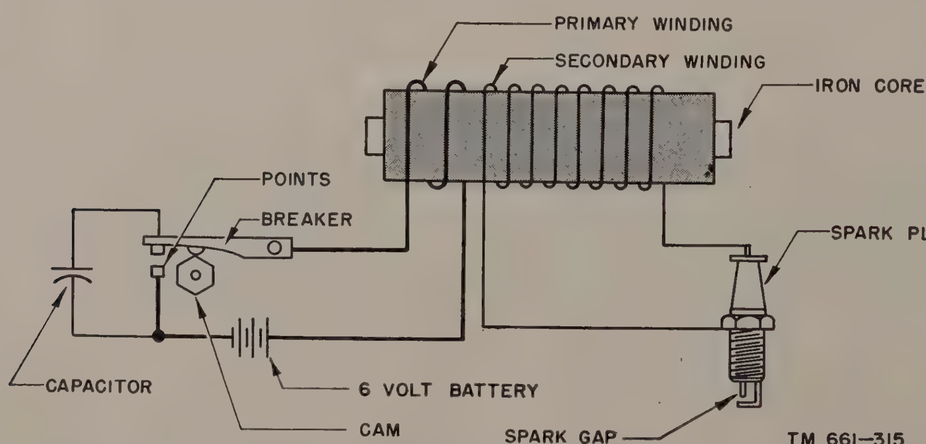


Figure 153. Ignition system.

current in the circuit is increased or decreased, each inductor individually opposes the change in current, and the current then chooses that path that offers least opposition. This path is the path of least inductance. A smaller amount of current also flows in each of the other paths or inductors. The total inductance of inductors connected in parallel is then somewhat smaller than the smallest inductance just as the total resistance of resistors connected in parallel is smaller than the smallest resistance. The formula for calculating total inductance of inductors connected in parallel therefore is similar to the formula for calculating the total resistance of resistors in parallel. *If there is no coupling between inductors connected in parallel, the total inductance is equal* RECIPROCAL OF THE *to the sum of the reciprocals* of the individual inductances (B of fig. 154). For example, if the parallel inductances in B of figure 154 are 2, 6, and 3 henrys, the total inductance is:

$$L_{Total}=\frac{1}{\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}}$$

$$=\frac{1}{\frac{1}{2}+\frac{1}{6}+\frac{1}{3}}$$

$$=\frac{1}{\frac{3+1+2}{6}}$$

$$=\frac{1}{\frac{6}{6}}$$

$$=\frac{6}{6}$$

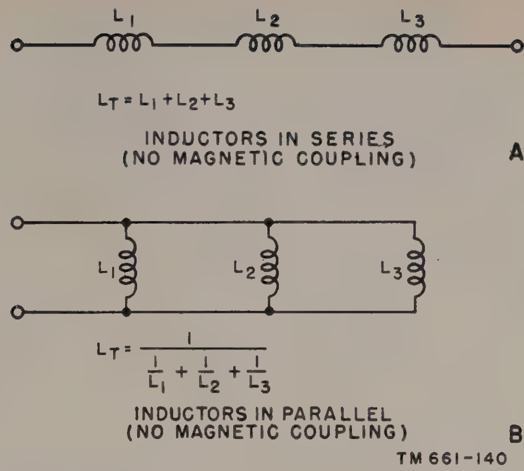
$$=1 \text{ henry.}$$

c. To avoid complicated fractions, it is simpler to use the product over the sum method just as with resistors in parallel. The total inductance of two inductors connected in parallel, and not magnetically coupled, is equal to their product divided by their sum:

$$L_{\text{Total}}=\frac{L_1\times L_2}{L_1+L_2}.$$

Applying this formula to the above example, the total inductance of the 6- and 3-henry inductors in parallel is—

$$L_{\text{Total}}=\frac{6\times 3}{6+3}=\frac{18}{9}=2 \text{ henrys.}$$



A. Inductors in series.
B. Inductors in parallel.

Figure 154.

Combining this inductance with the remaining parallel inductance of 2 henrys, using the same method, the total inductance is—

$$L_{\text{Total}}=\frac{2\times 2}{2+2}=\frac{4}{4}=1 \text{ henry.}$$

This result is the same as that arrived at by the use of the reciprocal formula and involves less complicated fractions.

d. When there is magnetic coupling between the inductors, however, the above formulas must be modified since the total inductance will be more or less, depending on whether the magnetic fields of the two inductors are in a direction such as to aid or oppose each other. A of figure 155 shows two inductors connected in series with aiding magnetic fields. B of figure 155 shows the same two inductors connected in series with opposing magnetic fields.

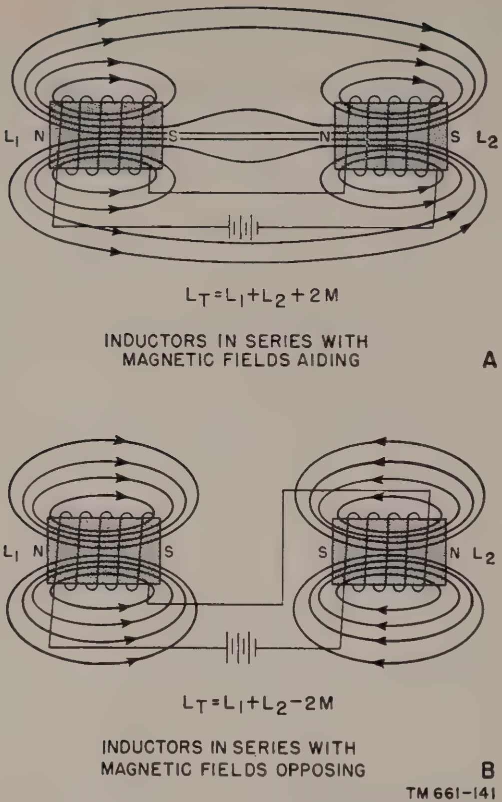
e. The effect of the magnetic coupling is to add or subtract mutual inductance to or from the total series inductance. For two inductors connected in series with their magnetic fields aiding, the total inductance is—

$$L_{\text{Total}}=L_1+L_2+2M,$$

where L equals the total inductance, L_1 equals one of the inductances, L_2 equals the other inductance, and M equals the mutual inductance between L_1 and L_2.

For two inductors connected in series with their magnetic fields opposing, the total inductance is—

$$L_{\text{Total}}=L_1+L_2-2M.$$



A. Aiding fields. B. Opposing fields.

Figure 155.

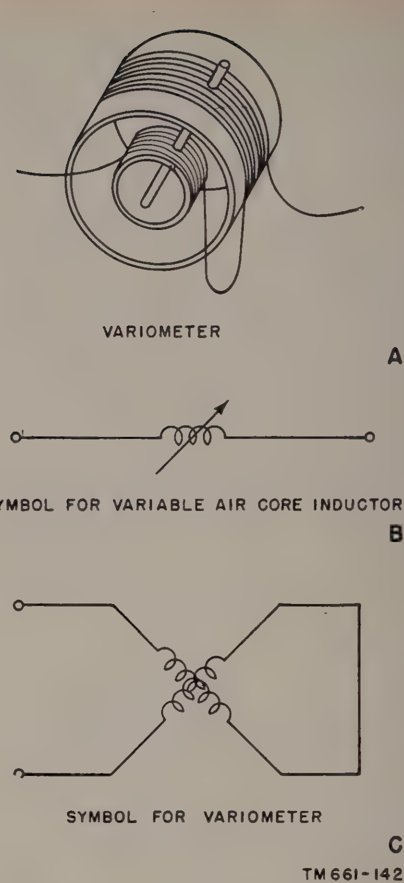


Figure 156. Variable inductors.

f. The fact that the over-all inductance of two series-connected magnetically coupled inductors can be varied by making their fields aid or oppose each other is taken advantage of in the construction of a variable inductor called a variometer. The variometer (A of fig. 156) usually consists of two coils connected in series, and is so constructed that one coil may be rotated within the other. Depending on the relative position of the coils with respect to each other, their fields aid or oppose each other and their over-all total inductance is consequently variable. The maximum *variation* of inductance is four times the mutual inductance.

g. The symbol for a variable air-core inductor is shown in B of figure 156. It is generally used to indicate any type of variable air-core inductor including the variometer. However, when it is desired to specifically indicate a variometer, the symbol in C of figure 156 is used.

160. Summary

a. A circuit is said to have self-induction if any change in current flowing in the circuit gives rise to an emf tending to prevent this change.

b. A circuit is said to have *self-inductance* if it possesses the property explained in *a* above.

c. A circuit has an inductance of 1 henry when a current change of 1 ampere per second causes a cemf of 1 volt to be induced in it. Expressed as a formula,

$$\textit{Inductance}\ \mathrm{L}\ \text{(in henrys)} = \frac{\textit{induced voltage}\ \mathrm{e}\ \text{(in volts)}}{\textit{rate of change of current}\ \text{(in amperes per second)}}$$

d. The voltage induced by self-induction is

$$e = -L\frac{di}{dt}$$

where e equals the induced voltage,
L equals the inductance, and
$\frac{di}{dt}$ equals the rate of change of current.

e. The inductance of a coil depends on:
(1) The number of turns.
(2) The cross-section of the coil.
(3) The type of core.
(4) The mean length of core or coil.

f. If an inductive circuit, the sum of the *IR* drops and induced voltages at any instant equals the impressed or applied voltage, in equation form

$$e = IR + L\frac{di}{dt}$$

where e is the applied voltage,
I is the current at any instant,
R is the total resistance in the circuit,
L is the inductance of the circuit, and
$\frac{di}{dt}$ is the rate of change of current.

g. As the amount of current in an inductive circuit increases, the rate of increase becomes smaller.

h. Increasing the inductance in a circuit decreases the rate at which current will rise and increases the time necessary for the current to reach its final steady value.

i. As the amount of current in an inductive circuit decreases, the rate of decrease of current becomes smaller.

j. Breaking an inductive circuit can cause the development of a very high induced voltage.

k. The mutual inductance between two coils is 1 henry when a current change of 1 ampere per second in the primary coil induces an emf for 1 volt in the second coil. As a formula,

$$\textit{Mutual inductance } M \text{ (in henrys)} = \frac{\textit{induced voltage } e \textit{ in secondary} \text{ (in volts)}}{\textit{rate of change of current in primary} \text{ (in amperes per second)}}$$

l. The voltage induced in the secondary by mutual induction is $e = M\frac{di}{dt}$

where:
e equals the induced voltage in the secondary,
M equals the mutual inductance, and
$\frac{di}{dt}$ equals the rate of change of current in the primary.

m. The mutual inductance between two coils depends on—

(1) The number of turns in the primary coil.
(2) The number of turns in the secondary coil.
(3) The relative position of the two coils.
(4) The magnetic permeability of the medium between the coils.

n. If all the lines of force from the primary coil cut or link all the turns of the secondary coil, and all the lines of force from the secondary link all the turns of the primary, the two coils are said to have maximum, 100 percent, or unity coupling. A high, medium, and small percentage of coupling is referred to as tight, medium, and loose coupling, respectively.

o. Increasing the degree of coupling between two coils increases their mutual inductance.

p. The total inductance of several inductors connected in *series*, with no magnetic coupling between them, is equal to the sum of the individual inductances. As a formula,

$$L_{Total} = L_1 + L_2 + L_3.$$

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q. If there is no coupling between inductors connected in *parallel*, the total inductance is equal to the sum of the reciprocals of the individual inductances. As a formula,

$$L_{Total} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}.$$

r. The total inductance of two inductors connected in *parallel*, and not magnetically coupled, is equal to their product divided by their sum. As a formula,

$$L_{Total} = \frac{L_1 \times L_2}{L_1 + L_2}.$$

s. For two inductors connected in *series with their magnetic fields aiding*, the total inductance is—

$$L_{Total} = L_1 + L_2 + 2M$$

where L is the total inductance,
L_1 is one of the inductors,
L_2 is the other inductor, and
M is the mutual inductance between L_1 and L_2.

t. For two inductors connected in *series with their magnetic fields opposing*, the total inductance is—

$$L_{Total} = L_1 + L_2 - 2M$$

161. Review Questions

a. What is self-induction?

b. Why is the voltage induced by self-induction called a cemf?

c. Why is it that the current in an inductive circuit does not immediately increase to its final value?

d. Define inductance.

e. What does $\frac{di}{dt}$ represent?

f. What is the formula for the voltage induced by self-induction? Give an example using the formula.

g. What are the factors that determine the inductance of a coil?

h. Draw a curve to show how current increases in an inductive circuit, decreases in an inductive circuit.

i. In an inductive circuit, what is the equation that gives the relation between the IR drops, the induced voltages, and the applied voltage?

j. In A of figure 145, at the instant the switch is closed, what is the voltage across the resistor? Across the coil? At the instant that the current rises to 6 amperes, what is the IR drop, the voltage across the coil, and the rate at which the current is increasing?

k. As the current rises in an inductive circuit, does the rate of increase of current remain constant, increase, or decrease?

l. How does increasing the amount of inductance in an inductive circuit affect the rate at which the current will rise? Affect the time necessary for the current to reach its final steady value?

m. When the switch in A of figure 147 is instantaneously opened at X and closed at Y, what causes the induced emf in the coil? What happens to the current in the circuit consisting of the coil and the resistor? Is there more voltage across the resistor on the coil? Explain. Give the equation that shows the relationship of these two voltages.

n. If, in B of figure 147, the current falls to 4 amperes, what is the IR drop across the resistor, the voltage across the coil, and the rate at which the current is decreasing?

o. If in an inductive circuit the inductance is increased, will the rate at which the current decreases remain constant, increase, or decrease?

p. Why is it dangerous to break an inductive circuit?

q. What is a choke?

r. Distinguish between self-inductance and mutual inductance.

s. Define mutual inductance in terms of henrys.

t. What is the formula for the voltage induced in the secondary by mutual induction? If the rate of change of current in the primary is 6 amperes per second, and the mutual inductance is 8 henrys, what is the magnitude of the voltage induced in the secondary?

u. What four factors does mutual inductance depend on?

v. Does the mutual inductance of two parallel coils increase or decrease as they are brought nearer each other?

w. What is maximum coupling? Unity coupling? 100-percent coupling? Tight, medium, and loose coupling?

x. As the coupling increases, does the mutual coupling increase or decrease?

y. What is the formula for the total inductance of several inductors connected in series with no magnetic coupling between them? If the values of these inductors are 6, 9, and 2 henrys, respectively, what is the total inductance?

z. What is the formula for several inductors connected in parallel with no magnetic coupling? For two inductors?

aa. Three parallel inductors without magnetic coupling, have inductance values of 6, 3, and 6 henrys. Using the product over sum method find the total inductance. Is this value smaller than any of the three inductors?

ab. What is the formula for two inductors connected in series with their magnetic fields aiding? Opposing?

ac. What is a variometer? What is the symbol for a variable air-core inductor? For a variometer?

APPENDIX I
POWER, WORK, AND DIMENSION

1. Energy

One of the fundamental laws of nature is that energy is conserved. In other words, the total *energy* in the universe is a constant. This *natural law* has been verified experimentally. Of course, it is true that energy may assume different forms. For example, when coal is used for heating purposes, the internal energy of the coal is converted into heat energy. The work done in rubbing two bodies together is evidenced as generated heat and the electrical energy delivered to a motor is changed into the mechanical energy of rotation. Modern theory seems to indicate that there is an amount of energy associated with every object and that its mass is merely a different aspect of its energy. This theory tells us that a 1-pound body is equivalent to the tremendous energy of 4.12×10^{23} ergs, an amount of energy capable of raising the temperature of 674,000,000 tons of water 1° C. This mass-energy equivalence is the principle underlying the successful development of the atomic bomb. To summarize, we may say that *energy may change form but can be neither created nor destroyed.*

Note. For a definition of an erg, see paragraph 2 of this appendix.

2. Work

a. Definition of Work. The concept of work requires for its definition two very basic factors—that of force and that of distance. The work done in displacing an object by the application of a force directed along the displacement is the product of the force and the displacement. Symbolically, this is written in the form—

$$W = F \times l$$

where W = work done on the body,
F = force in the *direction of* l exerted on the body, and
l = displacement of the body.

This can also be written as—

$$work = force\ times\ distance.$$

If the distance is expressed in feet and the force in pounds, the work done is in terms of pound-feet. For example, if a 10-pound weight is lifted a distance of 1 foot the work done is 10 pound-feet. In order to explain all the terms used in the formula, a series of illustrative examples will be given in this appendix.

b. Units of Work. In the cgs system, the work done by a force of 1 dyne acting through a distance of 1 centimeter (dyne-centimeter) is called an *erg*. However, for practical work, the erg is much too small to be of any value. Therefore, a unit which is 10,000,000 times as great is used and is called a *joule*. This definition is not as arbitrary as it seems, since the joule is also the amount work done in moving a coulomb of electricity through a potential of 1 volt. A coulomb represents 6,280,000,000,000,000,000 (6.28×10^{18}) electrons. In the cgs system, the unit of heat is the *calorie* which is defined as the amount of heat required to raise 1 gram of water from 15° to 16° C. Now, since heat is also identified as energy, there should be a relationship between the calorie and the erg. Actually, this is so, and it has been found experimentally that a calorie is equivalent to 41,850,000 ergs. In other words, the performance of this number of ergs of work is required to raise the temperature of 1 gram of water from 15° C. to 16° C., or 1°. This gives an indication of the smallness of the erg. It follows that a calorie is equivalent to 4.185 joules.

(1) *Example No. 1.* In raising the 100-pound weight in figure 157 10 feet against the force of gravity, an amount of work equal to 100 pounds $\times$ 10 feet = 1,000 pound-feet has been performed. Notice that the force required to raise the object is equal and opposite to its weight and that the force is in the direction of the displacement.

(2) *Example No. 2.* Suppose the 100-pound weight shown in figure 158 is being dragged along a rough, horizontal surface. In this case, due to the friction be-

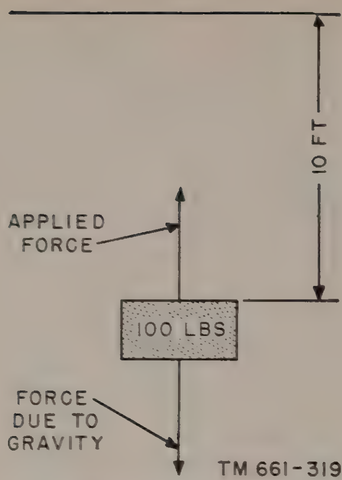


Figure 157. Work must be done in moving the 100-pound weight against the force of gravity.

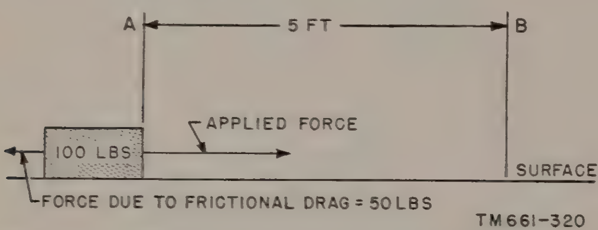


Figure 158. Work must be done in moving the 100-pound weight against the force due to frictional drag.

tween the surface and the weight, there will be a drag which tends to prevent the weight from moving. In other words, the effects of friction can be replaced by a retarding force. The amount of drag will depend on the roughness of the surface. In the figure, this drag is equal to 50 pounds. Thus, the applied force required to move the object is 50 pounds in a direction opposite to the drag. The work done in moving the weight 5 feet is, therefore, 50 pounds × 5 feet = 250 pound-feet.

c. Explanation. At this point the student should raise a very legitimate question. Since the total energy in the universe is constant, and since work is energy, then what has become of the 250 pound-feet of energy which was used in moving the 100-pound weight (fig. 158) from *A* to *B*? The answer is that the frictional drag is exactly 50 pounds, and in applying the force of 50 pounds through a distance of 5 feet, the 250 pound-feet of energy is used in overcoming the friction. This work will appear as heat generated by the rubbing of the two surfaces. In other words, if the weight is displaced a distance of 1 foot, then at least an amount of work equal to 50 × 1 pound-feet must be supplied. This work will appear as generated heat which could be used to heat a container of water. In this case the work performed will have been transformed into the extra energy of the water whose temperature has been increased. Another plausible question is this. Suppose that a force greater than 50 pounds is applied, say 60 pounds. Then, in a distance of 10 feet, the work performed is 60 pounds × 10 feet = 600 pound-feet. Since only 500 pound-feet (50×10) is required to overcome friction, what has happened to 100 pound-feet of work? At position *A*, the weight had no velocity, it was at a standstill, but at *B* it has a definite velocity. (It cannot come to a standstill at *B*, since applied force exceeds retarding force and motion must be present.) Now, obviously, a body has energy because of its velocity. That this is true can be demonstrated by a simple experiment (fig. 159).

A of figure 159 shows a nail ¼ foot long about to be driven into a wall. Suppose the force resisting penetration of the nail to be 10 pounds. After being struck by a moving object, the nail has been driven a distance of ⅛ foot into the wall (B of

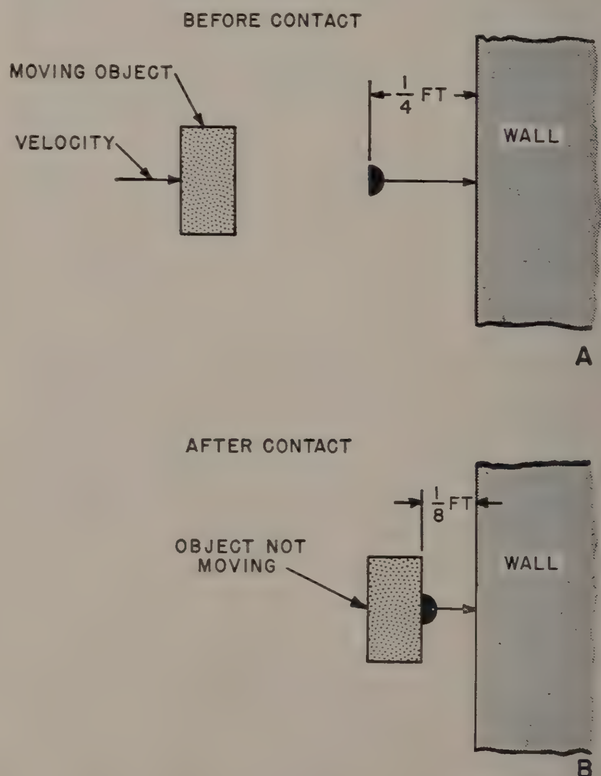


Figure 159. A body has energy because of its velocity.

fig. 159). The work that has been done is 10 pounds × ⅛ foot=1.25 pound-feet. Of course, the object is no longer moving since it has given up all of its energy in driving the nail. Thus because of the velocity of the object, work was performed. Therefore, any body has energy by virtue of its velocity. It follows by the law of conservation of energy (par. 1) that in order to impart a velocity to any body, work must be done.

d. In figure 160 a 100-pound weight moves in a slot (the slot confines the weight to horizontal

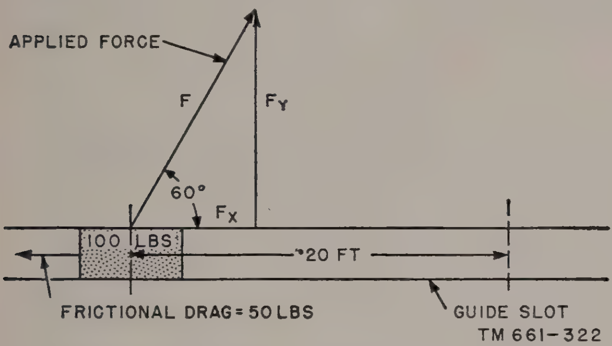


Figure 160. Applied force is at an angle of 60° with respect to the direction the weight can be moved (horizontal travel).

movement only). A force F the direction of which makes an angle of 60° with the horizontal is applied. How large must the force F be in order to just move the weight and how much work is done in a displacement of 20 feet?

Solution. Since the frictional drag is 50 pounds, the force F_x, which is the horizontal component of F, must be 50 pounds. Now, from the diagram we see that

$$\frac{F_x}{F} = \textit{cosine of } 60°.$$

Then

$$F = \frac{F_x}{\textit{cosine } 60°} = \frac{50\ \textit{lbs.}}{1/2} = 100 \text{ lbs.}$$

Thus, we see that if the force F is applied in the direction shown, it must have a magnitude of 100 pounds, despite the fact that the frictional drag is only 50 pounds. From the definition given in *a* above, we see that the work done is 50 lb.×20 ft.=1,000 lb.-ft. This is so because the component of force in the direction of the displacement is only 50 pounds. The work is not 100 lb.×20 ft.=2,000 lb.-ft. since the weight *does not* move in the direction of F but in that of F_x, the horizontal component. (The student not familiar with the *sine* and *cosine* functions of trigonometry may refer to any standard text on this subject.) This example serves to illustrate the well-known fact that if the direction of rope-pull on a sled makes a very large angle with the horizontal, a very large pull is required on the rope in order to start the sled. Finally, if the pull is at right angles to the horizontal, no amount of force can produce motion, since all the force is directed upward and none is left to produce movement along the horizontal. This is shown in figure 161.

Suppose that in figure 161 an additional force of 50 pounds is directed along the horizontal. Then as we have seen, the 100-pound weight will move. However, the force F of 10 pounds is constantly directed along the vertical and has no component in the direction of displacement. Thus, if the weight moves 5 feet, the 50-pound force does an

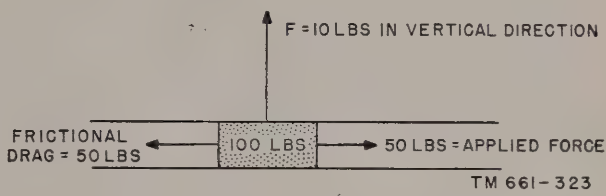


Figure 161. Force (F) cannot produce movement of weight.

amount of work equal to 250 pound-feet whereas the 10-pound force *does no work* since its component along the direction of travel is zero. Here we have an example of a 10-pound force being exerted through a distance but no work being done. This was the reason for using "the force directed along the displacement" in the definition of work given in *a* above.

3. Power

a. Power is the *time rate* at which work is done, and is measured in foot-pounds per second. Expressed as an equation,

$$\textit{power} = \frac{\textit{work}}{\textit{time}}$$

The following example will serve to illustrate the difference between work and power. When a 100-pound bag of sand is dragged 10 feet, 1,000 foot-pounds of work is performed. If this work is performed in 5 seconds, the power required is—

$$\textit{power} = \frac{\textit{work}}{\textit{time}} = \frac{1{,}000\ \textit{ft-lb}}{5\ \textit{sec}} = 200 \text{ ft-lb/sec.}$$

If the 100-pound bag is dragged 10 feet in 10 seconds, the power required is—

$$power=\frac{work}{time}=\frac{1{,}000\ \ ft\text{-}lb}{10\ sec}=100\text{ ft-lb/sec.}$$

b. Power is also measured in hp (horsepower) units. One hp equals 550 foot-pounds per second or 33,000 foot-pounds per minute.

c. The unit of electrical power is the *watt*. One watt of power is required to do 1 joule of work in 1 second. Expressed as an equation,

$$watt=\frac{joule}{second}$$

$$746\ watts=1\text{ mechanical hp, and}$$

$$1\ watt=.00134\text{ hp.}$$

d. Electrical circuits are referred to in terms of *voltage, amperage,* and *wattage* rather than in units of joules. Therefore, it will be convenient to form an equation which expresses the power in watts in terms of volts and amperes rather than in joules/sec as follows:

joule=1 *coulomb moved through a potential difference of* 1 *volt.*

Dividing both sides of the equation by time in seconds,

$$\frac{joule}{second}=\frac{1\ coulomb\ moved\ through\ a\ potential\ difference\ of\ 1\ volt}{1\ second}$$

$$\frac{1\ coulomb}{1\ second}=1\ ampere.$$

Substituting,

$$\frac{1\ joule}{1\ second}=1\ ampere\ moved\ through\ a\ potential\ difference\ of\ 1\ volt$$

$$\frac{joule}{second}=watt.$$

Therefore,

1 watt=1 *ampere moved through a potential difference of* 1 *volt.*

In other words, *when an ampere of current flows between two points, the difference of potential of which is* 1 *volt,* 1 *watt of power is being expended between those two points.*

e. Thus, *the power expended in any circuit is the product of the voltage and the current flowing in that circuit.*

Expressed as an equation,

$$P=E\times I,\text{ or }E=P/I,\text{ or }I=P/E$$

where P is the power in watts,
I is the current, and
E is the potential difference in volts.

This formula is usually remembered as

$$watts=volts\times amperes.$$

Since the watt is a very small unit of power, we commonly use the kw (kilowatt), which is 1,000 watts.

$$kilowatt=\frac{volts\times amperes}{1{,}000}$$

Figure 162 illustrates two methods of connecting meters in a circuit for the purpose of measuring current and voltage.

Examples:

(1) If an electric motor requires 10 amperes at 110 volts, what is the power consumed?

$$P=E\times I=10\times 110=1{,}100\ watts,\text{ or }1.1\ kw.$$

(2) If a battery charger operates at 100 volts and consumes 600 watts, what must be the smallest fuse which will maintain operation?

$$I=P/E=\frac{600}{100}=6\text{-}ampere\ fuse.$$

(3) What voltage will be measured at the terminals of a 500-watt generator when 10 amperes are being drawn?

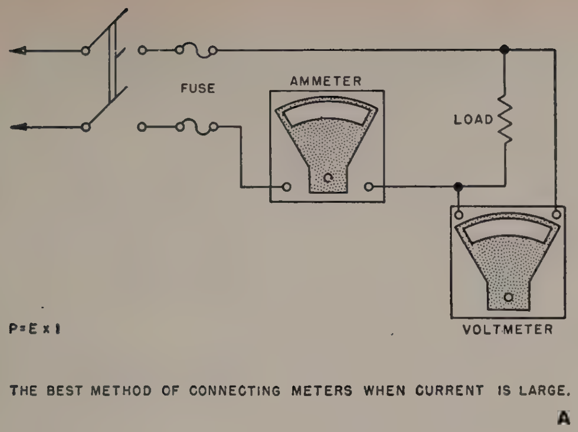
$$E=P/I=\frac{500}{10}=50\ volts.$$

f. By the use of these power formulas, one of the three quantities of current, power, or voltage can be found if the other two quantities are known.

g. The load of a machine or apparatus is the power which it delivers. However, in the field of radio, the word *load* also refers to the device which consumes the power delivered by the generator.

h. Power also may be referred to in terms of resistance and may be shown by substituting the Ohm's law value of resistance in the power formula

$$P=E\times I, I=E/R, P=\frac{E\times E}{R},\text{ and }P=E^2/R.$$



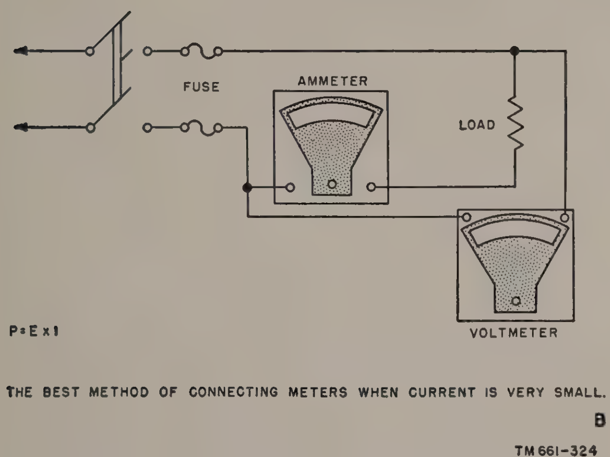


Figure 162. Measurement of current and voltage.

The latter formula also may be transposed as

$$E=\sqrt{PR} \text{ or } R=E^2/P.$$

The following problems will serve to illustrate the application of these formulas:

(1) A resistor radiates 20 watts of power in the form of heat when connected to a 20-volt d-c source. What is the resistance value of the resistor?

$$R=E^2/P=\frac{400}{20}=20.$$

(2) What is the wattage rating of a 120-volt bulb which has a resistance of 144 ohms?

$$P=E^2/R=\frac{(120)^2}{144}=100 \textit{ watts}.$$

(3) What is the resistance of an electric iron which operates on 120 volts and consumes 1,000 watts?

$$R=E^2/P=\frac{(120)^2}{1,000}=\frac{14,400}{1,000}=14.4 \textit{ ohms}.$$

i. Since power is the time rate of doing work, it follows that the greater the length of time the power is consumed, the greater will be the total power consumed. Electric power is purchased commercially in watthours (watts$\times$hours) or kw hours $\frac{(\text{watts}\times\text{hours})}{1,000}$.

A 100-watt lamp requires 100 watts of power for proper operation and consumes 100 watthours of power in 1 hour, 200 watthours in 2 hours, etc. In terms of kw hours, the lamp uses 100/1,000=.1 kw hour of power in 1 hour; in 10 hours, the lamp uses 10 times as much, or 1 kw hour of power.

Examples:

(1) A kw hour meter reads .09 kw in 10 hours. What is the average rate of consumption?

$$.09 \text{ kw}=90 \text{ watts}$$

$$\frac{90}{10}=9 \text{ watt/hour}.$$

(2) If a 2-hp motor is connected to the power line and operated for 10 hours continuously, what will be the power consumed in kw hours?

746 watt=1 hp

1,492 watts=1.492 kw

$$\begin{array}{r} 1.492 \text{ kw} \\ \times 10 \text{ hours} \\ \hline 14.92 \text{ kw hours} \end{array}$$

4. Power Losses

a. The most common loss of power in electrical work is that which is dissipated in the form of heat when current flows through a resistance. This power loss is sometimes called the I^2R loss or copper loss, and is always present when current is flowing. The heat is usually dissipated into the air and lost, but it can be utilized as in the case of the electric oven, toaster, soldering iron, or the filament of a vacuum tube. It may be calculated by the following formula:

$$P=I^2R \text{ or } P=E^2/R.$$

b. Power losses in resistors are an important consideration in communications work. Resistors are used to reduce the supply voltage in plate, screen, and filament circuits of radio sets. For example, if the 12-volt battery of a combat car is used to light a 6-volt radio tube which draws .3

ampere of current, a series resistor will be necessary. The value of this resistor may be found as follows: Since the voltage drop or IR drop necessary is 6 volts and the current drawn by the tube is .3 ampere, the resistance will be—

$$R=E/I=6/.3=20 \textit{ ohms.}$$

The power rating should be—

$$P=\frac{E^2}{R}=\frac{36}{20}=1.8 \textit{ watts, or}$$

$$P=I^2R=.3\times.3\times20=1.8 \textit{ watts.}$$

Therefore, any 20-ohm resistor with a wattage rating of 1.8 watts or greater may be used. If a resistor of smaller wattage rating is used, it will overheat or burn out and cause an open circuit. This wattage rating indicates the safe wattage that the resistor will radiate in the form of heat in free air without becoming damaged. Resistors that are placed underneath a radio chassis may cause trouble because of poor radiation of heat. To avoid this possibility, radio sets are well ventilated. In large transmitters, it is common practice to use ventilating blowers to cool power tubes and other components.

c. Electric motors have losses due to friction and resistance of windings. Therefore, the mechanical output can never equal the electrical input. The output of any power-consuming device divided by the input and multiplied by 100 will give its power efficiency in percent. (No machine can be 100 percent efficient.)

$$\textit{Efficiency}=\frac{\textit{Output}}{\textit{Input}}\times100.$$

(1) If a 1-hp motor draws 6 amperes of current at 220 volts, what is the efficiency of the motor?

$$\textit{Efficiency}=\frac{\textit{Output}}{\textit{Input}}\times100=\frac{746}{220\times6}\textit{ watts}\times100=$$

$$\frac{74600}{1320}=56.51\%$$

(2) A certain tube when operating in a circuit draws 20 milliamperes current at 150 volts. What power is dissipated?

$20 \textit{ ma}=.02 \textit{ ampere}$

$P=E\times I$

$P=150\times.02=3 \textit{ watts.}$

(3) If 45 kw is supplied to a motor and its output is found to be 50 hp, what is the efficiency of the motor?

$45 \text{ kw}=45{,}000 \textit{ watts}=\textit{input}$

$50 \text{ hp}=50\times746=37{,}300 \textit{ watts}=\textit{output}$

$$\textit{efficiency}=\frac{\textit{output}}{\textit{input}}\times100=\frac{37{,}300}{45{,}000}\times100=83\%.$$

5. Problems

a. An electric iron operates at 110 volts and consumes 3.5 kw hours in 5 hours of continuous operation. What is the resistance of the iron?

3.5 kw hours=3500 watt hours in 5 hours

$$\frac{3500}{5}=700 \textit{ watts per hour}$$

$$P=\frac{E^2}{R} \text{ or } R=\frac{E^2}{P}=\frac{110^2}{700}$$

$$=\frac{110\times110}{700}=\frac{121}{7}=17.3 \textit{ ohms.}$$

b. An electric refrigerator operates for 10 minutes out of each hour. If the motor delivers ½ hp and takes 5 amperes at 110 volts from the line, how much power will it consume in 30 days; what is the efficiency of the motor, and what will be the cost of operation at \$.05 per kw hour? The answers are computed as follows:

(1) Input $=P=EI=110\times5=550$ watts

Output $=\frac{1}{2}$ hp $=\frac{746}{2}=373$ watts

24 hours $\times$ 30 days $=$ 720 hours

10 min./1 hour $=10/60=1/6$ hour

$\frac{1}{6}\times720=120$ hours total

$120\times550=66{,}000$ watthours or 66 kw hours consumed in 30 days.

(2) Efficiency $=\frac{\text{output}}{\text{input}}\times100=\frac{373}{550}\times100=$ 67.8%.

(3) $66\times.05=\$3.30$.

6. Theory of Units and Dimensions

The fundamental dimensions of physics are those of *length, mass,* and *time,* abbreviated *L, M, T.* Of course systems of units may be used

for each of these quantities—for example, length may be measured in centimeters, feet, or inches, and mass may be expressed in grams, pounds, etc., and time in seconds, hours, minutes, etc. For a thorough understanding of physics, the student must understand the difference between *dimension* and the *unit* in which it is expressed. A dimension is a conception independent of the unit in which it is expressed. To understand this refer to figure 163.

a. The *distance* from A to B is the same regardless of whether we expressed it in inches or feet. In other words, the separation of A and B is a quantity independent of the units used to measure

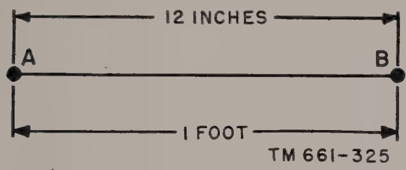


Figure 163. Distance measured in feet and inches.

it. From this we see that the inch unit has a *magnitude* only 1/12 that of the foot. Twelve-inch units laid side by side could reach from A to B and similarly 1 foot unit would do the same. Thus we are led to the ideas of magnitude of a unit and dimension of a unit.

b. Suppose we wish to express a distance L in terms of units M and N. Suppose the M unit has a magnitude m and the N unit has a magnitude n. Then by other previous considerations,

$$L=a\times n$$

$$L=b\times m$$

where a is the *number* of M units required to measure L and b is the *number* of N units required to measure L. Therefore since L is the same in both cases; $a\times n=b\times m$.

$$\therefore \frac{a}{b}=\frac{m}{n}.$$

This equation states that the number of units required to cover L varies inversely as the magnitude of the unit. This is illustrated in the previous example where $12\times$(magnitude of the inch unit)$=$ $1\times$(magnitude of the foot unit). Here, $a=12$, $n=$magnitude of the inch unit, $b=1$, $m=$magnitude of the foot unit.

c. It might be expected that length, mass, and time would be sufficient to provide the dimensions of all physical quantities. However this is not found to be the case. Before showing why this is so let us find the dimensions of force. Sir Isaac Newton concluded from his experiments in gravitation, that the unbalanced force acting on a body was proportional to its acceleration. This can be written as—

$$f=ma$$

where f=force acting on the body.
a=acceleration of the body.
m=a constant of proportionality called *mass*.

To illustrate what is meant by acceleration, consider the following example: An object has a velocity of 10 cm/sec. By applying a force the velocity is changed from 10 cm/sec to 15 cm/sec in 2 seconds. The rate of increase of velocity is therefore

$$\frac{15\,\frac{cm}{sec}-10\,\frac{cm}{sec}}{2\ sec}=\frac{5\,\frac{cm}{sec}}{2\ sec}=2.5\,\frac{cm}{(sec)\,(sec)}$$

Or stated otherwise, the rate of change is 2.5 cm/sec in 1 second. This is called the *acceleration* of the body. Thus acceleration is the *time rate* of *change* of velocity. From this example, it is seen that the dimension of acceleration is—

$$\frac{(length)}{(time)^2}=\frac{L}{T^2}$$

Returning to Newton's law, we see that the dimension of force is—

$$\frac{(mass)\times(length)}{(time)^2}=ML/T^2.$$

Consider the equation giving the force between two electric charges (Coulomb's law, ch. 2).

$$F=\frac{q_1\times q_2}{\epsilon d^2}$$

q_1 and q_2=the electric charges,
d=the distance between charges,
ϵ=a constant characterizing the medium in which the charges exist, and
F=the force of repulsion or attraction between the charges.

Now the dimension of F is $\frac{ML}{T^2}=MLT^{-2}$. The dimension of $d=L$.

Let the dimension of charge be Q and the dimension of ϵ be E.

Solving for the dimension of charge, we get: (dimension of charge) = (dimension of force) (dimension of ϵ) (dimension d)^2. Substituting the proper dimensions in terms of mass, length, and time under the radical sign gives:

$$Q=\sqrt{\frac{ML}{T^2}\times L^2\times \epsilon}=M^{1/2}L^{3/2}T^{-1}\epsilon^{1/2}.$$

But what is ϵ in terms of length, mass, and time? This cannot be found until we know that of Q which, in turn, depends on that of ϵ. What is the way out of this dilemma? Simply, that we must give up the idea that all physical quantities can be described dimensionally in terms of L, M, and T. Because of this, physicists have introduced a fourth fundamental dimension—that of *charge*.

From the above equation, ϵ is equal to—

$$\epsilon=Q^2M^{-1}L^{-3}T^2=\frac{Q^2T^2}{M\times L^3}.$$

Thus the permittivity of a medium has the dimension,

$$\frac{(charge)^2\times(time)^2}{(mass)\times(length)^3}$$

7. Concept of an Equation

Surely no rational being has ever attempted to equate horses to cows; this apparently nonsensical statement however forms the heart of this paragraph. What is meant by saying that

$$a=b$$

Would this *equation* make sense if a represented 10 horses and b represented 10 cows? The answer is, of course, "No." Despite the fact that the *number* of horses is equal to the *number* of cows, horses are not cows. Thus in an equation, not only must the left side be *numerically* equal to the right side, but the *dimension* of the left must be the same as that of the right. So if in an equation which we suppose to represent a certain physical quantity, the dimension yielded by the equation is not the dimension of this quantity, the equation is wrong.

a. Example 1. Suppose we were told that the energy dissipated in a resistance (R) in time (t) by the passage of current (I) is given by the expression $W=I^2R^2t$, and we have no previous knowledge of the I^2R law. In this case we should check this equation as follows:

Energy has the dimension of work which is (*force*)$\times$(*distance*). Therefore, the dimension of work is—

$$\frac{ML}{T^2}\times L=\frac{ML^2}{T^2}$$

If the equation $W=I^2R^2t$ were correct, the right side would have to have the same dimension: thus, since current is voltage/resistance, the right side has the dimension—

$$\frac{(voltage)^2}{(resistance)^2}\times(resistance)^2\times(time)=(voltage)^2\times(time)$$

From chapter 3 we know that voltage is work per unit charge so that the dimension of voltage is $\frac{(work)}{(charge)}$.

From this we see that the right side has the dimension

$$\frac{M^2L^2}{T^4}\times\frac{1}{Q^2}\times T=\frac{M^2L^2}{T^3Q^2}$$

where Q is the fourth dimension of charge which we have agreed to introduce. Comparing our two results we see that if the equation were true,

$$\frac{ML^2}{T^2}=\frac{M^2L^2}{T^3Q^2}\text{ or }Q=\sqrt{\frac{M}{T}}$$

which is a contradiction since the dimension of charge cannot, as we have seen, be expressed in terms of M, L, and T alone. Thus the equation cannot possibly be correct.

b. Example 2. An engineering company through experiment decides that the amount of frictional resistance offered to the passage of a body through oil is given by—

$$f=\frac{kmv^2}{l}A$$

f=frictional retarding force,
m=mass of the body,
v=velocity of the body,
A=cross-sectional area of the body,
l=length of the body parallel to the flow, and
k=a constant having no dimension (a pure numeric)

Is this formula correct? By the previous procedure, the right side must have the dimension of force $\frac{ML}{T^2}$.

Since area has the dimension $(\text{length})^2$ and velocity that of $\frac{(length)}{(time)}$, substituting in the left of the formula results in the dimension—

$$\frac{(mass)\ \ (length)^2\ \ (length)^2}{(time)^2\ \ (length)}.$$

Symbolically, we get—

$$\frac{M\times L^4}{T^2\times L}=\frac{ML^3}{T^2}.$$

(Notice that k does not appear since we are only interested in the dimension of the right side and k has no dimension.)

But force has the dimension—

$\frac{ML}{T^2}$ which does not agree with the formula.

Thus, the equation is not correct.

c. SUMMARY. The important points of this appendix may be listed as follows:

(1) The fundamental dimensions of mechanics are length, mass, and time.

(2) The relationship between unbalanced force acting on a body, mass of the body, and its acceleration is given by Newton's law—

$$f=ma.$$

(3) A dimension is independent of the units used to measure it.

(4) In electrostatics it is found that the dimensions of length, mass, and time are not sufficient to determine that of charge; the dimension of charge is added to these and also considered fundamental.

(5) For an equation to be true, both sides must be numerically equal and dimensionally the same.

APPENDIX II

CHEMICAL REACTIONS IN LEAD-ACID CELL

1. Chemical Reactions

In chapter 8, the various chemical reactions that take place in a lead-acid cell are described in general terms. This appendix gives a more complete description of these chemical reactions.

a. The complete equation for the chemical reaction of a lead-acid cell is—

$$\underset{\text{(spongy lead)}}{Pb} + \underset{\text{(lead peroxide)}}{PbO_2} + \underset{\text{(sulphuric acid)}}{2H_2SO_4} \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftarrows}} \underset{\text{(lead sulphate)}}{2PbSO_4} + \underset{\text{(water)}}{2H_2O} \tag{1}$$

However, let us consider this equation in parts in order to analyze what takes place at the negative and positive plates during both the discharge and charge cycles. At the negative plate during discharge, the spongy lead (Pb) partially dissolves due to the action of the sulphuric acid (H_2SO_4) and is said to ionize. During ionization, each spongy lead atom loses 2 electrons, and is changed to a lead ion.

$$\underset{\text{(spongy lead)}}{Pb} \longrightarrow \underset{\text{(lead ion)}}{Pb^{++}} + \underset{\text{(electrons)}}{2e^-} \tag{2}$$

The negative electrons flow from the negative plate, through the external load, to the positive plate. A flow of negative electrons is electricity. The electron flow constitutes the electric current drawn from the cell to the load to do useful work. The lead atom in the PbO_2 molecule of the positive plate is able to absorb the two electrons returning from the external load, and thus becomes Pb^{++}.

$$\underset{\text{(lead atom of PbO}_2\text{)}}{Pb^{++++}} + \underset{\text{(electrons)}}{2e^-} \longrightarrow \underset{\text{(lead)}}{Pb^{++}} \tag{3}$$

Therefore, the action at the positive plate breaks the lead peroxide molecule (PbO_2) into Pb^{++} and O_2. While the previous actions are taking place, the sulphuric acid molecule (H_2SO_4) breaks up into H^+ ions and SO_4^{++} ions. The H^+ ions from the H_2SO_4 and O_2 from the PbO_2 are free to unite and form water (H_2O).

$$\underset{\text{(hydrogen ions)}}{4H^+} + \underset{\text{(lead peroxide)}}{PbO_2} + \underset{\text{(electrons)}}{2e^-} \longrightarrow \underset{\text{(lead ions)}}{Pb^{++}} + \underset{\text{(water)}}{2H_2O} \tag{4}$$

It can be seen from the above equations, that at both the negative and positive plates there are lead ions (Pb^{++}). At both plates the lead ions (Pb^{++}) unite with the sulphate ions (SO_4^{--}) to form lead sulphate ($PbSO_4$).

$$\underset{\text{(lead ions)}}{Pb^{++}} + \underset{\text{(sulphate ions)}}{SO_4^{--}} \longrightarrow \underset{\text{(lead sulphate)}}{PbSO_4} \tag{5}$$

The chemical reaction at the negative plate, therefore, is the action expressed in equations (2) and (5). Combining them we have

$$\underset{\text{(spongy lead)}}{Pb} + \underset{\text{(sulphate ions)}}{SO_4^{--}} \longrightarrow \underset{\text{(electrons)}}{2e^-} + \underset{\text{(lead sulphate)}}{PbSO_4} \tag{6}$$

The reaction at the positive plate is the action expressed in equations (4) and (5); consolidating them gives

$$\underset{\text{(lead peroxide)}}{PbO_2} + \underset{\text{(electrons)}}{2e^-} + \underset{\text{(hydrogen ions)}}{4H^+} + \underset{\text{(sulphate ions)}}{SO_4^{--}} \longrightarrow \underset{\text{(water)}}{2H_2O} + \underset{\text{(lead sulphate)}}{PbSO_4} \tag{7}$$

By adding the actions of equations (6) and (7) we have the reactions at both plates:

$$\underset{\text{(spongy lead)}}{Pb} + \underset{\text{(lead peroxide)}}{PbO_2} + \underset{\text{(sulphuric acid)}}{2H_2SO_4} \longrightarrow \underset{\text{(lead sulphate)}}{2PbSO_4} + \underset{\text{(water)}}{2H_2O} \quad (8)$$

The equation above is the equation for the discharge cycle of a lead-acid cell and it will be noted that it is the same as equation (1) (fig. 83). Plates *A*, *B*, and *C* show the relative conditions of a fully charged, a discharging, and a discharged cell.

b. In order to charge a lead-acid secondary cell, the chemical process just described must be reversed. This is accomplished by introducing a steady direct current in the proper direction. The plus terminal of the cell to be charged should be connected to the plus terminal of the battery charger and its minus terminal to the minus terminal of the charger. In this manner, a current is forced into the battery against the current that the battery would normally supply. It goes without saying, that the electromotive force of the charger must be greater than that of the battery. During charging, at the positive plate, the water (H_2O) and the lead sulphate ($PbSO_4$) break up, forming lead peroxide (PbO_2), hydrogen ions (H^+), sulphate ions (SO_4^-) and two negative electrons ($2e^-$). This equation is—

$$\underset{\text{(water)}}{2H_2O} + \underset{\text{(lead sulphate)}}{PbSO} \rightarrow \underset{\text{(lead peroxide)}}{PbO_2} + \underset{\text{(hydrogen ions)}}{4H^+} + \underset{\text{(sulphate ions)}}{SO_4^{--}} + \underset{\text{(electrons)}}{2e^-} \quad (9)$$

The hydrogen ions and the sulphate ions combine to form sulphuric acid (H_2SO_4). The negative plate gains the two electrons lost by the positive plate.
Therefore, at the negative plate, the lead atom of the lead sulphate ($PbSO_4$) gets back its two electrons and changes back into spongy lead. Consequently, we have—

$$\underset{\text{(lead sulphate)}}{PbSO_4} + \underset{\text{(electrons)}}{2e^-} \rightarrow \underset{\text{(spongy lead)}}{Pb} + \underset{\text{(sulphate ions)}}{SO_4^{--}} \quad (10)$$

The combined total reaction at both plates is expressed by adding equations (9) and (10)—

$$\underset{\text{(lead sulphate)}}{2\ PbSO_4} + \underset{\text{(water)}}{2H_2O} \rightarrow \underset{\text{(spongy lead)}}{Pb} + \underset{\text{(lead peroxide)}}{PbO_2} + \underset{\text{(sulphuric acid)}}{2H_2SO_4} \quad (11)$$

The above equation is the same as equation (1) reading from right to left. The changing chemical conditions of the charging cycle is depicted in D of figure 83. In addition to the reactions shown in equations (9), (10), and (11), the passing of a direct current through the cell causes a chemical reaction of the water, which is present in the electrolyte. The water ionizes slightly into H^+ and OH^- ions. The positive (H^+) ions are attracted to the negative plate, where they receive a negative electron and become a hydrogen atom. Each hydrogen (H) atom units with another hydrogen atom to form a molecule of hydrogen gas (H_2).

$$\underset{\text{(hydrogen ions)}}{2H^+} + \underset{\text{(electrons}}{2e^-} \rightarrow \underset{\text{(hydrogen gas)}}{H_2} \quad (12)$$

At the positive plate, the $4OH^-$ ions lose 4 electrons and break down into water and oxygen molecules. Thus—

$$\underset{\text{(OH ions)}}{4OH^-} - \underset{\text{(electrons)}}{4e^-} \rightarrow \underset{\text{(water)}}{2H_2O} + \underset{\text{(oxygen gas)}}{O_2} \quad (13)$$

The combined result for the action at both plates is, therefore,

$$\underset{\text{(water)}}{2H_2O} \rightarrow \underset{\text{(hydrogen gas)}}{2H_2} + \underset{\text{(oxygen gas)}}{O_2} \quad (14)$$

2. Electrolysis

It is in order to consider carefully the above reaction, which is known as the *electrolysis* of water. It explains certain effects that you may have observed while watching batteries being charged. Equation (14) shows that, during charging, two times as many hydrogen bubbles will appear at the negative

plate as oxygen bubbles at the positive plate. If one should have a doubt about which pole is plus and which pole is minus, it can be determined by observation during charging. The minus pole can be identified because twice as many bubbles will be seen there. The electrolysis action explains another fact. For this reason, the water level of the cell is lowered, and distilled water must be added from time to time. Also, this is a good place to add a word of caution with regard to charging batteries. The oxygen gas and hydrogen gas that trickle off can form a *dangerous* explosive mixture which ignites in the presence of a spark. For this reason, battery charging rooms should be well ventilated and smoking should be prohibited.

APPENDIX III

APPLICATION OF KIRCHHOFF'S LAWS TO THE SOLUTION OF D-C NETWORKS

1. Kirchhoff's Laws

In chapters 6 and 9 of this manual, various applications of Ohm's law in solving for voltage, current, and resistance in simple series and parallel circuits have been presented. However, this law alone is not sufficient for solving many of the complex circuits or networks that may be encountered in practice. The more complex networks require the application of Kirchhoff's laws which are stated as follows:

a. LAW No. 1. *The algebraic sum of the voltage rises encountered in tracing around any closed loop is zero.*

b. LAW No. 2. *The algebraic sum of the currents arriving at or leaving any junction point is zero.*

2. Voltage Drop and Voltage Rise

According to Ohm's law, the voltage drop or voltage rise across any resistor is equal to the product of the current through the resistor and the magnitude of the resistor. Or,

$$E = I \times R.$$

Before Kirchhoff's laws are explained, a few words must be said with regard to voltage drop and voltage rise.

a. Let us suppose that we have a resistance R, through which an electron current flows in the direction shown in figure 164. From our work in electrostatics, we know that *electrons move from points of lower potential to points of higher potential.* Thus point A is at a lower potential than point B. This means that there is a voltage rise in going from point A to point B. However, in a number of technical books, current is *assumed* to be a movement of positive charge and, when this assumption is made, it is said that positive charge moves from point B to point A. Since positive charge goes from points of higher potential to points of lower potential, B must be at a higher potential than A. The results are the same in either case.

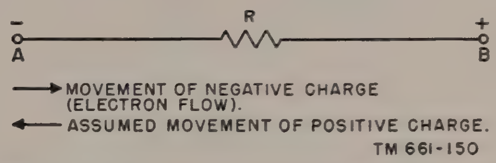


Figure 164. Electron flow versus assumed current flow.

b. When applying Kirchhoff's laws, it does not matter whether we assume current to be a flow of positive charge or negative charge if the following rule is observed: In tracing through a resistance in the direction of assumed current flow, a *voltage drop* is encountered if positive current is assumed and a *voltage rise* if negative current is assumed. As an illustration, consider figure 165. Using Ohm's law, we know that an electron current of 10 amperes flows in this circuit. The direction of current flow is from the negative terminal to the positive terminal of the battery, as shown by the solid arrow. However, positive current can be assumed to flow in the direction shown by the dotted arrow. Note that the polarity of the voltage across the 10-ohm resistance is the same in both cases. *In this appendix current will be assumed to be positive.*

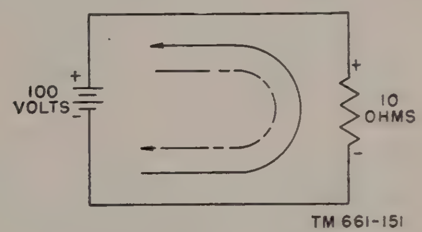


Figure 165. Voltage drop and voltage rise.

3. Explanation of Kirchhoff's Laws

a. LAW No. 1: *The algebraic sum of the voltage rises around any closed loop is zero.* To see what this law means, refer to figure 166 and assume that the current flow is in the direction shown by the solid arrow. Starting at A, trace around the loop in the direction of current flow:

A–B: voltage rise $=+100$ volts.

B–C: voltage rise $=-10I$ since with positive current assumed, there is actually a drop in voltage.

C–D: voltage rise $=-5I$.

D–E: voltage rise $=+50$ volts.

These are all the voltage rises in the loop, and their sum by Kirchhoff's first law must be zero.

Therefore, $+100-10I-5I+50=0$

$$150=15I$$

$$I=\frac{150}{15}=+10 \text{ amperes.}$$

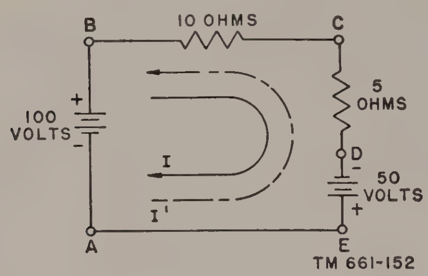


Figure 166. Example of Kirchhoff's first law.

The plus sign indicates that a positive charge would actually flow in the assumed direction. Now suppose that the current direction were assumed to be reversed, as shown by the dotted line. Let us trace around the circuit in the same direction as before:

A–B: voltage rise $=+100$ volts.

B–C: voltage rise $=+10I^1$ since with negative current C is at a higher potential than B.

C–D: voltage rise $=+5I^1$

D–E: voltage rise $=+50$ volts.

Therefore, $+100+10I^1+5I^1+50=0$

$$15I^1=-150$$

$$I^1=-10 \text{ amperes.}$$

The negative sign indicates that positive charge actually does not flow in the direction assumed. In other words, we have not gotten two different answers for the current in the circuit. A current of $+10$ amperes in one direction is the same as a current of -10 amperes in the other direction. From this example, the following points should now be understood clearly.

(1) In tracing through a resistance in the direction of assumed positive current flow, a voltage drop is encountered.

(2) The direction of *assumed* current flow is immaterial as long as one is consistent.

(3) If the *assumed* direction is incorrect, the current solved for will be negative.

b. Law No. 2: *The algebraic sum of the currents toward or away from a junction point must be zero.* This statement can be restated as follows: The amount of current entering a junction point must be equal to the amount of current leaving it. This law is actually a consequence of a more general one known as the *principle of conservation of charge.* As an example consider figure 167. A current of 1 ampere flows toward point A and a current of 5 amperes flows away from point A. What is the current flowing from A to B? We

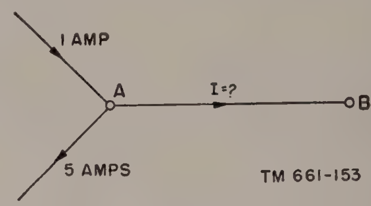


Figure 167. Example of Kirchhoff's second law.

have 1 ampere toward A and -5 amperes toward $-A$ (— sign indicates that it actually flows away from A.).

$$-I+1-5=0.$$

$$I=-4 \text{ amps.}$$

This means that I does not actually flow away from A but toward it. This is obvious since 4 amperes is needed together with the 1 ampere to give 5 amperes leaving A. Also, we have 5 amperes away from A, -1 ampere away from A, and I amperes away from A. Therefore, $-1+5+I=0$, and $I=-4$ as before.

Summarizing, we may say:

(1) The sum of the currents toward a junction point (the currents are added with their proper signs) equals the sum of the currents away from the point (again with their proper signs).

(2) It is immaterial what directions are assumed for the various currents, as long as one is consistent.

4. Application of Kirchhoff's Laws

a. Example No. 1, Parallel Combination (fig. 168). Around Loop $A\ B\ C\ D\ E\ F\ A$, we obtain (by Kirchhoff's first law)—

$$E-I_1R_1=0, \text{ or } I_1=E/R_1.$$

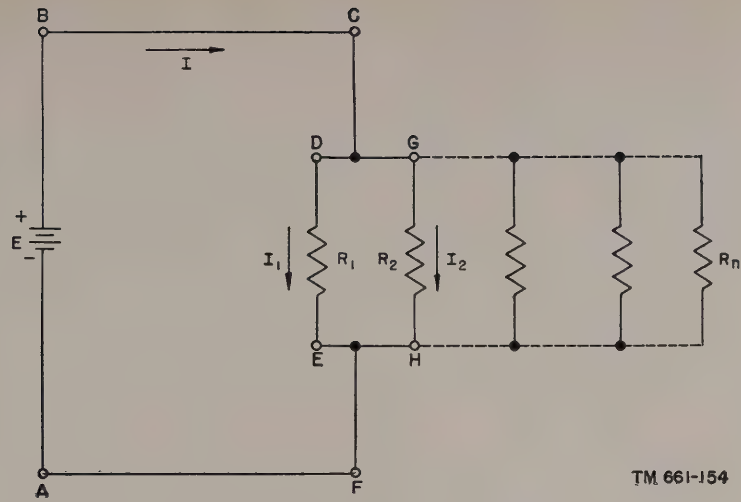


Figure 168. Network of parallel resistances.

Around loop $A\ B\ C\ G\ H\ F\ A$, we obtain—

$$E-I_2R_2=0, \text{ or } I_2=E/R_2.$$

From Kirchhoff's second law, $I=I_1+I_2$.

Therefore, $I=\frac{E}{R_1}+\frac{E}{R_2}=E\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$,

or, $\frac{E}{I}=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$. This means that the resistances in parallel are equivalent to one resistance of value

$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}.$$

Suppose we had any number of resistances in parallel, say R_n.

Then, as before $I_1=\frac{E}{R_1}$, $I_2=\frac{E}{R_2}$, $I_n=\frac{E}{R_n}$

$$I=\frac{E}{R_1}+\frac{E}{R_2}+\cdots\cdots\frac{E}{R_n}=E\left(\frac{1}{R_1}+\frac{1}{R_2}+\cdots\cdots\frac{1}{R_n}\right)$$

or, $\frac{E}{I}=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots\cdots\frac{1}{R_n}}$.

In other words, if the entire network of resistances in parallel were placed in a closed box (fig. 169) the entire combination would look to the battery like a single resistance of value.

$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots\cdots\frac{1}{R_n}} \qquad \text{(a)}$$

Thus whether one resistance of value (a) or the parallel combination were in the box of figure 169 it would not matter as far as the source of emf was concerned. This is the reason for speaking of (*a*) as the equivalent resistance of the parallel combination. To be more specific, suppose $R_1=1$ ohm, $R_2=2$ ohms, and $R_3=5$ ohms, and that they are in parallel; then the equivalent resistance of the combination is, by (a)—

$$\frac{1}{\frac{1}{1}+\frac{1}{2}+\frac{1}{5}}=\frac{1}{1+.5+.2}=\frac{1}{1.7}=.588 \text{ ohm.}$$

If $E=100$ volts, the total current would be—

$$\frac{100}{.588}=170 \text{ amps.}$$

b. Example No. 2, Series-Parallel Combination. It is desired to find I, I_1 and I_2 of figure 170. There are two ways of doing this problem:

(1) The first way is by replacing the parallel combination with its equivalent resistance and then solving the resulting series circuit for I. Now, 10 ohms and 15 ohms in parallel gives—

$$\frac{1}{\frac{1}{10}+\frac{1}{15}}=\frac{150}{25}=6 \text{ ohms.}$$

The circuit now reduces to that shown in figure 171, where the voltage drop from A to B is the same as the drop across the

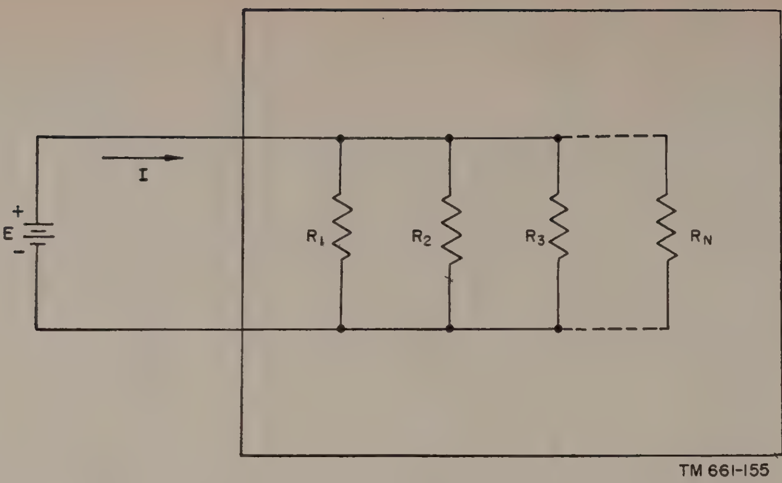


Figure 169. Parallel resistance network placed in closed box.

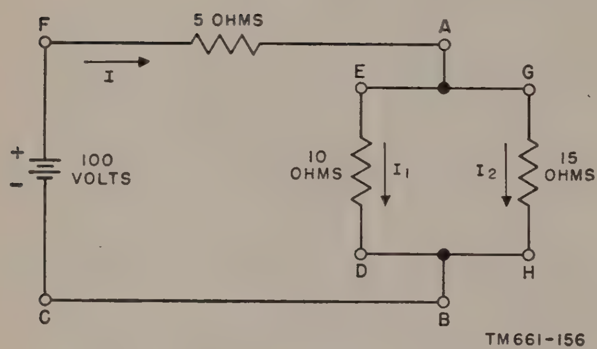


Figure 170. Series-parallel combination.

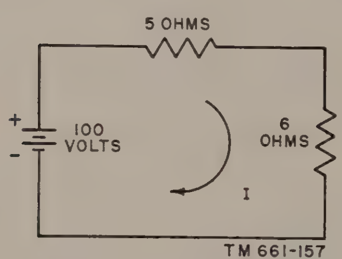


Figure 171. Parallel combination of figure 170 replaced by equivalent circuit.

6-ohm resistor and is $6\times 9.09=54.54$ volts. Thus $I_1=\frac{54.44}{10}=5.454$ amps, and $I_2=\frac{54.54}{15}=3.64$ amps. To check, we see if $I_1+I_2=I$ (Kirchhoff's second law); $I_1+I_2=9.094$ which checks with $I=9.09$.

(2) The second method makes use of Kirchhoff's first law directly.

Going around loop $C\ F\ A\ E\ D\ B\ C$, we obtain—

$$+100-5I-10I_1=0.$$

Around loop $C\ F\ A\ G\ H\ B\ C$, we obtain—

$$100-5I-15I_2=0.$$

Also, from the second law, we must have $I=I_1+I_2$. Therefore, we have three equations:

(a) $5I+10I_1+0\times I_2=100$
(b) $5I+0\times I_1+15I_2=100$
(c) $I_1+I_2=I$.

For I, substitute I_1+I_2 in (a) and (b) to obtain—

$$15I_1+5I_2=100 \text{ (d)}$$
$$5I_1+20I_2=100 \text{ (e)}$$

Multiply (d) by 4 to obtain—

$$60I_1+20I_2=400$$
$$5I_1+20I_2=100.$$

Subtract the bottom equation from the top equation. The result is—$55I_1+0=300$. Therefore, $I_1=\frac{300}{55}=\frac{60}{11}=5.45$ amperes.

Substituting this value of I_1 in (e) we obtain—

$$27.25+20I_2=100$$

Therefore, $20I_2=100-27.25=72.75$

Or, $I_2=\dfrac{72.75}{20}=3.6375=3.64$ amperes.

$I=I_1+I_2=5.45+3.644=9.09$ amperes.

Despite the fact that both of these methods give the same results, the first is definitely shorter. *However, the second method is of general applicability and can be used to solve any complex circuit.* In many circuits, as we shall see later, the equivalent resistance cannot be found directly, thus eliminating method one.

5. Two Equations in Two Unknowns

In paragraph 4, it was found necessary to solve a system of three equations in three unknowns. In paragraph 6, a method will be given for solving such systems. This paragraph gives the method for solving a system of two equations in two unknowns.

$$15I_1+4I_2=100 \text{ (a)}$$
$$13I_1+3I_2=400 \text{ (b)}$$

I_1 and I_2 may represent the currents in a circuit. To eliminate I_2, multiply (a) by 3, and (b) by 4.

$$45I_1+12I_2=300 \text{ (c)}$$
$$52I_1+12I_2=1600 \text{ (d)}$$

Then subtracting (c) from (d),

$$7I_1+0=1300.$$

Therefore, $I_1=\dfrac{1300}{7}=185\ \dfrac{5}{7}=185.71.$

Substitute this value of I_1 in equation (c). This yields—

$$8357+12I_2=300.$$

Therefore, $I_2=\dfrac{-8357+300}{12}=\dfrac{-8057}{12}=-671.4.$

Now generalizing a bit, let us try to find a solution of any two simultaneous equations of the form—

$$aI_1+bI_2=A \qquad (1)$$
$$cI_1+dI_2=\text{B} \qquad (2)$$

a, b, c, d, A, B are any arbitrary numbers. For example, in the previous problem (equation (c)),

$$a=45,\ b=12,\ A=300$$
$$c=52,\ d=12,\ B=1600$$

Following the same method as in the problem, we will eliminate I_2 by multiplying (1) by d, and (2) by b, and then subtract the two results. This gives, after multiplication—

$$(ad)I_1+(db)I_2=dA \qquad (3)$$
$$(cb)I_1+(bd)I_2=bB. \qquad (4)$$

Subtracting (4) from (3) gives—

$$(ad-bc)I_1+dbI_2-bdI_2=dA-bB$$
$$(ad-bc)I_1=dA-bB$$
$$I_1=\frac{dA-bB}{ad-bc}.$$

Substituting this value of I_1 in (2) we get—

$$\frac{c(dA-bB)}{(ad-bc)}+dI_2=B.$$
$$\therefore dI_2=B-\frac{c(dA-bB)}{ad-bc}$$
$$=\frac{adB-bcB-cdA+cbB}{ad-bc}$$
$$=\frac{(ad)B-(cd)A}{ad-bc}$$
$$I_2=\frac{d(aB-cA)}{d(ad-bc)}=\frac{aB-cA}{(ad-bc)}.$$

Therefore the system of equations—

$$aI_1+bI_2=A,$$
$$cI_1+dI_2=B.$$

has for solutions,

$$I_1=\frac{dA-dB}{ad-bc}$$
$$I_2=\frac{aB-cA}{ad-bc}.$$

Notice that the denominator has the same value for both I_1 and I_2. The solutions can be remembered very easily in the following way: Suppose the coefficients of equations (1) and (2) are written in the following way:

$$D=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

and suppose that we start at the upper left corner and multiply in the direction of the arrows and subtract the last product. Thus, we get

$ad-cb$, which is the denominator of our solutions. To remember the numerators, use the following rule. In D substitute for the coefficients of the unknown, the quantities appearing on the right side of the equations. Expand as explained above. Thus, to get the numerator of I_1, we replace a and c in D by A and B to obtain:

$$\begin{vmatrix} A & b \\ B & d \end{vmatrix} - dA - bB.$$

For I_2, the substitution gives—

$$\begin{vmatrix} a & A \\ c & B \end{vmatrix} = aB - cA.$$

Our solutions can now be expressed in the following forms:

$$I_1 = \begin{vmatrix} A & b \\ B & d \end{vmatrix} \Big/ \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$I_2 = \begin{vmatrix} a & A \\ c & B \end{vmatrix} \Big/ \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

The quantity $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is called the *determinant* of the system of equations (1) and (2) and has the value $(ad-bc)$ according to the rule given for expanding it. As an application of these rules, consider the example solved previously:

$$15I_1+4I_2=100$$

$$13I_1+3I_2=400$$

$$D=\begin{vmatrix} 15 & 4 \\ 13 & 3 \end{vmatrix}=3\times15-4\times13=45-52=-7.$$

The numerator of $I_1=\begin{vmatrix} 100 & 4 \\ 400 & 3 \end{vmatrix}=300-1600=-1300.$

The numerator of $I_2=\begin{vmatrix} 15 & 100 \\ 13 & 400 \end{vmatrix}=15\times400-13\times100=6000-1300=4700.$

Therefore, $I_1=\dfrac{-1300}{-7}=+185.71,$

and $I_2=\dfrac{4700}{-7}=-671.43.$

The answers, of course, are the same. The advantage of this method is that it provides a very systematic method of solving the equations.

6. Three Equations in Three Unknowns

The solution of three equations in three unknowns can always be reduced to the solution of two equations in two unknowns. To show this, an example will be worked out.

$$2I_1+3I_2+11I=100 \qquad (1)$$

$$I_1+7I_2+4I_3=200 \qquad (2)$$

$$5I_1+2I_2+I_3=300 \qquad (3)$$

To eliminate I_1, first multiply equation (1) by 1 and (2) by 2 to obtain—

$$2I_1+3I_2+11I_3=100 \qquad (1')$$

$$2I_1+14I_2+8I_3=400 \qquad (2')$$

Then subtract (1′) from (2′) to obtain—

$$11I_2-3I_3=300 \qquad \text{(a)}$$

Again, multiply (2) by 5 and (3) by 1. This yields—

$$5I_1+35I_2+20I_3=1{,}000 \qquad (2'')$$

$$5I_1+2I_2+I_3=300 \qquad (3'')$$

Subtract (3″) from (2″) to obtain—

$$33I_2+19I_3=700 \qquad \text{(b)}$$

(a) and (b) together constitute two equations in two unknowns. Thus,

$$11I_2-3I_3=300 \qquad (3)$$

$$33I_2+19I_3=700 \qquad (4)$$

In other words, using (1) and (2), I_1 was eliminated and using (2) and (3), I_1 was again eliminated. But (3) and (4) can be solved by the method already explained.

$$D=\begin{vmatrix} 11 & -3 \\ 33 & 19 \end{vmatrix}=11\times19-(-3\times33)=209+99=308.$$

Numerator of $I_2=\begin{vmatrix} 300 & -3 \\ 700 & 19 \end{vmatrix}=300\times19-(-3\times 700)=5700+2100=7800.$

Numerator of $I_3=\begin{vmatrix} 11 & 300 \\ 33 & 700 \end{vmatrix}=11\times700-33\times300=7700-9900=-2200.$

Therefore, $I_2=\dfrac{7800}{308}=25.3$

and $I_3=\dfrac{-2200}{308}=-7.14.$

Substituting the values of I_2 and I_3 in (1) we get—

$$2I_1+3\times(25.3)+11\times(-7.14)=100.$$

Therefore, $2I_1+75.9-78.5=100.$

$$2I_1-2.6=100$$
$$2I_1=102.6$$
$$I_1=51.3 \text{ amperes.}$$

The answers to our problem are—

$$I=51.3,$$
$$I_2=25.3, \text{ and}$$
$$I_3=-7.14.$$

7. Problems

It was stated before that Kirchhoff's first law could be applied to many cases where equivalent resistances could not be found easily. A series of problems now will be given and solved.

a. EXAMPLE No. 1. In figure 172, two batteries with their internal resistances are shown supplying a resistance load of 100 ohms. We are required to find—

(1) I_L, I_1, I_2,
(2) power delivered to the 100-ohm load,
(3) power dissipated in .1-ohm and .3-ohm resistances, and
(4) power delivered by each battery.

b. SOLUTION TO EXAMPLE No. 1. Going around loop *A B C D E F A*, we obtain from Kirchhoff's first law—

$$-100I_L+10-.1I_1=0.$$

Going around loop *A B C F A*, yields—

$$-100I_L+50-.3I_2=0.$$

Also, $I_L=I_1+I_2$, by applying Kirchhoff's second law to junction *C*.

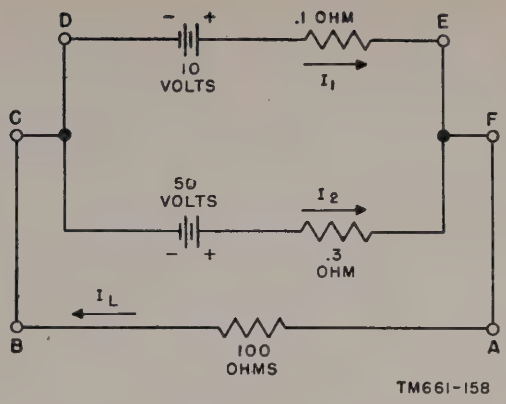


Figure 172. Kirchhoff's laws: example No. 1.

We then have the three equations—

$$+100I_L+.1I_1=+10 \qquad (1)$$

$$+100I_L+.3I_2=+50 \qquad (2)$$

$$I_1+I_2=I_L. \qquad (3)$$

Substituting I_1+I_2 in (1) and (2) gives—

$$100.1I_1+100I_2=10 \qquad (1')$$

$$100I_1+100.3I_2=50. \qquad (2')$$

For this system,

$$D=\begin{vmatrix} 100.1 & 100 \\ 100 & 100.3 \end{vmatrix}=10{,}040.03-10{,}000=40.03$$

Numerator of $I_1=\begin{vmatrix} 10 & 100 \\ 50 & 100.3 \end{vmatrix}=1003-5000=-3997$

Numerator of $I_2=\begin{vmatrix} 100.1 & 10 \\ 100 & 50 \end{vmatrix}=5005.0-1000=4005.$

$$\therefore I_1=\frac{-3997}{40.03}=-99.8 \text{ amperes.}$$

$$I_2=\frac{4005}{40.03}=100.00 \text{ amperes.}$$

$$I_L=I_1+I_2=-99.8+100.0=.20 \text{ ampere.}$$

Power delivered to 100-ohm load—$I_L 2\times R_L=.0625\times100=6.25$ watts.

Power dissipated in .1-ohm resistor$=(-99.8)^2\times.1=997.0$ watts.

Power dissipated in .3-ohm resistor$=(100.00)^2\times.3=3{,}000$ watts.

Note that since I_1 is negative, current (positive current) actually flows into the 10-volt battery. This means that the 50-volt battery is charging the 10-volt battery. Thus the amount of power delivered to the 10-volt battery $=(10\times 99.2)=$ 992.0 watts. We can look at this differently. From previous work, we know that the power delivered by a battery is $E\times I$ when I is either the *electron current* flowing *into* the positive terminal or the *positive current* flowing away from the positive terminal. Now, in this case, $E=10$ volts and $I=-99.2$ amperes away from the positive terminal. The power *delivered* by the 10-volt battery is $10\times(-99.8)=-998$ watts.
The power delivered by the 50-volt battery is $50\times 100.05=5002.5$ watts. Now the total power absorbed by the circuit is—

997.0 (.1-ohm resistor)
3000.0 (.3-ohm resistor)
+998.0 (10-volt battery)
+6.25 (100-ohm load)
=5001.25 watts.

The efficiency of any system is defined as—

$$eff=\frac{\textit{power delivered to load}}{\textit{total power delivered}}\times 100.$$

In the present problem, the

$$=eff\frac{6.25}{5002.5}\times 100=.125 \text{ percent.}$$

This figure shows an absurdly inefficient system as a result of using two batteries in parallel whose generated voltages are markedly different (10 volts and 50 volts). *When operating in parallel, batteries should be as nearly identical as possible.*

c. Example No. 2. In figure 173:

(1) Find I_1, I_2, I_3.

(2) Find the total power delivered to the circuit, and the amount delivered by each battery.

(3) Find the *equivalent resistance* of the circuit looking in from (A and B), (C and and D), (E and F).

Note. Currents have been assumed to flow out of the plus terminals of all batteries. These currents may turn out to be negative.

d. Solution to Example No. 2. Going around loop ① clockwise gives—

$$+200-10\ I_1+5\ I_3-50-I_1=0, \text{ or}$$
$$150=11\ I_1+0\times I_2-5\ I_3$$

Going around loop ② clockwise gives—

$$+50-5\ I_3-50\ I_2-20\ I_2+100-100\ I_2=0, \text{ or}$$
$$150=0\times I_1+170\ I_2+5\ I_3.$$

Also, by the second law,

$$I_1+I_3=I_2.$$

The 3 equations are—

$$11\ I_1+0\times I_2-5\ I_3=150 \qquad \text{(a)}$$
$$0\times I_1+170 I_2+5 I_3=150 \qquad \text{(b)}$$
$$I_1-I_2+I_3=0 \qquad \text{(c)}$$

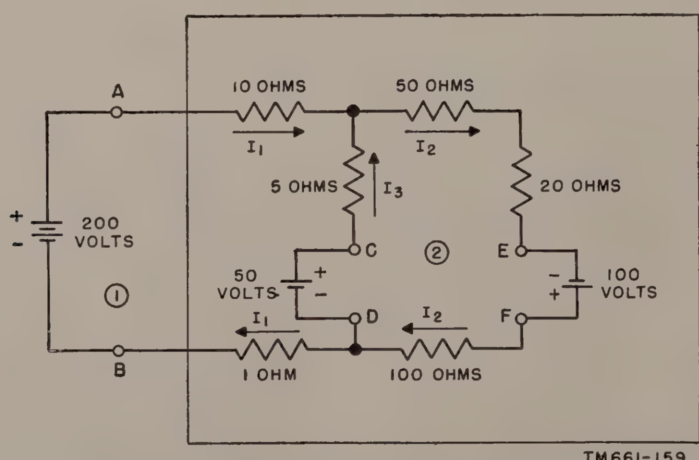


Figure 173. Kirchhoff's laws: example No. 2.

Substituting for I_2 in (a) and (b) gives—

$$11\ I_1-5\ I_3=150$$

$$170\ I_1+175\ I_3=150$$

$$D=\begin{vmatrix} 11 & -5 \\ 170 & 175 \end{vmatrix}=1925-(-850)=2775.$$

$$\text{Numerator of } I_1=\begin{vmatrix} 150 & -5 \\ 150 & +175 \end{vmatrix}=26{,}250-(-750)=27{,}000.$$

$$\text{Numerator of } I_3=\begin{vmatrix} 11 & 150 \\ 170 & 150 \end{vmatrix}=1650-25{,}500=23{,}850.$$

Thus, $I_1=\dfrac{27{,}000}{2775}=+9.73$ amperes,

$I_3=\dfrac{23{,}850}{2775}=-8.6$ amperes, and

$I_2=I_1+I_3=9.73-8.6=1.13$ amperes.

Power delivered by a 200-volt battery $=200\times I_1=200\times 9.73=1{,}946$ watts

Power delivered by a 50-volt battery $=50\times I_3=50\times(-8.6)=-430.0$ watts. (It actually absorbs power.)

Power delivered by a 100-volt battery $=100\times I_2=100\times 1.13=113$ watts.

Total power delivered to the circuit $=1946+113=2{,}059$ watts.

Equivalent resistance looking into terminals (A and B)

$$=\frac{200\text{v}}{I_1}=\frac{200}{9.73}=20.5 \text{ ohms.}$$

Equivalent resistance looking into terminals (C and D)

$$=\frac{50}{I_3}=\frac{50}{-8.6}=-5.81 \text{ ohms.}$$

Equivalent resistance looking into terminals (E and F) $=\dfrac{100}{I_2}=\dfrac{100}{1.13}=88.4$ ohms.

The -5.81 ohms means, of course, that the 50-volt battery does not deliver power but, instead, absorbs power.

8. Loop Current Method of Solving Networks

The reader by now has undoubtedly noticed that an equation was required for every independent loop in a network. Two loops are said to be independent if, in tracing around one of them, elements (resistances and batteries) are encountered which do not belong to the other. In general, a loop L_1 is said to be independent of loops L_2 L_3 . . . L_n, if there are elements in L_1 not found in either L_2, L_3 . . . L_n. To illustrate this idea, refer to figure 174. In this circuit, we actually have 3 loops: $A\ B\ E\ F\ A$, $A\ B\ C\ D\ E\ F\ A$, and $B\ C\ D\ E\ B$. However, only two are independent (only two independent equations can be written down). Note that in going around loops $ABEFA$ and $B\ C\ D\ E\ B$, all the elements in loop $A\,B\,C\,D\,E\,F\,A$ are covered. To show this more clearly, the loop equations will be written.

$$100-I_1-3I_3=0 \quad (A\ B\ E\ F\ A) \qquad (1)$$

$$-2I_2-50+3I_3=0 \quad (B\ C\ D\ E\ B) \qquad (2)$$

Now we might think that going around loop $A\,B\,C\,D\,E\,F\,A$ would give another equation. Thus,

$$100-I_1-2I_2-50=0 \qquad (3)$$

or $\quad 50-I_1-2I_2=0,$

but adding (1) and (2) gives $50-I_1-2I_2=0$, showing that (3) could have been gotten from (1) and (2). Therefore, the information contained in (3) is already contained in (1) and (2). In other words, going around loop $A\ B\ C\ D\ E\ F\ A$ gave no new information. This shows exactly what is meant by *independent* loops. Going around each loop gives additional information (another equation) not already obtainable from the others. *Therefore, the number of simultaneous equations to solve (after using the second law) is equal to the number of independent loops in the network.*

a. The loop-current method of writing down the loop equations automatically makes use of Kirchhoff's second law. To illustrate the method, the equation of the previous problem will now be written down on the *loop-current* basis: I_1 and I_2 (fig. 175) are called the loop currents. This is

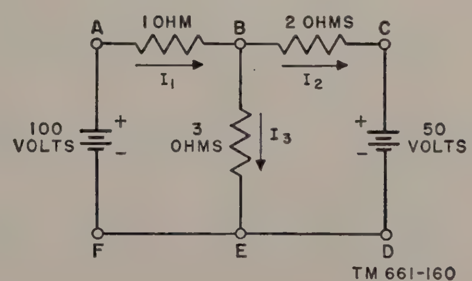


Figure 174. Loop current method.

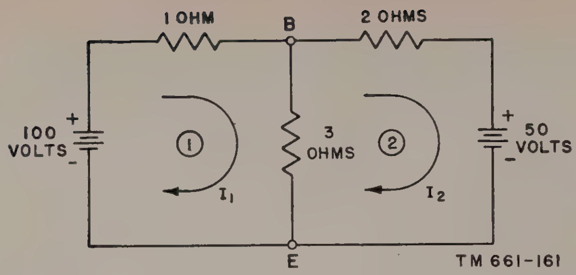


Figure 175. Forming equations by using loop current method.

not to be understood to mean that I_1, for example, flows through every element of loop ①. Obviously the current from B to E through the 3-ohm resistor is I_1-I_2. (This is what is meant by saying that the second law is used automatically.) From loop ①,

$$100-I_1-3\ (I_1-I_2)=0$$

or

$$100=4I_1-3I_2.$$

From loop ②,

$$-2I_2-50-3\ (I_2-I_1)=0$$

or

$$-50=-3I_1+5I_2.$$

The two equations are therefore—

$$100=4I_1-3I_2 \qquad \text{Loop ①}$$

$$-50=3I_1+5I_2 \qquad \text{Loop ②}$$

b. There are several important points to notice about the equations for loops ① and ② above.

(1) In equation 1 obtained from loop ①, the coefficient of I_1 is the *total series resistance* of loop 1 (1 ohm+3 ohms).

(2) The coefficient of I_2 in ① is the negative of the resistance coupling from ① to ② (− 3 ohms).

(3) The same applies to equation 2.

(4) On the left of each equation is the emf in each loop acting in the direction of assumed current flow. The 100-volt battery acts clockwise, and −50 volts acts clockwise in loop ② and, therefore, in the direction I_2.

(5) (1), (2), (3), and (4) hold only if all loop currents are assumed to flow clockwise in each loop.

c. The procedure for writing the loop equation for any number of loops can now be stated on the following form: Draw as many loop currents as there are independent loops. The directions of all these currents are clockwise. In writing the equation for *any loop*, the coefficient of the corresponding loop current is the *total series resistance of the loop;* the other loop currents enter the equation multiplied by the *negative of the resistance* which couples them into the loop for which the equation is being written. Equate this equation to the emf in the loop under consideration acting in the direction of assumed loop current (clockwise).

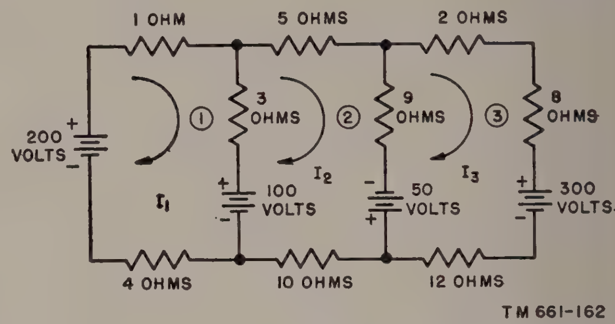


Figure 176. Loop current method; example No. 1.

9. Examples of Loop Current Method for Solving Networks

a. EXAMPLE No. 1. In figure 176, note that there are 3 independent loops and that all loop currents are drawn clockwise.

The resistance coupling loop ① to ② is 3 ohms.
The resistance coupling loop ② to ③ is 9 ohms.
The resistance coupling loop ① to ③ is 0 ohms.
The total emf in loop ① acting in the direction of I_1 is: +200−100=+100 volts. For loop ②, this emf is: 100+50=150 volts. For loop ③ this emf is: −50−300=−350 volts.
The total series loop resistance for ① is: 1+3+4=8 ohms.
The total series loop resistance for ② is: 3+5+9+10=27 ohms.
The total series loop resistance for ③ is: 9+2+8+12=31 ohms.
The loop equations are:

$$100=8I_1-3I_2+0I_3 \qquad \text{Loop ①}$$
$$150=-3I_1+27I_2-9I_3 \qquad \text{Loop ②}$$
$$-350=0I_1-9I_2+31I_3 \qquad \text{Loop ③}$$

The student should convince himself that he fully understands how to write down the loop equations by working out as many examples as possible. With a little practice, the equations can be written down on sight for any network no matter how complex.

b. Example No. 2. In figure 177, there are seven independent loops and seven equations will be required. These will be written down *on sight* as follows:

$$
\begin{array}{rlllllll}
150= & 16I_1 & -10I_2 & & & & & \\
70= & 10I_1 & +36I_2 & -20I_3 & -4I_5 & & & \\
-320= & & -20I_2 & +64I_3 & -30I_4 & & -11I_6 & \\
-50= & & & -30I_3 & +45I_4 & & & -I_7 \\
-10= & & -4I_2 & & & +24I_5 & -15I_6 & \\
-100= & & & -11I_3 & & -15I_5 & +36I_6 & -2I_7 \\
510= & & & & -I_4 & & -2I_6 & +78I_7
\end{array}
$$

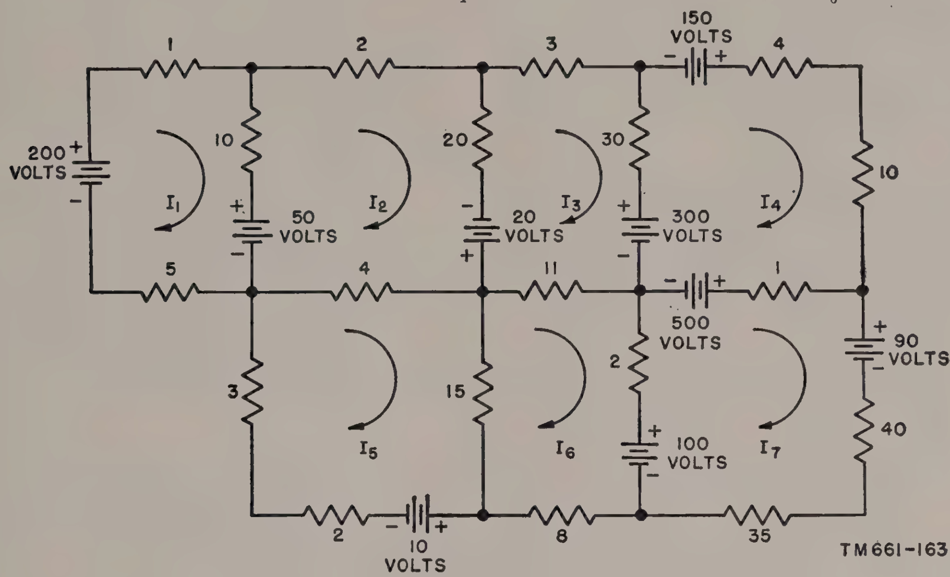


Figure 177. Loop-current method; example No. 2.

Of course, the solution of these seven simultaneous equations still presents a formidable problem. Since any network no matter how complex can be thought of as a system of loops, this method finds its greatest application in the analyses of general networks. Of course, it can also be used to advantage in the solution of particular problems.

10. The Principle of Superposition

Suppose a beam is supported on two pivots A and B (fig. 178) and that a force F_1 is applied to the center of the beam. The beam will deflect a certain amount. Now the force F_1 is removed and a force F_2 is applied. Again, the beam will deflect. Let both F_1 and F_2 be applied simultaneously; what is the deflection? Provided neither F_1 nor F_2 nor F_1+F_2 is great enough to tear the beam (no permanent deformation) it will be found that the new deflection is the sum of those which existed when both F_1 and F_2 acted alone. This is in essence the principle of *superposition*. This principle applies equally well to electrical circuits composed of resistors and batteries. The resistors must be *bilateral* and *linear*. This means that the resistor must offer the same resistance to current regardless of the direction of current flow, and its resistance must be independent of the amount of current flowing through it. Of course the circuits ordinarily encountered in d-c work are of this type and are the only ones which will concern us. A network composed only of such elements is said to be a *linear* and *bilateral* network. This automatically excludes the use of diodes or other vacuum tubes in a circuit, since a diode allows current to pass freely in one direction (looks like a short circuit) and allows very little current to flow in the other direction (an open circuit). It is obviously not bilateral. With these preliminaries out of the way, the principle

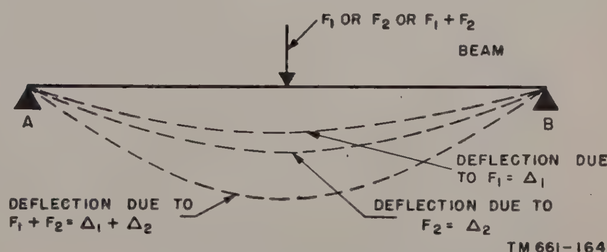


Figure 178. Principle of superposition.

of superposition as applied to electric circuits can now be stated: In any network containing resistances and batteries, the current through any part of the circuit can be found in the following way:

a. Short all batteries except one, leaving only their internal resistances in the circuit.

b. Find the current through the part under consideration.

c. Do this for each battery present.

d. Add all the currents found, taking into consideration their signs. This sum is the current that flows through the part of the circuit *when all batteries are acting simultaneously*.

11. Application of the Principle of Superposition

a. Example No. 1. Consider the problem of the two batteries in parallel (fig. 179) which has

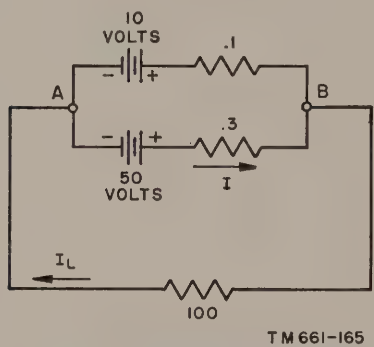


Figure 179. Principle of superposition applied to an electric circuit.

already been solved in paragraph 7*a*. The current through the 100-ohm resistor was found to be .2 ampere. Let us find this same current by superposition.

(1) First, short the 10-volt battery. The circuit is then connected as shown in figure 180.

The .1 ohm and 100 ohms in parallel give:

$\frac{1}{\frac{1}{.1}+\frac{1}{101}}=\frac{1}{10.01}=\cong .1$ ohm ($\sim$ means approximately) $\therefore I=\frac{50}{.3+.1}=\frac{50}{.4}=\frac{500}{4}=$ 125 amperes.

Drop in the .3-ohm resistor $=.3\times125=$ 37.5 volts.

Voltage drop from B to $A=50-37.5=$ 12.5 volts.

$I_L=\frac{12.5}{100}=.125$ ampere.

(2) Now, short out the 50-volt battery. The circuit is then connected as shown in figure 181. The .3 ohm and 100 ohms in parallel give—

$$\frac{1}{\frac{1}{.3}+\frac{1}{100}}=\frac{1}{\frac{10}{3}+.01}\cong\frac{3}{10}.3 \text{ ohm}$$

$$I=\frac{10}{.1+.3}=\frac{10}{.4}=\frac{100}{4}=25 \text{ amperes.}$$

The drop in the .1-ohm resistor is $.1\times 25=2.5$ volts.

The voltage drop from B to A is $10-2.5=7.5$ volts.

The current $I_L=\frac{7.5}{100}=.075$ ampere.

Therefore, from our principle, the current through the 100-ohm resistance when both batteries act together is $.125+.075=.200$ ampere, which checks with the previous answer.

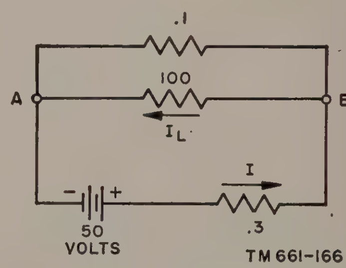


Figure 180. Figure 179 with 10-volt battery shorted.

b. Example No. 2. Consider the problem of paragraph 7*c* where the value of current I_3 through the 5-ohm resistor was found to be -8.6 amperes. This problem also can be solved by superposition.

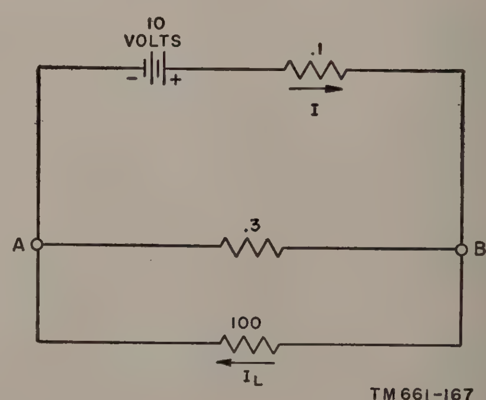


Figure 181. Figure 179 with 50-volt battery shorted.

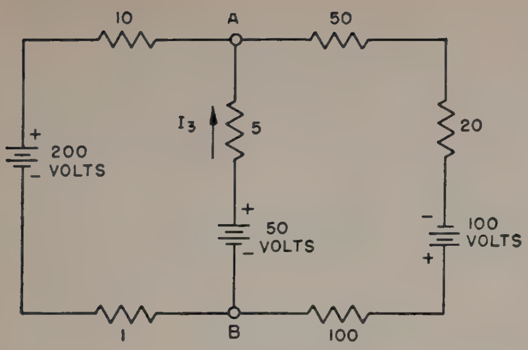


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Figure 182. Principle of superposition: example No. 2.

Note. The circuit of figure 182 is the same as that shown in figure 173.

(1) First, short the 200-volt and 100-volt batteries. The circuit is then connected as shown in figure 183. Combining the two parallel branches gives—

$$\frac{1}{\frac{1}{11}+\frac{1}{170}}=\frac{1}{.091+.0059}=\frac{1}{.097}=10.42$$

ohms.

$$I_3=\frac{50}{15.42}=3.24 \text{ amperes.}$$

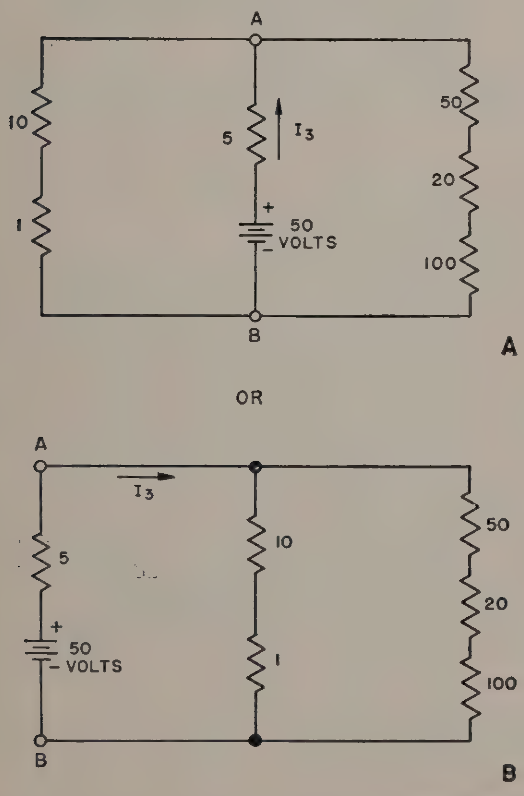

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Figure 183. Figure 182 with 200-volt and 100-volt batteries shorted.

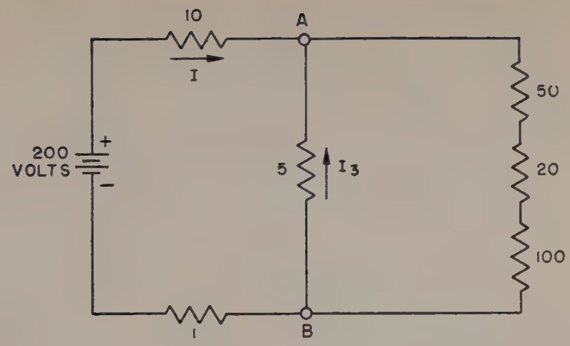

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Figure 184. Figure 182 with 50-volt and 100-volt batteries shorted.

(2) Next, short the 50-volt and 100-volt batteries and the circuit is connected as shown in figure 184.

The two parallel branches give—

$$\frac{1}{\frac{1}{5}+\frac{1}{170}}=\frac{1}{.2+.006}\cong 5 \text{ ohms}$$

$$\therefore I=\frac{200}{16}=12.5 \text{ amperes.}$$

The voltage drop in the 10-ohm resistor= $10\times 12.5=125$ volts. ∴ Drop in 1-ohm resistor=12.5 volts.

Drop from A to $B=200-12.5-125=200-137.5=62.5$ volts.

$$I_3=-\frac{62.5}{5}=-12.5 \text{ amperes.}$$

(3) Finally, short the 50-volt and 200-volt batteries, and the circuit is connected as shown in figure 185.

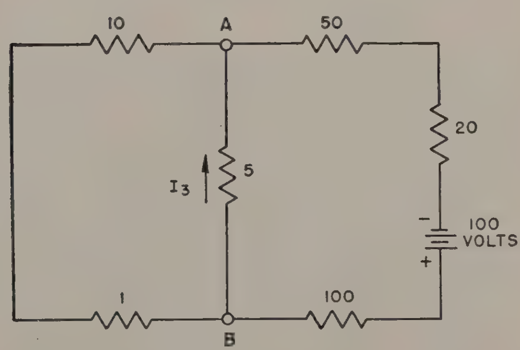

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Figure 185. Figure 182 with 50-volt and 200-volt batteries shorted.

The parallel branches give—

$$\frac{1}{\frac{1}{11}+\frac{1}{5}}=\frac{1}{.091+.2}=\frac{1}{.291}=3.44 \text{ ohms.}$$

The voltage drop from B to A=100 −.577 (170)=100−98=2 volts.

$$I=\frac{100}{173.43}=.577 \text{ ampere.}$$

The current that flows through the 5-ohm resistor when all batteries act together is, by superposition, the sum of the currents (I_3) found in (1), (2), and (3) above. Thus, $I_3=3.29-12.5+.4=-12.5+3.69=8.77$ amperes. We see that this checks very well with the current of −8.6 amperes as determined in paragraph 7c. The .17-ampere difference is a result of slide-rule error.

12. Thevenin's Theorem

a. Suppose we have a network which may be composed of any number of batteries and resistors. The network is placed in a box (fig. 186)

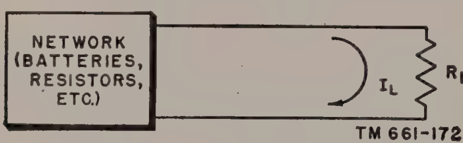

Figure 186. Network placed in box.

for purposes of analysis. Two leads are bought out to which a load resistance is connected. What is the current I_L through R_L?. Two facts must be realized.

(1) The value of I_L will, in general, depend on the network being used (the value of batteries and resistors in the box).

(2) I_L will also depend on the value of R_L (obviously).

b. Now, suppose the following experiment is performed: A variable battery and an ammeter are placed in series with R_L as shown in figure 187. Also a voltmeter is placed across the source of voltage. Then, the variable source of emf is varied until the ammeter reads zero. Let the voltmeter reading be E_0. By superposition, we know that the current through the load R_L with the source on the circuit is the sum of two currents:

(1) That due to the variable source alone, all sources in the box being shorted (only their internal resistance left in the circuit). Call this I'_L.

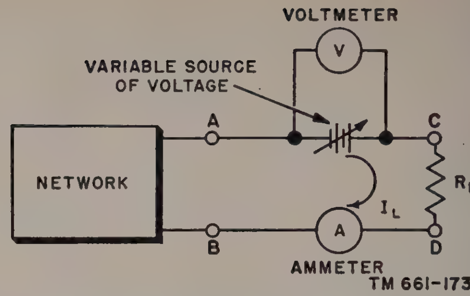

Figure 187. Figure 186 with variable battery, ammeter, and voltmeter added.

(2) That due to the network alone with the variable source shorted (sources assumed to be ideal-zero in internal resistance). But this current is the one which would flow through R_L if it were connected directly across the output terminals (*A* and *B*) of the network and is the current we are interested in determining. Call this current I_L. Since the ammeter reads zero, the sum of these two currents (when the voltmeter reads E_0) is zero. This, $I''_L=I_L+I'_L=0$. $I_L=-I'_L$. But I'_L is the current that flows through R_L with all sources in the network shorted. When all these batteries are shorted, the network presents between (*A*, *B*) a resistance of value R_0 (any combination of resistances can be reduced to one resistance). Therefore, $I'_L=\frac{E_0}{R_0+R_L}$. This is shown in figure 188. With all batteries in the network shorted, the network presents a resistance of value R_0 to the terminals *A* and *B*. When the voltmeter reads E_0, the current through the load R_L is zero. Now apply Kirchhoff's first law to the loop *BACDB*. This gives: $-V_{BA}+E_0=0$ (V_{BA}=drop from *B* to *A*). Therefore, $E_0=V_{BA}$. But V_{BA} is the voltage measured between terminals *A* and *B* on *open circuit* (since ammeter reads zero) and is of course independent of R_L (it is measured with R_L disconnected). This now

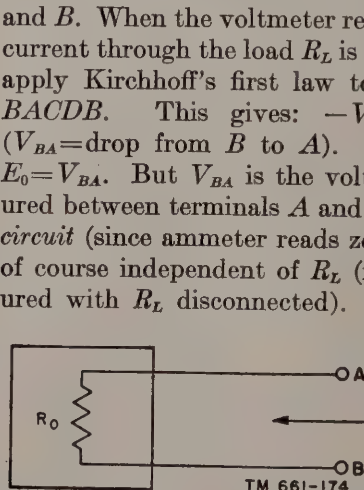

Figure 188. Resistance looking back from terminals A and B with all batteries shorted.

gives the result shown in figure 189. E_0 is the open circuit voltage of the network between terminals A and B. (E_0 is the voltage indicated by a voltmeter connected between A and B with R_L disconnected.) R_0 is the short-circuit resistance of the network between terminals A and B (it is measured with all batteries in the network shorted and R_L disconnected). Of course, for any given network, E_0 and R_0 are constants and can be measured in the laboratory. Once E_0 and R_0 are determined, the above question can be used to determine the current I_L for any value R_L. This theorem will now be applied to the two problems previously solved by Kirchhoff's first law and by superposition.

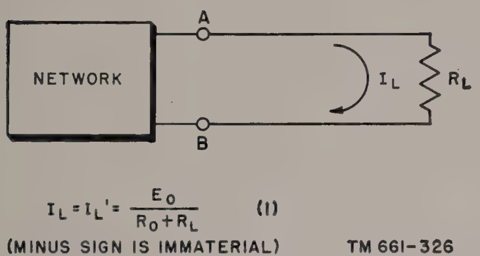

Figure 189. Thevenin's theorem.

13. Examples Showing Use of Thevenin's Theorem

a. Example No. 1. In figure 190, the 100-ohm load is R_L and the two batteries in parallel constitute the network. To find I_L, the open-circuit voltage between A and B must be determined (100-ohm load disconnected) and the short-circuit resistance R_0 looking into A and B (all batteries shorted) and the 100-ohm load disconnected.

(1) To find E_0 with the 100-ohm load removed, the circuit is connected as shown in figure 191. Obviously, there will be only the current I. $I=\frac{40}{.4}=100$ amperes. The drop in the .3-ohm resistance$=100\times .3=30$ volts. The voltage drop from B to $A=50-30=20$ volts$=E_0$.

(2) To find R_0, short all batteries and connect the circuit as shown in figure 192.

$$R_0=\frac{1}{\frac{1}{.1}+\frac{1}{.3}}=\frac{1}{10+\frac{10}{3}}=\frac{3}{40}=.075 \text{ ohm.}$$

$$I_L=\frac{E_0}{R_0+R_L}=\frac{20}{100+.075}\simeq .2 \text{ ampere,}$$

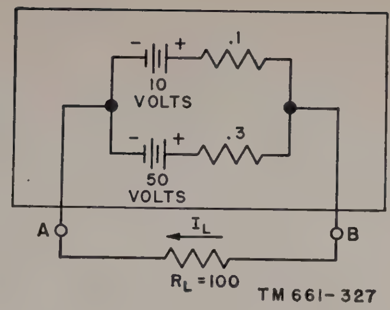

Figure 190. Application of Thevenin's theorem: example No. 1.

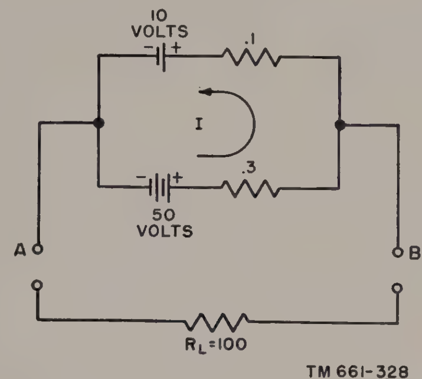

Figure 191. Figure 190 with R_L removed.

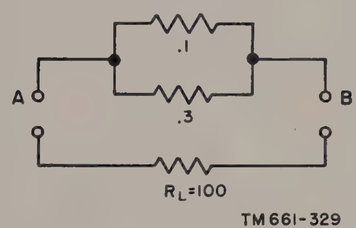

Figure 192. Figure 191 with all batteries shorted.

which agrees with the answer already found.

b. Example No. 2. The problem is to find I_3 (−8.6 amperes) by the use of *Thevenin's theorem* (fig. 193). In this case $R_L=5$ ohms, E_0 is the open-circuit voltage between A and B, and R_0 is the resistance seen between A and B with all batteries shorted (of course, the 5-ohm resistor is disconnected while finding these quantities).

(1) To find E_0 with the 5-ohm resistor disconnected, the circuit becomes a simple series circuit connected as shown in figure 194.

$$I=\frac{300}{181}=1.66 \text{ ampere.}$$

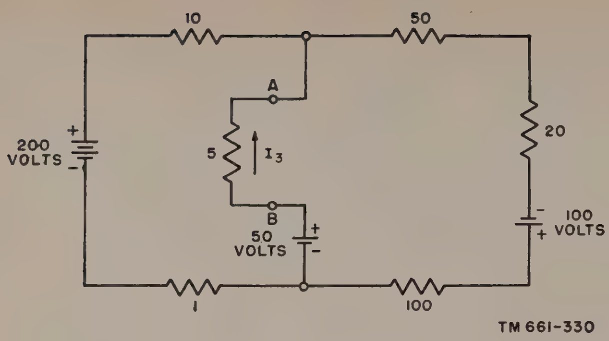

Figure 193. Thevenin's theorem: example No. 2.

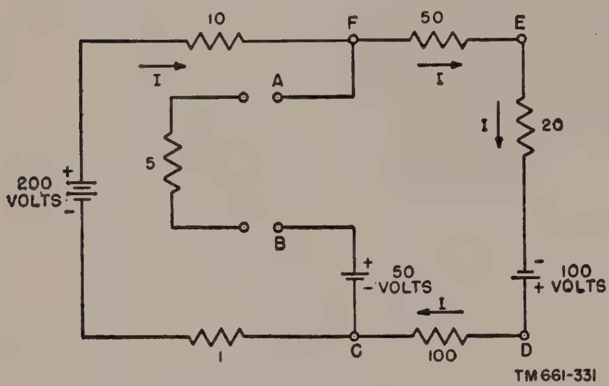

Figure 194. Figure 193 with the 5-ohm resistor disconnected.

The voltage drop from B to A is $V_{BC}+V_{CD}+V_{DE}+V_{EF}$.

$V_{BC}=50$; $V_{CD}=-100\times1.66=-166$ volts.

$V_{DE}=100-20\times1.66=100-33.20=66.80$ volts.

$V_{EF}=-50\times1.66=-83.00$ volts.

Thus $V_{BA}=50-166+66.8-83=-249+116.8=-132.2$ volts.

Therefore, $E_0=-132.2$ volts.

(2) To find R_0 with all batteries shorted, the circuit is connected as shown in figure 195. Between A and B, 11 ohms are in parallel with 170 ohms. The total resistance is

$$\frac{1}{\frac{1}{11}+\frac{1}{170}}=\frac{1}{.091+.00588}$$

$$=\frac{1}{.097}=10.3=R_0$$

This gives for I_L, $\frac{-132.2}{10.3+5}=\frac{-1.32.2}{15.3}$

$=-8.65$ amperes.

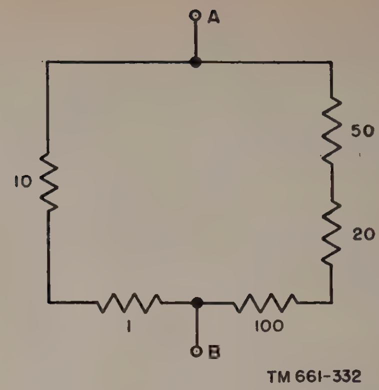

Figure 195. Figure 194 with all batteries shorted.

c. EXAMPLE NO. 3. An interesting application of Thevenin's theorem is in finding the conditions under which a network delivers maximum power to a load (fig. 196). The load R_L is connected across the output terminals of the network. What must be the value of R_L in order that it may absorb maximum power from the system? By Thevenin's theorem, the load current $I_L=\frac{E_0}{R_0+R_L}$ where E_0=open-circuit voltage of network and R_0=short-circuit resistance of the network. The power absorbed by $R_L=I_L^2R_L=P_L=\frac{E_0^2}{(R_0+R_L)^2}\times R_L$. Of course, E_0 and R_0 are constants of the network;

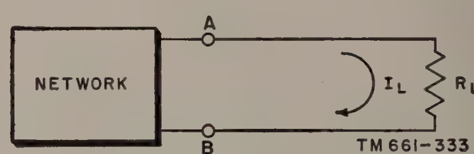

Figure 196. Network connected to load R_L.

the only way this power can be changed is by changing R_L. The problem is to determine R_L so that P_L has its largest possible value.

Expanding,

$$P_L=\frac{E_0^2\times RL}{R_0^2+2R_0R_L+R_L^2}=\frac{E_0^2}{\frac{R_0^2}{R_L}+2^R{}_0+R_L}$$

$$=\frac{E_0^2}{\frac{R_0^2}{R_L}+R_L+2^R{}_0}.$$

Now

$$\frac{R_0^2}{R_L}+2^R{}_0+R_L=\frac{(R_0-\sqrt{R_L})^2}{(\sqrt{R_L})}+4^R{}_0$$

as is easily verified by expanding:

$$P_L = \frac{E_0^2}{\left(\frac{R_0}{\sqrt{R_L}} - \sqrt{R_L}\right)^2 + 4R_0}.$$

If P_L is to have its largest value, the denominator of this equation must have its smallest value. This will be so when—

$$\left(\frac{R_0}{\sqrt{R_L}} - \sqrt{R_L}\right)^2 = 0, \quad \text{or} \quad \frac{R_0}{\sqrt{R_L}} - \sqrt{R_L} = 0$$

or $R_0/\sqrt{R_L} = \sqrt{RL}$, giving $R_0 = \sqrt{R_L} \times \sqrt{R_L} = R_L$

When $R_L = R_0$, $P_L = P$ maximum$= \frac{E_0^2}{4R_0}$

We have therefore just proved: in order to deliver maximum power, a network must be connected to a load whose resistance equals the short-circuit resistance of the network. This is known as *matching* and is extremely important in electrical work. Let us apply this result to a battery whose internal resistance is R, as shown in figure 197. In this case, R_0 is R. *Therefore, in order to deliver maximum power, a battery must be connected to a load the value of which equals its internal resistance.*

Under these conditions, $I_L = \frac{E}{R+R} = \frac{E}{2R}$

The power delivered to the load is $\frac{E^2}{4R^2} \times R = \frac{E^2}{4R}$.

The power dissipated in the internal resistance of the battery is also $\frac{E^2}{4R}$.

The efficiency of the system is—

$$\frac{output}{input} \times 100 = \frac{E^2/4R}{\frac{E^2}{4R} + \frac{E^2}{4R}} \times 100 = \frac{1}{2} \times 100 = 50 \text{ percent.}$$

When maximum power is being delivered, the efficiency of the system is only 50 percent. Also, the current under this condition is $\frac{E}{2R}$; R is usually very small for good batteries and, consequently, the current is tremendous. These are the two main reasons for not operating batteries at the maximum power point.

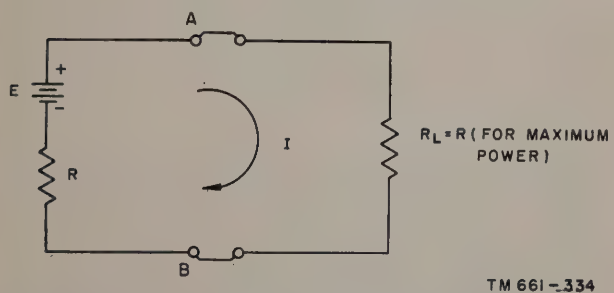

Figure 197. Load R_L connected to battery which has internal resistance R.

14. Delta-to-wye Transformation

A of figure 198 shows the resistors connected in delta, and B of figure 198 shows three resistors connected in wye. Very often in the solution of problems, delta or wye configurations are met and their solutions can be most simply obtained if some way of transforming a delta into a wye, or vice versa, is known.

a. What is meant by saying that wye is *equivalent* to a delta or a delta is *equivalent* to a wye? Suppose the delta were put in a closed box, and three leads were brought out as shown in figure 199. Having no knowledge of the contents of the box, the following three voltage and current measurements can be made. A voltage is to be applied to terminals *a* and *b* and the current in line (a) reads (*c*, floating), as shown in figure 200. The rate $\frac{V}{I}$ is the *equivalent resistance* of the network between terminals *a* and *b*. Similarly the same can be done for the other pairs of terminals

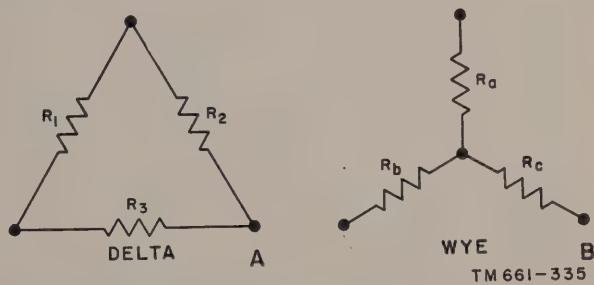

Figure 198. Resistors connected in delta and wye.

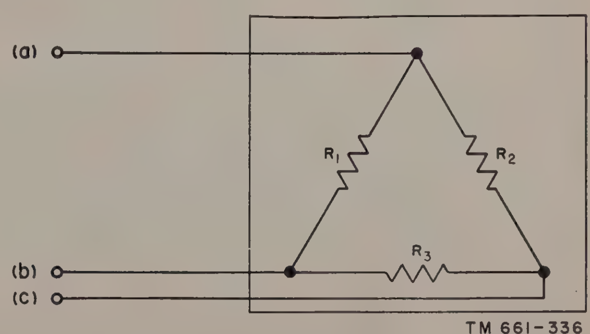

Figure 199. Resistors connected in delta placed in a box.

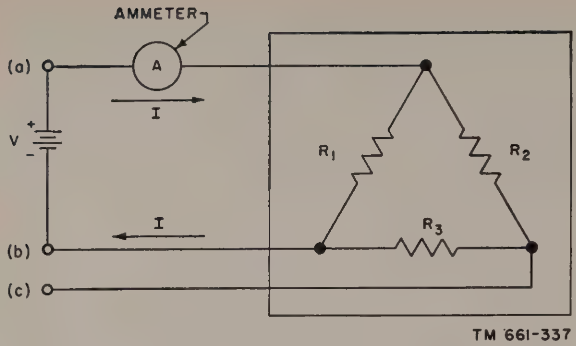

Figure 200. Figure 199 with voltage applied between terminals a and b.

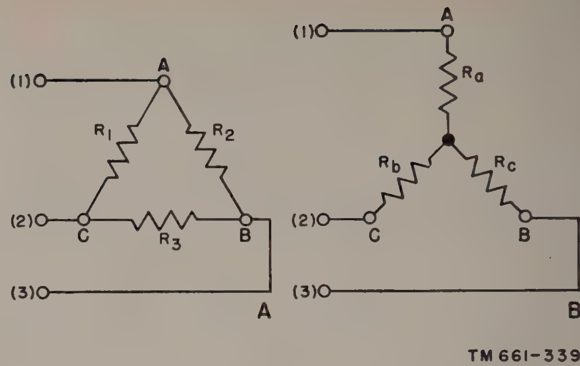

Figure 202. Resistors connected in delta equivalent to resistors connected in wye.

(*a* and *c*) and (*b* and *c*). In other words, the ③ voltage and ③ current measurements yield three values of resistance: R_{ab}, R_{bc}, R_{ac}. This is the maximum knowledge about the network that can be gotten *without opening the box* and looking at the circuit. Obviously, if another network is put in the box and connected between the leads (*a*, *b*, and *c*) and the same values of R_{ab}, R_{bc}, and R_{ac} are obtained, the two networks are *equivalent* as far as any external measurements are concerned. For example, if a wye is placed in the box (fig. 201) conditions for equivalence require electrically

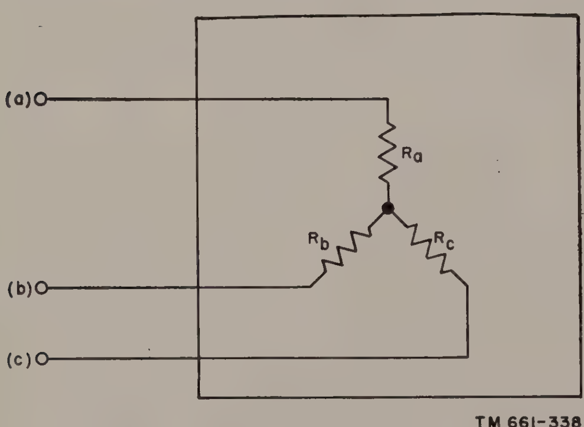

Figure 201. Resistors connected in wye placed in a box.

that it behave the same between any two terminals as the corresponding delta. These ideas make it very simple to determine a wye equivalent to a delta (fig. 202). The resistance of the delta between terminals (1) and (2) should equal that of the wye between the same terminals. This gives—

$$\frac{R_1(R_2+R_3)}{R_1+R_2+R_3}=R_a+R_b$$

for terminals (2) and (3),

$$\frac{R_3(R_1+R_2)}{R_1+R_2+R_3}=R_b+R_c$$

for (1) and (3)

$$\frac{R_2(R_1+R_3)}{R_1+R_2+R_3}=R_a+R_c.$$

b. Note that there are three simultaneous equations of R_a, R_b, and R_c, the legs of the wye. The solution of these equations yields—

Delta-to-*wye*

$$R_a=\frac{R_1R_2}{R_1+R_2+R_3}$$

$$R_b=\frac{R_1R_3}{R_1+R_2+R_3}$$

$$R_c=\frac{R_2R_3}{R_1+R_2+R_3}$$

Wye-to-*delta*

$$R_1=\frac{R_aR_b+R_bR_c+R_cR_a}{R_c}$$

$$R_2=\frac{R_aR_b+R_bR_c+R_cR_a}{R_b}$$

$$R_3=\frac{R_aR_b+R_bR_c+R_cR_a}{R_a}$$

As an application of these formulas, two examples will be worked out in the following paragraph.

15. Examples of Delta-to-wye Transformation

a. Example No. 1. For the circuit shown in figure 203, find the current delivered by the 10-volt battery.

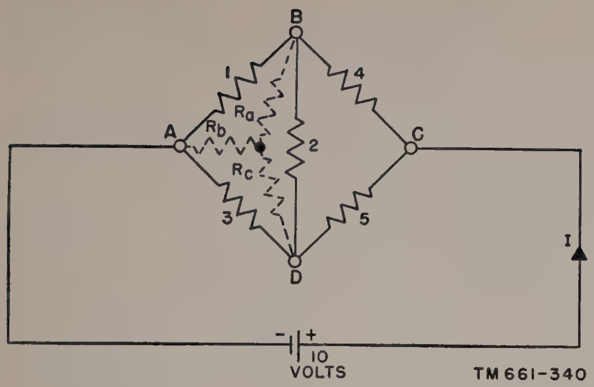

Figure 203. Delta-to-wye transformation: example No. 1.

Solution: To solve this problem, the delta *A B D* will be changed into a wye (dotted lines). From the formula given,

$$R_a=\frac{1\times 2}{1+2+3}=\frac{2}{6}=\frac{1}{3}\text{ ohm.}$$

$$R_b=\frac{1\times 3}{1+2+3}=\frac{3}{6}=\frac{1}{2}\text{ ohm.}$$

$$R_c=\frac{2\times 3}{1+2+3}=\frac{6}{6}=1\text{ ohm.}$$

The current now becomes as shown in figure 204. The total resistance of the circuit is ½-ohm in series with 4⅓ ohms in parallel with 6 ohms. The parallel combination gives—

$$\frac{1}{\frac{1}{4\frac{1}{3}}+\frac{1}{6}}=\frac{1}{\frac{3}{13}+\frac{1}{6}}=\frac{1}{.231+.1665}=\frac{1}{.398}=2.51\text{ ohms.}$$

Total resistance (between *A* and *C*)$=.5+2.51=$ 3.01 ohms. $I=\frac{10}{3.01}=3.32$ amperes.

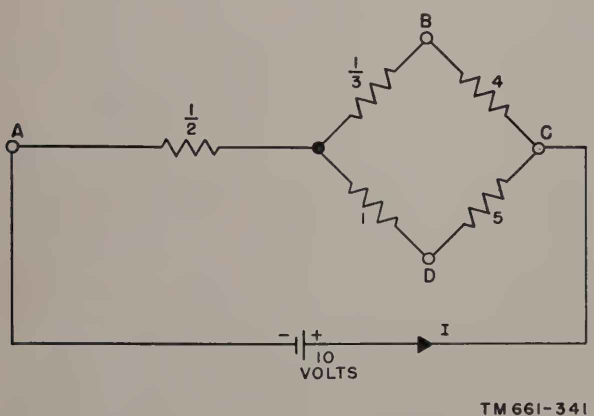

Figure 204. Figure 203 after delta A B D has been transformed to wye.

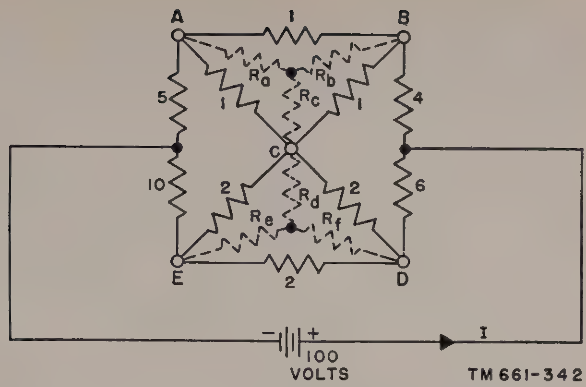

Figure 205. Delta-to-wye transformation: example No. 2.

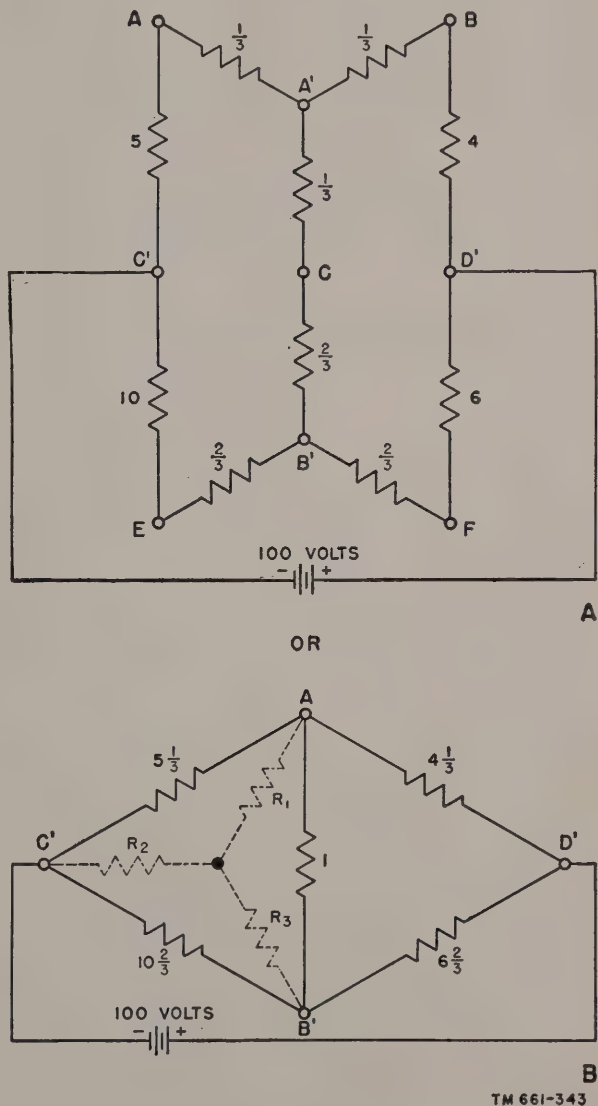

Figure 206. Figure 205 after A B C and C D E have been transformed.

b. EXAMPLE No. 2. In figure 205, find the current delivered by the 100-volt battery. The solution is as follows:

(1) Delta $A\ B\ C$ is reduced to a wye.

$$R_a=\frac{1\times 1}{1+1+1}=\frac{1}{3}, R_b=\frac{1}{3} \text{ ohm}, R_c=\frac{1}{3} \text{ ohm}.$$

(2) Delta $C\ D\ E$ is reduced to a wye:

$$R_d=R_f=R_e=\frac{2\times 2}{2+2+2}=\frac{2}{3} \text{ ohms}.$$

(3) The circuit can now be drawn as shown in figure 206 or

(4) The delta $A^1\ B^1\ C^1$ can now be reduced to a wye.

$$R_1=\frac{\frac{16}{3}\times 1}{5\frac{1}{3}+10\frac{2}{3}+1}=\frac{\frac{16}{3}}{17}=\frac{16}{51}=.313 \text{ ohm}.$$

$$R_3=\frac{1\times\frac{32}{3}}{17}=\frac{32}{51}=.628 \text{ ohm}.$$

$$R_2=\frac{5\frac{1}{3}\times 10\frac{2}{3}}{17}=\frac{\frac{16}{3}\times\frac{32}{3}}{17}=3.35 \text{ ohms}.$$

(5) The circuit then can be redrawn as shown in figure 207. The parallel combination gives—

$$\frac{1}{\frac{1}{4.64}+\frac{1}{7.30}}=\frac{1}{.215+.137}=\frac{1}{.352}=.284.$$

(6) Total resistance between A and B= 3.35+.284=3.634 ohms.

$$I=\frac{100}{3.634}=27.5 \text{ amperes}.$$

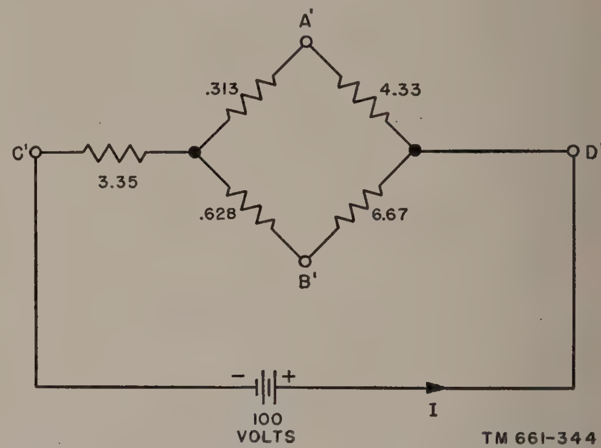

Figure 207. Figure 206 after A^1 B^1 C^1 has been transformed to wye.

APPENDIX IV
MISCELLANEOUS

1. Greek Alphabet

Name	Capital	Small	Used to designate
alpha	A	α	Angles, area, coefficients, and attentuation constant.
beta	B	β	Angles and coefficients.
gamma	Γ	γ	Electrical conductivity and propagation constant.
delta	Δ	δ	Angles, increment, decrement, and determinants.
epsilon	E	ϵ	Dielectric constant, permittivity, and base of natural logarithms.
zeta	Z	ζ	Coordinates.
eta	H	η	Efficiency, hysteresis, and coordinates.
theta	Θ	ϑ θ	Angles and angular phase displacement.
iota	I	ι	Coupling coefficient.
kappa	K	κ	
lambda	Λ	λ	Wavelength.
mu	M	μ	Permeability, amplification factor, and prefix *micro*.
nu	N	ν	
xi	Ξ	ξ	
omicron	O	o	
pi	Π	π	Pi=3.1416.
rho	P	ρ	Restivity and volume charge density.
sigma	Σ	σ ς	Summation.
tau	T	τ	Time constant and time-phase displacement.
upsilon	Υ	υ	
phi	Φ	ϕ φ	Magnetic flux and angles.
chi	X	χ	Angles.
psi	Ψ	ψ	Dielectric flux.
omega	Ω	ω	Resistance in ohms and angular velocity.

2. Symbols

Quantity	Symbol	Equation	Practical unit	Subrationalized mks.
Length	l		centimeter	Meter.
Distance	d		centimeter	Meter.
Mass	m			Kilogram.
Time	t		second	Second.
Velocity	ν	$\nu=l/t$	cm/sec	Meter/sec.
Acceleration	α	$\alpha=\nu/t$	cm/sec^2	Meter/sec^2.
Force	F	$F=\eta\ \alpha$		$\frac{\text{joule}}{\text{meter}}$=newton.
Work	W	$W=F\ l$	joule	Joule.
Power	P	$P=w/t$	watt	Watt.
Permittivity of medium	ϵ		$\frac{1}{(9\times10^{N})}$ farad/cm	$\frac{1}{(36\pi\times10^{9})}$ farad/meter
Charge	q	$F=q_1\ q_2/E\ r^2$	coulomb	Coulomb.

Quantity	Symbol	Equation	Practical unit	Subrationalized mks.
Capacitance	C	$C=q/v$	farad	Farad.
Potential difference	V or E	$V=\frac{W}{q}$	volt	Volt.
e. m. f.	e	$e=-d\,\phi/d\,t$	volt	Volt.
Current	I	$I=d\,q/d\,t$	ampere	Ampere.
Resistance	R	$R=V/I$	ohm	ohm.
Resistivity	P		ohm/cm	Ohm/meter.
Conductance	G	$G=1/R$	mho	mho.
Conductivity	γ	$\gamma=1/\mu$	mho/cm	mho/meter.
Permeability	μ		10^{-9} henry/cm	$\frac{4\pi\times10^{-7}\text{ henry}}{\text{meter}}$.
Reluctivity	ν	$\nu=1/\mu$		
Pole strength	m	$F=m_1\,m_2/\mu\,r^2$		Weber.
Magnetomotive force	F		$\frac{1}{4}\,\pi$ ampere turn	Ampere turn.
Magnetizing force	H	$H=F/l$	$\frac{1}{4}\,\pi$ ampere turn	Ampere turn/m.
Magnetic flux density	B	$B=\mu\,H$	weber/cm²	Weber/meter².
Magnetic flux	ϕ	$\phi=BA$	weber or volt-sec	Weber = volt-sec.
Reluctance	R	$R=F/\phi$	$\frac{\frac{1}{4}\pi\text{ ampere turn}}{\text{weber}}$	$\frac{\text{ampere turn}}{\text{weber}}$.
Inductance	L	$L=e/(dI/dt)$	henry	Henry.

3. Conversion Factors

To convert—	Into—	Multiply by—	Conversely multiply by—
Ampere-hours	Coulombs	3,600	2.778×10^{-4}.
Amperes per sq. cm	Amperes per sq. inch	6.452	.155.
Ampere turns	Gilberts	1.257	.7958.
Ampere turns per cm	Ampere turns per inch	2.54	.3937.
Btu (British thermal unit)	Foot-pounds	778.3	1.285×10^{-3}.
Btu	Joules	1,054.8	9.48×10^{-4}.
Btu	Kilogram-calories	.252	3.969.
Btu	Horsepower-hours	3.929×10^{-4}	2,545.
Centigrade	Fahrenheit	(C° x 9/5) + 32	(F° − 32) x 5/9.
Circular mils	Square centimeters	5.067×10^{-6}	1.973×10^{5}.
Circular mils	Square mils	.7854	1.273.
Cubic inches	Cubic centimeters	16.39	6.102×10^{-2}.
Cubic inches	Cubic feet	5.787×10^{-4}	1,728.
Cubic inches	Cubic meters	1.639×10^{-5}	6.102×10^{4}.
Cubic meters	Cubic feet	35.31	2.832×10^{-2}.
Cubic meters	Cubic yards	1.308	.7646.
Degrees (angle)	Radians	1.745×10^{-2}	57.3.
Dynes	Pounds	2.248×10^{-6}	4.448×10^{5}.
Ergs	Foot-pounds	7.367×10^{-8}	1.356×10^{7}.
Feet	Centimeters	30.48	3.281×10^{-2}.
Foot-pounds	Horsepower-hours	5.05×10^{-7}	1.98×10^{6}.
Foot-pounds	Kilogram-meters	.1383	7.233.
Foot-pounds	Kilowatt-hours	3.766×10^{-7}	2.655×10^{6}.
Gauss	Lines per sq. inch	6.452	.155.
Grams	Dynes	980.7	1.02×10^{-3}.
Grams	Ounces (avoirdupois)	3.527×10^{-2}	28.35.
Grams per cm	Pounds per inch	5.6×10^{-3}	178.6.
Grams per cubic cm	Pounds per cu. inch	3.613×10^{-2}	27.68.
Grams per sq. cm	Pounds per sq. foot	2.0481	.4883.
Horsepower (550 ft.-lb. per sec.)	Foot-lb. per minute	3.3×10^{4}	3.03×10^{-5}.
Horsepower (550 ft.-lb. per sec.)	Btu per minute	42.41	2.357×10^{-2}.
Horsepower (550 ft.-lb. per sec.)	Kg-calories per minute	10.69	9.355×10^{-2}.
Horsepower (Metric) (542.5 ft.-lb. per sec.).	Horsepower (550 ft.-lb. per sec.)	.9863	1.014.

To convert—	Into—	Multiply by—	Conversely multiply by—
Inches	Centimeters	2.54	.3937.
Inches	Mils	1,000	.001.
Joules	Foot-pounds	.7376	1.356.
Joules	Ergs	10^7	10^{-7}.
Kilogram-calories	Kilojoules	4.186	.2389.
Kilograms	Pounds (avoirdupois)	2.205	.4536.
Kg per sq. meter	Pounds per sq. foot	.2048	4.882.
Kilometers	Feet	3,281	3.048×10^{-4}.
Kilowatt-hours	Btu	3,413	2.93×10^{-4}.
Kilowatt-hours	Foot-pounds	2.655×10^6	3.766×10^{-7}.
Kilowatt-hours	Joules	3.6×10^6	2.778×10^{-7}.
Kilowatt-hours	Kilogram-calories	860	1.163×10^{-3}.
Kilowatt-hours	Kilogram-meters	3.671×10^5	2.724×10^{-6}.
Liters	Cubic meters	.001	1,000.
Liters	Cubic inches	61.02	1.639×10^{-2}.
Liters	Gallons (liq. US)	.2642	3.785.
Liters	Pints (liq. US)	2.113	.4732.
Meters	Yards	1.094	.9144.
Meters per min	Feet per min	3.281	.3048.
Meters per min	Kilometers per hr	.06	16.67.
Miles (nautical)	Kilometers	1.853	.5396.
Miles (statute)	Kilometers	1.609	.6214.
Miles per hr	Kilometers per min	2.682×10^{-2}	37.28.
Miles per hr	Feet per minute	88	1.136×10^{-2}.
Miles per hr	Kilometers per hr	1.609	.6214.
Poundals	Dynes	1.383×10^4	7.233×10^{-5}.
Poundals	Pounds (avoirdupois)	3.108×10^{-2}	32.17.
Sq inches	Circular mils	1.273×10^6	7.854×10^{-7}.
Sq inches	Sq centimeters	6.452	.155.
Sq feet	Sq meters	9.29×10^{-2}	10.76.
Sq miles	Sq yards	3.098×10^6	3.228×10^{-7}.
Sq miles	Sq kilometers	2.59	.3861.
Sq millimeters	Circular mils	1,973	5.067×10^{-4}.
Tons, short (avoir 2,000 lb.)	Tonnes (1,000 Kg.)	.9072	1.102.
Tons, long (avoir 2,240 lb.)	Tonnes (1,000 Kg.)	1.016	.9842.
Tons, long (avoir 2,240 lb.)	Tons, short (avoir 2,000 lb)	1.120	.8929.
Watts	Btu per min	5.689×10^{-2}	17.58.
Watts	Ergs per sec	10^7	10^{-7}.
Watts	Ft-lb per minute	44.26	2.26×10^{-2}.
Watts	Horsepower (550 ft-lb per sec.)	1.341×10^{-3}	745.7.
Watts	Horsepower (metric) (542.5 ft-lb per sec).	1.36×10^{-3}	735.5.
Watts	Kg-calories per min	1.433×10^{-2}	69.77.

4. Abbreviations

ac	alternating current
a-c	alternating-current
AWG	American Wire Gage
B & S	Brown and Sharpe (wire gage)
Btu	British thermal unit
cemf	counter electromotive force
cgs	centimeter-gram-second
cm	centimeter
dc	direct current
d-c	direct-current
DPDT	double-pole, double-throw
E	voltage
emf	electromotive force
ft.-lb	foot-pound
hp	horsepower
I	current
JAN	Joint Army-Navy
kv	kilovolt
kw	kilowatt
L	inductance
ma	milliampere
mv	millivolt
R	resistance
v	volt

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